Kalman Filtering for Positioning and Heading Control of Ships and Offshore Rigs

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ESTIMATING THE EFFECTS OF WAVES, WIND, AND CURRENT
Ships and offshore rigs perform missions that require tight motion control. During the past 30 years, there has been an increasing demand for higher accuracy and reliability of ship motion-control systems to such an extent that it is difficult to conceive of operations in which motion-control systems are not an essential part, or even an enabling factor. Modern marine vessels are equipped with sophisticated motion-control systems that have different objectives depending on the particular operations being performed. Some of these control objectives include position and heading regulation, path following, trajectory tracking, and wave-induced motion reduction.

The stochastic nature of environmentally induced forces has led to extensive use of the Kalman filter for estimating ship-motion-related quantities and for filtering disturbances, both of which are of paramount importance for implementing marine motion-control systems. In this article, we introduce the elements of a ship motion-control system, describe the models used for position-regulation and course-keeping control, and discuss how the Kalman filter is used for wave filtering, that is, removing the oscillatory wave-induced motion from velocity and positioning measurements.

SHIP MOTION CONTROL AND WAVE FILTERING

Ships and offshore rigs are exposed to wind, waves, and current forces. The requirements of the various operations performed by the vessel, together with the characteristics of the environmental forces, define the modes of operation and the objectives of the motion-control system. Figure 1 illustrates the components of a ship motion-control system [1], [2]. The guidance system generates a smooth and feasible desired reference trajectory described in terms of position, velocity, and acceleration. The trajectory is generated by algorithms that use the ship’s actual and desired position and a mathematical model of the ship together with information regarding missions, operator decisions, weather, and fleet operations. The control system processes motion-related signals to infer the state of the ship, to filter disturbances, and to generate an appropriate command for the actuators so as to reduce the difference between the actual and desired ship trajectories. The controller may have multiple modes of operation depending on the type of mission performed, for example, course keeping, positioning, and roll- and pitch-motion damping. For some ships and particular operations, the desired control action can be delivered in several ways due to overactuation, which provides increased reliability to actuator faults. Thus, multiple combinations of actuator demands can yield the same control action. In these cases, the control system must also solve a control allocation problem based on an optimization criteria [2]. The navigation system, which provides reliable measurements of position and heading, collects information from the various sensors, such as GPS, speed log, compass, gyros, radar, and accelerometers. The navigation system also performs signal quality checking and transforms the measurements to a common reference frame used by the control and guidance systems [1]–[3].

Environmental forces due to waves, wind, and current are considered disturbances to the motion-control system. These forces, which can be described in stochastic terms, are conceptually separated into wave- and low-frequency components.

Waves produce a pressure change on the hull surface, which in turn induces forces. These pressure-induced forces have an oscillatory component that depends linearly on the wave elevation. Hence, these forces have the same frequency as that of the waves and are therefore referred to as wave-frequency forces. Wave forces also have a component that depends nonlinearly on the wave elevation [4], [5]. Nonlinear wave forces are due to the quadratic dependence of the pressure on the fluid-particle velocity induced by the passing of the waves. If, for example, two sinusoidal incident waves have different frequencies, then their quadratic relationship gives pressure forces with frequencies at both the sum and difference of the incident wave frequencies as well as zero-frequency or mean forces. Hence, the nonlinear wave forces present both lower and higher frequencies than the wave frequencies. The mean wave forces cause the vessel to drift. The forces with a frequency content at the difference of the wave frequencies can have low-frequency content, which can lead to resonance in the horizontal motion of vessels with mooring lines or under positioning control [5]. The high-frequency, wave-pressure-induced forces, which can be described in stochastic terms, are conceptually separated into wave- and low-frequency components.

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FIGURE 1 The basic components of a modern ship motion-control system can be grouped into three subsystems. The navigation system processes information from the motion sensors to provide reliable signals related to the actual trajectory of the vessel. The guidance system provides desired feasible trajectories based on information related to the vessel state and mission in progress. The control system processes the information coming from the navigation and guidance system to produce forces that correct deviations of the vessel trajectory from the desired vessel trajectory. (Picture of courtesy of Austal Ships, Australia.)
The requirements of the various operations performed by the vessel define the modes of operation and the objectives of the motion-control system.

forces, which have frequency content at the sum of the wave frequencies, are normally too high to be considered in ship motion control, but these forces can contribute to structural vibration in the hull. For further details about wave loads and their effects on ship motion, see [4] and [5].

Like waves, wind and current induce forces due to pressure variation on the vessel structure. Wind forces have a mean component and a random fluctuating component due to gusts. In ship motion control, only the mean wind forces are compensated since the frequency of gusts is often outside the bandwidth of the vessel response [2]. Current-induced forces affect vessels requiring positioning control and vessels at mooring. These forces have low-frequency content. The variations of these forces can be induced by changes in the speed and direction of the current relative to the vessel.

Wind, current, and nonlinear wave forces that are of interest to ship motion control are referred to as low-frequency forces. Figure 2 shows a typical example of observed motion in marine surface vessels. The motion components derived from the inducing forces are thus referred to as the low-frequency motion and wave-frequency motion. Due to the characteristics of ship motion depicted in Figure 2, the objective of a ship motion-control system may be to control only the low-frequency motion, only the wave-frequency motion, or the total motion, that is, the sum of the low- and wave-frequency motions [1]–[3].

In low-to-medium sea states, the frequency of oscillations of the linear wave forces do not normally affect the operational performance of the vessel. Hence, controlling only low-frequency motion avoids correcting the motion for every single wave, which can result in unacceptable operational conditions for the propulsion system due to power consumption and potential wear of the actuators. Operations that require the control of only the low-frequency motion include dynamic positioning, heading autopilots, and thruster-assisted position mooring. Dynamic positioning refers to the use of the propulsion system to regulate the horizontal position and heading of the vessel. In thruster-assisted position mooring, the propulsion system is used to reduce the mean loading on the mooring lines. Additional operations that require the control of only the low-frequency motion include slow maneuvers that arise, for example, from following underwater remotely operated vehicles. Operations that require the control of only the wave-frequency motions include heave compensation for deploying loads at the sea floor [6] as well as ride control of passenger vessels, where reducing roll and pitch motion helps avoid motion sickness [3].

The control of only low-frequency motion is achieved by appropriate filtering of the wave-frequency components from the position and heading measurements and estimated velocities. This filtering is performed before the signals are passed on to the controller as indicated in Figure 1.

Early course-keeping autopilots used a proportional controller with a deadband nonlinearity. The deadband provided an effect similar to wave filtering since it delivered a null control action until the control signals were large enough to be outside the deadband. The amount of deadband in the autopilot could be changed, and this setting was called “weather” since the size of the deadband was selected by the operator based on weather conditions [7], [8]. Other systems used lowpass and notch filters, which introduced significant phase lag, and thus performance degradation when a high gain control is required. An alternative to traditional filtering consists of using a wave-induced motion model and an observer to separate the wave motion from the low-frequency motion. For this approach, which requires a spectral factorization model of the disturbances, the Kalman filter is the preferred observer [9]. The Kalman filter can use measurements from multiple sensors with different levels of accuracy to produce estimates of the ship velocities, which are not measured in most ship-positioning applications.

In the remainder of this article, we focus on two operations where only low-frequency motion control is required, dynamic positioning and course keeping, and we present

![FIGURE 2 Example of motion of a marine vessel. The total motion can be thought of as the superposition of a low-frequency (LF) and a zero-mean oscillatory wave-frequency (WF) component. This plot shows the total motion and low-frequency components of the heading angle of a vessel during a course change.](image-url)
the models commonly used to design wave filtering based on Kalman filters. For an account of the development of these control applications, see “Historical Aspects of Ship Motion-Control Systems Related to Autopilots and Dynamic Positioning.”

MODELS FOR DYNAMIC POSITIONING AND KALMAN FILTER DESIGNS

The advent of the global positioning system (GPS) in the 1980s opened the possibility for the design of accurate automatic systems for station-keeping (position regulation) and low-speed maneuvering of ships and offshore rigs. Vessels operating in this mode rely on feedback information from position measurements and heading. These measurements are used in a motion-control system that operates in the three planar degrees of freedom, surge (forward motion), sway (transverse motion), and yaw (rotation about the vertical axis, also called heading).

The position of the vessel is normally measured by a differential GPS, while the heading is measured by a gyrocompass. Additional types of sensors are usually available to ensure reliability of the positioning system, namely, inertial measurement units, hydro-acoustic position sensors, taut wires, and laser sensors. Examples of these sensors are illustrated in Figure 3.

Mathematical Modeling of Vessel Dynamics for Positioning Control Systems

To describe the motion of a ship, two reference frames are used, a local geographical Earth-fixed frame and a body-fixed frame, which is attached to the vessel. The components of the position-orientation vector $\eta = [n, e, \psi]^T$ are
the north-east positions \((u, v)\) of the vessel relative to the local geographical frame and the yaw or heading angle \(\psi\) relative to the north. The components of the velocity vector \(\mathbf{v} = [u, v, r]^T\) are the surge and sway velocities \((u, v)\) and the yaw rate \(r\). These variables are depicted in Figure 4. The body-fixed velocities \(u\) and \(v\) are the time derivatives of the position of the origin of the body-fixed frame relative to the origin of the local geographical frame expressed in the body-fixed frame. Similarly, the yaw rate \(r\) is a component of the angular velocity of the body-fixed frame with respect to the local geographical frame expressed in the body-fixed frame. For ship-positioning and heading control, the translational motion is assumed to be confined to the horizontal plane, and thus the angular velocity has only one component, namely, \(r\), which the rotation rate about the axis perpendicular to the horizontal plane.

A model for vessel dynamics can be expressed as

\[
\dot{\mathbf{y}} = R(\psi) \mathbf{v},
\]

\[
(M_{RB} + M_A) \mathbf{v} + C_{RB}(\mathbf{v}) \mathbf{v} + d(V_o, \gamma_o) = \tau_{\text{control}} + \tau_{\text{wind}} + \tau_{\text{waves}}.
\]

The kinematic transformation (1) relates the body-fixed velocities to the time derivative of the positions in the local geographical frame. This transformation is given by the rotation matrix

\[
R(\psi) = \begin{bmatrix}
\cos \psi & -\sin \psi & 0 \\
\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{bmatrix}, \quad R^{-1}(\psi) = R^T(\psi).
\]

For further details about kinematic models used in marine control systems and transformations, see [10].

The terms on the right-hand side of (2) represent the vectors of forces due to control, wind, and waves, respectively. These forces are directed along the body-fixed directions \(x_b\) and \(y_b\), and the moment is taken about the vertical axis \(z_b\), so that \(\tau = [X, Y, N]^T\), where \(X\) is the surge force, \(Y\) is the sway force, and \(N\) is the yaw moment. Table 1 summarizes this notation and indicates the reference frames in which the variables are expressed.

The positive-definite, rigid-body mass matrix \(M_{RB}\) and the skew-symmetric Coriolis-centripetal matrix \(C_{RB}(\mathbf{v})\) are given by
The objective of a ship motion-control system may be to control only the low-frequency motion, only the wave-frequency motion, or the total motion.

\[
M_{RB} = \begin{bmatrix}
m & 0 & 0 \\
0 & m & mx_g \\
0 & mx_g & I_z
\end{bmatrix},
\]

\[
C_{RB}(\nu) = \begin{bmatrix}
0 & 0 & -m(x_g r + v) \\
0 & 0 & mu \\
m(x_g r + v) & -mu & 0
\end{bmatrix}, \quad (4)
\]

where \(x_g\) denotes the longitudinal position of the center of gravity of the vessel relative to the body-fixed frame. The Coriolis-centripetal terms appear as a consequence of expressing the equations of motion in body-fixed coordinates. This formulation simplifies the model since, when expressed in the body-fixed frame, the moments of inertia become time invariant and the direction of the actuator forces are simpler to describe. When a vessel operates under positioning control, the velocities are small. Hence, the Coriolis-centripetal terms \(C_{RB}(\nu)\nu\) in (2) can be ignored for control design.

When a vessel moves in the water, the changes in pressure on the hull are proportional to the velocities and accelerations of the vessel relative to the fluid. The coefficients used to express the pressure-induced forces proportional to the accelerations are called added-mass coefficients. These forces reflect the change in momentum in the fluid due to the vessel accelerations. The positive-definite hydrodynamic added mass matrix \(M_A\) is represented by

\[
M_A = \begin{bmatrix}
-X_u & 0 & 0 \\
0 & -Y_p & -Y_f \\
0 & -Y_f & -N_r
\end{bmatrix}, \quad (5)
\]

where the added-mass coefficients \(X_u, Y_p, Y_f,\) and \(N_r\) depend on the hull shape. Despite the terminology, note that only \(X_u\) and \(Y_p\) have mass units.

The term \(d(V_{rc}, \gamma_{rc})\) on the left-hand side of (2) represents the current and damping forces. These nonconservative forces, which reflect the transfer of energy from the vessel to the fluid, depend on the speed and direction of the current relative to the vessel is given by

\[
V_{rc} = \sqrt{u_{rc}^2 + v_{rc}^2} = \sqrt{(u - u_c)^2 + (v - v_c)^2}, \quad (6)
\]

\[
\gamma_{rc} = -\tan2(v_{rc}, u_{rc}), \quad (7)
\]

where \(u_c\) and \(v_c\) are the components of the current velocity in the vessel body-fixed frame. Thus, the angle of the current \(\gamma_{rc}\) is defined relative to the bow of the vessel.

It is common practice to write the current forces in surge, sway, and yaw as functions of nondimensional current coefficients \(C_X(\gamma_{rc}), C_Y(\gamma_{rc}), C_N(\gamma_{rc})\), [1], [2], that is,

\[
d(V_{rc}, \gamma_{rc}) = \frac{1}{2}\rho V_{rc}^2 \begin{bmatrix}
A_{F_c} & C_{X_c}(\gamma_{rc}) \\
A_{L_c} & C_{Y_c}(\gamma_{rc}) \\
A_{L_c} & C_{N_c}(\gamma_{rc})
\end{bmatrix}, \quad (8)
\]

where \(\rho\) is the water density, \(A_{F_c}\) and \(A_{L_c}\) are frontal and lateral projected areas of the submerged part of the hull, and \(L_{\text{ref}}\) is the length of the ship. Typical current coefficients for a dynamically positioned vessel are shown in Figure 5. These coefficients are determined experimentally based on scale models or using computational fluid dynamic models [2].

Unless the vessel under consideration is the subject of an extensive hydrodynamic analysis and scale-model testing, the current coefficients \(C_X(\gamma_{rc}), C_Y(\gamma_{rc}), C_N(\gamma_{rc})\) in (8) are difficult to estimate with accuracy. In such cases, it is common practice to simplify the model (8) in terms of a linear damping term and a bias term [33] of the form

\[
d(V_{rc}, \gamma_{rc}) = D\nu - R^T(\psi)b, \quad (9)
\]

FIGURE 5 Current coefficients of an offshore supply vessel determined using computational fluid dynamics. The current coefficients are normalized nondimensional forces in surge \(C_X\), sway \(C_Y\), and yaw \(C_N\). The normalization is based on current speed, water density, and the projected area of the vessel below the water line. These coefficients can also be determined experimentally from scale model tests.
Most of the vessels operating in the offshore industry worldwide use Kalman filters for velocity estimation and wave filtering.

where

$$D = D^T = \begin{bmatrix} D_{11} & 0 & 0 \\ 0 & D_{22} & D_{23} \\ 0 & D_{32} & D_{33} \end{bmatrix}, \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}. \quad (10)$$

The linear damping term also models the transfer of energy from the vessel to the fluid due to the waves that are generated as a consequence of the vessel motion.

Using (9), the simplified vessel model (1)–(2) becomes

$$\dot{\eta} = R(\psi)v, \quad (11)$$

$$M\ddot{v} + D\dot{v} = R^T(\psi)b + \tau_{\text{control}} + \tau_{\text{wind}} + \tau_{\text{waves}}, \quad (12)$$

$$b = 0, \quad (13)$$

where $M = M_{Rh} + M_d$ in (12). The bias term is constant in Earth-fixed coordinates, under the assumption of constant or slowly varying currents. This assumption is appropriate considering the tide periods relative to the ship dynamic response characteristics. The bias term must therefore be rotated to be included into the equation of motion (12). This rotation captures the effect that the current forces change with the heading of the vessel.

The bias is estimated by the observer shown in Figure 1, which allows the resulting force term in (12) to be compensated by the motion controller. This approach thus achieves integral action for positioning control since the bias represents the low-frequency disturbances [2].

**Vessel Parallel Model Formulation**

The model (11)–(13) is nonlinear due to the kinematic transformations. This model can be linearized dynamically by introducing the vessel parallel coordinates [2], which are defined in a reference frame fixed to the vessel with axes parallel to the Earth-fixed reference frame. The vessel parallel coordinates $\eta_p$ are thus defined by using the transformation

$$\eta_p := R^T(\psi)\eta, \quad (14)$$

where $\eta_p$ is the position-attitude vector expressed in body coordinates. For dynamic positioning applications, rotation about the $z$-axis is often slow. Therefore, $\dot{r} = 0$ and $R(\psi) \approx 0$ are good approximations. Consequently, (11)–(13) can be written in terms of $\eta_p$, and the resulting model, which is referred to as the vessel parallel reference model, is given by

$$\dot{\eta}_p = v, \quad (15)$$

$$M\ddot{v} + D\dot{v} = b_p + \tau_{\text{control}} + \tau_{\text{wind}} + \tau_{\text{waves}}, \quad (16)$$

$$b_p = 0, \quad (17)$$

where $b_p := R^T(\psi)b$. Since (15)–(17) is linear, a linear observer can be used to estimate the velocity $v$ and bias $b_p$ using the force vectors $\tau_{\text{control}}$, $\tau_{\text{wind}}$, and $\tau_{\text{waves}}$ along with $\eta_p$, which is known from the measurement of $\eta$ through (14).

**Wind Force Models**

The wind forces and moments can be represented in a similar way to the current forces and moments by means of nondimensional force coefficients [2], namely,

$$\tau_{\text{wind}} = \frac{1}{2} \rho \overline{V}_w^2 \begin{bmatrix} A_{Lw}C_{Xw}(\gamma_{rw}) \\ A_{Lw}C_{Yw}(\gamma_{uw}) \\ A_{Lw}C_{Nw}(\gamma_{uw}) \end{bmatrix}, \quad (18)$$

where $\rho$ is the air density, $A_{Lw}$ and $A_{Lw}$ are frontal and lateral projected wind areas, and $l_{rw}$ is the vessel’s overall length. The wind speed $V_{rw}$ and direction $\gamma_{rw}$ relative to the vessel are given by

$$V_{rw} = \sqrt{u_{rw}^2 + v_{rw}^2}, \quad (19)$$

$$\gamma_{rw} = -\arctan2(v_{rw}, u_{rw}), \quad (20)$$

with

$$u_{rw} = u - V_w \cos \beta_w, \quad (21)$$

$$v_{rw} = v - V_w \sin \beta_w. \quad (22)$$

Here $V_w$ and $\beta_w$ are the wind speed and direction in Earth-fixed coordinates. Typical wind coefficients $C_{Xw}(\gamma_{rw})$, $C_{Yw}(\gamma_{uw})$, and $C_{Nw}(\gamma_{uw})$ for an offshore vessel are shown in Figure 6.

The wind coefficients can be obtained either from computational fluid dynamics, model tests, or by scaling coefficients of similar vessels. For control design purposes, however, wind speed and direction measurements are often used for approximate feedforward compensation, and the errors associated with this compensation are modeled as a bias in (13) and (17). That is, the bias accounts for the simplified current forces as well as the wind forces. If feedforward compensation is not used, then a sluggish
To describe the motion of a ship, two reference frames are used, a local geographical Earth-fixed frame and a body-fixed frame, which is attached to the vessel.

response to changes in the direction of the wind relative to the vessel may result [2].

Models for Wave Loads and Wave-Induced Motion
As discussed above, wave forces are usually modeled as the sum of a linear and a nonlinear component, namely,

$$\tau_{\text{waves}} = \tau_{\text{waves}}^{\text{lin}} + \tau_{\text{waves}}^{\text{nlin}}.$$  

(23)

The linear oscillatory component has the same frequency as the wave elevation. The nonlinear component has both lower and higher frequency than the wave elevation. The linear and low-frequency nonlinear wave forces are relevant to ship motion control.

The low-frequency nonlinear wave forces are treated as an input disturbance and modeled by a bias term (17). That is, the bias term represents a combination of nonlinear wave and current. The linear wave forces, however, are usually transformed into an equivalent output disturbance. With this point of view, the measured position vector can be represented as

$$\eta_{\text{tot}} = \eta + \eta_{w},$$  

(24)

where $\eta_{\text{tot}}$ is the total position, $\eta_{w}$ is wave-induced position due to linear wave forces, and $\eta$ represents the low-frequency position due to control, wind, current, and nonlinear wave forces.

The sea surface elevation is typically modeled as the realization of a stationary Gaussian random process [34]. Stationarity can be considered for short periods between 20 min and 3 h depending on weather conditions, while Gaussianity depends on the wave height and depth [35]. The deeper the ocean at the location considered, the higher the waves for which Gaussianity can be assumed [34], [35]. Under these conditions, the wave elevation is completely characterized by the wave spectrum $S_{\zeta\zeta}(\omega)$, which changes with the sea state, namely, the dominant amplitudes and wave periods.

Under a linear response assumption, the wave-induced motion spectrum can be represented as

$$S_{\eta_{w}\eta}(\omega) = |H(j\omega)F(j\omega)|^2 S_{\zeta\zeta}(\omega).$$  

(25)

where $F(j\omega)$ represents the frequency response function from the wave elevation to the pressure-induced forces on the hull and $H(j\omega)$ represents the frequency response from force to motion [4], [5]. The frequency response function $F(j\omega)$, which is known as the force response amplitude operator (RAO) in naval architecture, depends on the shape of the hull and the angle from which the waves approach the vessel. The frequency response $H(j\omega)$, known as the force-to-motion RAO, depends on the shape of the hull and the mass distribution. These frequency response functions, which can be computed using hydrodynamic computer codes, are the basis of ship motion evaluation [3].

The spectrum of the wave-induced motion can be approximated using spectral factorization. That is, (25) can be approximated by the spectrum of the signal obtained using a linear filter driven by Gaussian noise $w(t)$ with a flat spectrum $S_{ww}$ over the frequencies of interest. Then,

$$S_{\eta_{w}\eta}(\omega) = |G(j\omega)|^2 S_{ww}.$$  

(26)

A second-order, noise-driven filter is usually appropriate for modeling wave-induced motion [1]–[3], namely,

![FIGURE 6 Wind coefficients of an offshore supply vessel determined using computational fluid dynamics. The wind coefficients are normalized forces in surge $C_X$, sway $C_Y$, and yaw $C_N$. The normalization is done in terms of wind speed, air density, and the projected area of the vessel above the waterline. These coefficients are determined experimentally from scale model tests for multiple wind directions measured by the wind angle $\gamma_{uw}$ relative to the bow of the ship.](image-url)
\[
G(s) = \frac{\omega_0^2 s}{s^2 + 2\xi \omega_0 s + \omega_0^2}.
\]

The parameters of the shaping filter (27) as well as the intensity \( S_{\text{sw}} \) of the noise depend on the sea state, the vessel shape, and the angle from which the waves approach the vessel.

A filter transfer function of the form (27) is considered for each degree of freedom. The combination of these transfer functions can be transformed into a single state-space model

\[
\begin{align*}
\dot{\xi} &= A_{\xi} \xi + E_{w} w, \\
\eta_{w} &= C_{w} \xi,
\end{align*}
\]

where the matrices \( A_{\xi}, B_{w}, \) and \( C_{w} \) depend on the chosen state-space realization adopted. Since a model for dynamic positioning has three degrees of freedom (surge, sway, and yaw), a second-order noise filter approximation yields a state vector \( \xi \) with six components. The state vector \( \xi \) may or may not have a physical interpretation depending on the particular state-space realization used.

**Linear Model for Dynamic Positioning Observer Design**

By combining the models of the vessel response together with the disturbance models discussed above, we obtain a model structure of the form

\[
\begin{align*}
\dot{\xi} &= A_{\xi} \xi + E_{w} w_{1}, \\
\eta_{p} &= \nu, \\
M \dot{\nu} + D \nu &= b_{p} + \tau_{\text{control}} + \tau_{\text{wind}} + w_{2}, \\
\nu &= \eta_{p} + C_{\xi} \xi + \nu.
\end{align*}
\]

The variables \( w_{j}, j = 1, 2, 3, \) are Gaussian noise vectors, which represent model uncertainty. Note that no uncertainty is associated with the kinematic equation (31) since this relation describes known geometrical aspects of motion. The measurement positions (34) are the sum of the wave motion \( C_{w} \xi \) and the motion \( \eta_{p} \) due to wind forces, current, and control. The measurement vector also contain some noise \( \nu \). The control forces usually have two components

\[
\tau = -\hat{\tau}_{\text{wind}} + B_{u} u,
\]

where \( \hat{\tau}_{\text{wind}} \) is an estimate of the wind forces implemented by using feedforward compensation and \( B_{u} u \) represents actuator forces. The wind feedforward term, which is proportional to the square of the measured wind velocity, depends on the vessel's projected area in the direction of the wind [2]. The vector \( u \) is the command to the actuators, which are assumed to have much faster dynamic response than the vessel; thus the coefficient \( B_{u} \) represents the mapping from the actuator command to the force generated by the actuator. For example, if the command to a propeller is the rotation speed, then the corresponding coefficient in \( B_{u} \) maps the speed to the generated thrust.

The resulting model for a dynamic positioning observer design is the 15th-order state-model

\[
\begin{align*}
\dot{x} &= Ax + Bu + Ew, \\
y &= Hx + \nu,
\end{align*}
\]

where \( x = [\xi^\top, \eta_{p}^\top, b_{p}^\top, \nu^\top]^\top \in \mathbb{R}^{15} \) is the state vector, \( u \in \mathbb{R}^{p} \) \((p \geq 3)\) is the control vector, \( w = [w_{1}, w_{2}, w_{3}]^\top \in \mathbb{R}^{3} \) represents the process noise vector, and

\[
\begin{align*}
A &= \begin{bmatrix} A_{w} & 0_{6 \times 3} & 0_{6 \times 3} & 0_{6 \times 3} \\
0_{3 \times 6} & I_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\
0_{3 \times 6} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\
0_{3 \times 6} & -M^{-1}D & M^{-1} & 0_{3 \times 3} \end{bmatrix}, \\
B &= \begin{bmatrix} 0_{6 \times 3} \\
0_{3 \times 3} \\
0_{3 \times 3} \\
M^{-1}B_{u} \end{bmatrix}, \\
H &= \begin{bmatrix} C_{w} & I_{3 \times 3} & 0_{3 \times 3} \end{bmatrix}, \\
E &= \begin{bmatrix} E_{w} \\
0_{3 \times 3} \\
I_{3 \times 3} \\
M^{-1}\end{bmatrix}.
\end{align*}
\]

**KALMAN FILTER DESIGN FOR SHIP DYNAMIC POSITIONING**

The model given in (36) forms the basis of a Kalman filter design. To implement the observer on a computer, the model needs to be discretized. The discretization has the form

\[
\begin{align*}
x(k + 1) &= \Phi x(k) + \Delta u(k) + \Gamma w(k), \\
y(k) &= Hx(k) + \nu(k),
\end{align*}
\]

where

\[
\begin{align*}
\Phi &= \exp(Ah), \\
\Delta &= A^{-1}(\Phi - I)B, \\
\Gamma &= A^{-1}(\Phi - I)E,
\end{align*}
\]

\( h \) is the sampling time, and the equivalent discrete-time noises \( w(k) \) and \( \nu(k) \) are Gaussian and white with zero mean. Appropriate sampling times can be determined from step responses in the various degrees of freedom of the vessel. As a rule of thumb, the sampling period is chosen to be in the range of 20–40 samples within the rise time of the force to velocity response of the fastest degree of freedom. For large offshore vessels and rigs, the sampling time is normally in the range of 100–500 ms. Small vessels with faster dynamics may require sampling times as low as 50 ms.

To implement a Kalman filter, the parameters of the model (40), (41) as well as the covariance of the state measurement noises in the model are necessary. The mass and damping parameters of the model can be initially estimated from hydrodynamic computations. Then, an update of the parameter estimates can be based on data obtained from tests performed in calm water [27]. The structure of the
model matrices (38), (39) determines the structure of the matrices of the discrete-time model (40), (41). Then, for example, standard grey-box, state-space prediction-error methods can be used to estimate the parameters [12].

The measurement noise is usually associated with sensors. After correcting the sensor bias, the covariance $\sigma^2_{w_i}$ of the measurement noise of the sensor $i$ can be estimated by the sample covariance from a data record taken while the vessel is at port with no motion. Since the noise of different sensors is often uncorrelated, the covariance matrix of the vector of measurement noise is chosen to be diagonal, that is,

$$R = \text{diag}(\sigma^2_{w_1}, \sigma^2_{w_2}, \ldots, \sigma^2_{w_p}).$$

The estimation of the covariance $Q$ of the state noise $w$ in (40) is more complex since it depends on the sea state, the heading of the vessel relative to the environmental disturbances, and how uncertain the model (40), (41) is [3]. This covariance matrix is chosen to be block diagonal, that is,

$$Q = \text{diag}(Q_1, Q_2, Q_3).$$

The matrix $Q_1 \in \mathbb{R}^{3 \times 3}$ is the covariance of the noise $w_{\text{hp}}$, which drives the noise filter representing linear wave-induced motion, $Q_2 \in \mathbb{R}^{3 \times 3}$ is the covariance of the noise $w_\text{n}$, which represents the uncertainty in the equation of motion, and $Q_3 \in \mathbb{R}^{3 \times 3}$ is the covariance of the noise $w_\text{b}$, which represents the uncertainty in the bias term that models the rest of the environmental forces, as indicated in (30)–(33).

The matrices $Q_2$ and $Q_3$ are chosen to be diagonal. The entries of the matrix $Q_2$ are taken as a fraction of the variance of the position measurement noises. The entries of $Q_3$ are high values. These choices provide a filter with an appropriate balance of the uncertainty in various parts of the model. The covariance $Q_1$ is estimated together with the parameters of the wave-frequency model (27) from data measured before and during the operation of the vessel. The parameters are re-estimated after significant changes in heading or at regular time intervals of 20 min, which is the time period for which the sea state can be considered to be stationary [3], [34], [35]. Since the vessel is in a positioning control mode, the total motion measured can be recorded and detrended to obtain an estimate of the wave-induced motion vector $\hat{y}_{\text{hp}}(k)$, or, equivalently, a first-order highpass filter can be used [14]. These data can then be used to estimate the parameters of the wave-induced motion model, for which it is convenient to consider the directly parameterized innovations form [12]

$$\dot{x}(k+1) = A_x(\theta) \dot{x}(k) + L_x(\theta) \epsilon(k),$$

$$\eta_{\text{hp}}(k) = C_x(\theta) \dot{x}(k) + \epsilon(k),$$

where $\theta$ is the vector of parameters to be estimated, and $\epsilon(k)$ is the vector of innovations. The parameter estimation can then be formulated as

$$\hat{\theta} = \arg \min_{\theta} \det \sum_{k=1}^{N} \epsilon(k, \theta) \epsilon(k, \theta)^T$$

with

$$\epsilon(k, \theta) = \hat{\eta}_{\text{hp}}(k) - C_x(\theta) \dot{x}(k),$$

$$\dot{x}(k+1) = A_x(\theta) \dot{x}(k) + L_x(\theta) \epsilon(k, \theta),$$

where $\eta_{\text{hp}}(k)$ is replaced by the estimate $\hat{\eta}_{\text{hp}}(k)$ obtained from detrending the measured data. Equations (48)–(50) comprise a standard prediction error estimation problem whose solution is related to the maximum likelihood estimate of the parameter vector $\theta$ [11], [12].

Once the parameters of the mode are estimated, the covariance $Q$, of the innovations can also be estimated from the sample covariance of the predictions errors. Then, the Kalman filter can be implemented with the innovation wave-frequency model, and thus we can chose $Q_1 = Q$. This choice entails no loss of information.

An alternative to the procedure described above consists of fixing the damping of the filters (27) to a value in the range 0.01 to 0.1 as suggested in [13] and estimate only the natural frequency and noise covariance [13], [14]. This estimation approach is summarized in [1], where recursive least squares is used for parameter estimation. A related approach, also based on recursive least squares, is given in [3].

From the procedures discussed above, all the parameters necessary to implement the Kalman filter for a dynamically positioned vessel are obtained. The Kalman filter can be formulated as follows [16]. The state estimate or correction is given by

$$\ddot{x}(k) = \ddot{x}(k) + K(k) [y(k) - H \ddot{x}(k)],$$

while the state prediction is given by

$$\ddot{x}(k+1) = \Phi \ddot{x}(k) + \Delta u(k).$$

The Kalman gain $K(k)$ is computed as

$$K(k) = \overline{P}(k) H^T (H \overline{P}(k) H^T + R)^{-1},$$

where $\overline{P}(k)$ satisfies the recursion

$$P(k) = (I - K(k) H)^{-1} \overline{P}(k),$$

$$\overline{P}(k+1) = \Phi P(k) \Phi^T + Q.$$  

The matrices $P(k)$ and $\overline{P}(k)$ are the conditional covariances of the estimation errors $x(k) - \ddot{x}(k)$ and $x(k) - \ddot{x}(k)$, respectively.

The algorithm (51)–(54) is a standard Kalman filter. The recursive form (54) may not be the best choice for real-time implementation since numerical issues may be associated with matrix inversion and covariance matrix propagation. These errors could result in covariance matrices that are not positive semidefinite. To address
these issues, various implementations of the equations are used to propagate and update the estimates and the covariance. These implementations, known as square root filtering, are algebraically equivalent to (54), but exhibit improved numerical precision and stability. For details see [16] and [17].

The filter (51)–(54) gives the state estimate

$$\dot{x}(k) = [\dot{\xi}(k)^T, \dot{\eta}_r(k)^T, \dot{\eta}_b(k)^T, \dot{\nu}(k)^T]^T.$$  \hspace{1cm} (55)

The components $\dot{\eta}_r(k)$ and $\dot{\nu}(k)$, which are the low-frequency motion components, are used by the controller, and in this way, the wave filtering is achieved. That is, the Kalman filter uses a model of the wave-frequency motion that facilitates estimation of the low- and wave-frequency components of the position and heading measurements. The bias estimate provided by the Kalman filter can be used in the controller to implement integral action unless the controller has an integrator; see [2] for further details on dynamic positioning control design.

Example of Kalman Filter Performance for a Vessel Under Positioning Control

Figure 7 shows simulation data of a Kalman wave filter on a 15-m fishing vessel under positioning control. In this simulation, the wave filter is switched on 500 s after the vessel is under positioning control. Then, at 600 s the vessel position is changed 10 m forward. Figure 7(a) shows the measured and wave-filtered surge position. Figure 7(b) shows the measured and wave-filtered surge velocity. Figure 7(c) shows the force generated by the controller.

During the first 600 s, while the wave filter is switched off, the wave-induced motion produces significant control action. Once the wave filter is switched on, the control action at wave-frequencies is reduced.

Observer Based on Nonlinear Models

A more advanced implementation of the observer results by using the nonlinear vessel model (1), (2), which includes the quadratic damping forces and current loads. This model can give better accuracy, but the resulting observer is computationally more intensive, and the parameters of the nonlinear model are more difficult to estimate. The filtering problem for this case can be solved using the extended Kalman filter. An alternative to the extended Kalman filter is to use a passivation design and Lyapunov theory. This approach is demonstrated in [33].

KALMAN FILTER DESIGN FOR SHIP COURSE-KEEPING AUTOPILOTS

Wave filtering for positioning systems must be implemented in three degrees of freedom, surge, sway, and yaw. When designing ship autopilots for automatic heading control, it is only necessary to consider a model and perform wave filtering for the yaw degree of freedom [1], [2], [8], [15]. In this section, we discuss the type of models that are normally used to implement observers for autopilot wave filtering and how the Kalman filter is used. For a discussion on the development of ship autopilots, see “Historical Aspects of Ship Motion-Control Systems Related to Autopilots and Dynamic Positioning.”

The Nomoto Model for Ship Heading Response

Autopilots are used to correct deviations from a desired equilibrium heading, and thus a linear model is sufficient for control design [15]. The response in yaw rate due to a small deviation in the angle of a control actuator, such as a rudder or the steering nozzle of a water jet, can be derived from (2) by isolating the yaw motion, which is given by

$$I_c \dot{\theta} - N_r r - N_d \delta = N_b \delta,$$  \hspace{1cm} (56)

where $I_c$ is the moment of inertia in yaw, $N_r$, $N_d$, and $N_b$ are hydrodynamic coefficients, $r$ is the yaw rate, and $\delta$ is the actuator angle. This model, which is known as the first-order Nomoto model [38], can be written as the transfer function

$$\frac{r(s)}{\delta(s)} = \frac{K}{1 + Ts}.$$  \hspace{1cm} (57)
When a vessel moves in the water, the changes in pressure on the hull are proportional to the velocities and accelerations of the vessel relative to the fluid.

The time constant and low-frequency gain are given by

\[ T = \frac{I_z - N_I}{-N_r}, \quad K = \frac{N_b}{N_r} \]

which can be estimated from trials in calm water.

Using the motion superposition assumption, as in the case of positioning control design, we model low-frequency environmental disturbances with a bias moment term in the equation of motion. Then, the state-space model can be written as

\[ \dot{y} = r, \]

\[ \dot{r} = -\frac{1}{T}r + \frac{1}{m} \tau_N + b, \]

\[ \dot{b} = 0, \]

where \( m = I_z - N_I \) and

\[ \tau_N = m \frac{K}{T} \delta = N_b \delta \]

denotes the control yaw moment.

**Autopilot Design**

For autopilot control design, it is common to design a proportional-integral-derivative controller with feedforward from wind and a smooth time-varying reference signal \( \psi_d(t) \) according to

\[ \tau_N(s) = -\frac{1}{T} \tau_{wind} + \frac{m \left( I_d - \frac{1}{T^2} \right)}{T} \tau_{FF} \]

\[ -m \left( K_p \dot{\psi} + K_d \dot{r} + K_i \int_0^t \dot{\psi}(\tau) d\tau \right), \]

where \( \tau_N \) is the controller yaw moment and \( \tau_{FF} \) is a feedforward term using the reference signal \( r_d = \psi_d \). The heading and yaw rate errors are denoted by \( \dot{\psi} = \psi - \psi_d \) and \( \dot{r} = r - r_d \), respectively. The control gains \( K_p, K_d, \) and \( K_i \) must be chosen such that the third-order linear error dynamics

\[ \dot{r} + \left( \frac{1}{T^2} + K_d \right) \dot{r} + K_i \dot{r} + K_p \int_0^t \dot{\psi}(\tau) d\tau = 0 \]

is asymptotically stable. Notice that the control law (64) depends on wind feedforward, where

\[ \dot{\tau}_{wind} = \frac{1}{2} \rho A L_{1w} C_N(\tau_{wind}). \]

The uncertainty in (66) has a low-frequency content, which is compensated by the integral action of the controller [2]. The rudder command is computed from the control input \( \tau_N \) as

\[ \delta = \frac{1}{N_b} \tau_N. \]

To implement the control law, both \( \psi \) and \( r \) are needed. Most ships have only compass measurements \( \psi \), and thus the turning rate \( r \) must be estimated. In addition, it is necessary to perform wave filtering such that the oscillatory wave-induced motions are avoided in the feedback loop.

**Models for Course Autopilots Observer Design**

As in the case of positioning, we consider the first-order, wave-induced motion as an output disturbance. Hence the measured yaw angle can be decomposed into

\[ \psi_{tot} = \psi + \psi_w, \]

where \( \psi \) is the response due to the control action and low-frequency disturbance and where \( \psi_w \) represents the first-order wave-induced motion.

To estimate \( \psi \) and \( r \), and thus wave filtering, the model used for a Kalman filter design is given by

\[ \dot{\xi}_w = \psi_w, \]

\[ \dot{\psi}_w = -\omega_0^2 \xi_w - 2 \zeta \omega_0 \psi_w + \omega_1, \]

\[ \dot{\psi} = r, \]

\[ \dot{r} = -\frac{1}{T} r + \frac{1}{m} (\tau_{wind} + \tau_N) + b + w_2, \]

\[ \dot{b} = w_3, \]

where \( \omega_0 \) and \( \zeta \) are the parameters of the filter (27) used to represent the wave-induced yaw motion, \( b \) is the bias term, and \( w_1, w_2, \) and \( w_3 \) are Gaussian white noises. The measurement equation is

\[ y = \psi + \psi_w + v, \]

where \( v \) represents zero-mean Gaussian white measurement noise introduced by the sensor. Notice that neither the yaw rate \( r \) nor the wave states \( \xi_w \) and \( \psi_w \) are measured. The resulting state-space model is
To implement a Kalman filter for a course autopilot, the model (75) can be discretized as in the case of dynamic positioning, and the parameters $T$ and $m$ of the response from the control forces can be estimated from tests performed in calm water. Then, the parameters $\omega_0$ and $\zeta$ of the first-order, wave-induced model and the covariance of the driving noise $w$ can be estimated from data collected during the operation of the vessel [3]. The procedure for the parameter estimation is similar to that discussed for dynamic positioning. The only significant difference is that the parameters of the first-order, wave-induced model must be re-estimated not only if significant heading changes occur but also if significant speed changes occur. Changes in the speed of the vessel produce a Doppler effect, and the frequencies of the waves seen from the vessel change with both the speed and direction of the waves relative to the vessel [3].

**Example Kalman Filter Design for Course-Keeping Autopilot**

To illustrate the performance of Kalman-filter-based wave filtering, we consider the case of an autopilot application taken from the Marine Systems Simulator (MSS) [40]. This simulation package implemented in Matlab-Simulink provides models of vessels and a library of Simulink blocks for heading autopilot control system design and blocks for a Kalman-filter-based wave filter from heading-only measurements.

The vessel considered is a 160-m mariner class vessel with a nominal service speed of 15 kn, or 7.7 m/s. The parameters of a complete and validated nonlinear model of the form (1)–(2) are given in [1]. From the step tests performed on the nonlinear model, a first-order Nomoto model

\[
\dot{x} = Ax + Bu + Ew, \quad y = Hx + v,
\]

where

\[
x = [\xi, \psi, r, b]^T,
\]

\[
u = \tau_{wind} + \tau_{N},
\]

\[
w = [w_1, w_2, w_3]^T,
\]

and

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 \\
-\omega_0^2 & -2\zeta\omega_0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}, \quad b = \begin{bmatrix}
0 \\
0 \\
m^{-1} \\
0 \\
\end{bmatrix}, \quad (80)
\]

\[
E = \begin{bmatrix}
0 & 0 & 0 \\
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}, \quad h^T = [0, 1, 1, 0, 0]. \quad (81)
\]

To implement a Kalman filter for a course autopilot, the model (75) can be discretized as in the case of dynamic positioning, and the parameters $T$ and $m$ of the response from the control forces can be estimated from tests performed in calm water. Then, the parameters $\omega_0$ and $\zeta$ of the first-order, wave-induced model and the covariance of the driving noise $w$ can be estimated from data collected during the operation of the vessel [3]. The procedure for the parameter estimation is similar to that discussed for dynamic positioning. The only significant difference is that the parameters of the first-order, wave-induced model must be re-estimated not only if significant heading changes occur but also if significant speed changes occur. Changes in the speed of the vessel produce a Doppler effect, and the frequencies of the waves seen from the vessel change with both the speed and direction of the waves relative to the vessel [3].
The objective of a positioning and heading control system is regulation while compensating only for the low-frequency environmental forces.

(57) is identified with the parameters \( K = 0.185 \) s\(^{-1}\) and \( T = 107.3 \) s. Based on the time constant, a sampling period of 0.5 s is chosen for the implementation of the Kalman filter. The standard deviation of the noise of the compass sensor is 0.5\(^\circ\). From a record of heading motion while the rudder is kept constant, the parameters of the first-order, wave-induced motion model are estimated, namely, \( \zeta = 0.1 \), \( \omega_0 = 1.2 \) rad/s, and the standard deviation of the noise driving the filter is \( \sigma_{v_1} = \sqrt{300} \) rad/s.

Using the above data, a Kalman filter is designed. Figures 8 and 9 show the performance of the Kalman filter. Figure 8(a) and (b) shows the true low-frequency heading angle and rate together with the Kalman filter estimates. Figure 8(c) shows the first-order, wave-induced heading angle component and its estimate. Figure 9 shows the performance of the control loop. Figure 9(a) shows the desired and the actual heading angle of the controlled vessel. Figure 9(b) depicts the rudder angle. In this figure, we can appreciate the effect of the wave filtering since the rudder angle has no motion at the wave frequency.

CONCLUSIONS

Ships and oil rigs perform operations that require accurate positioning and heading control. Vessel motion is affected by environmental forces, which can be separated into low- and wave-frequency forces. The low-frequency forces are caused by wind, current, and wave forces that depend nonlinearly on the wave elevation. The wave-frequency forces are caused by oscillatory pressure changes that depend linearly on the wave elevation.

The objective of a positioning and heading control system is regulation while compensating only for the low-frequency environmental forces. This mode of operation results in energy savings and prevents actuator wear. For example, in a 24-h positioning offshore operation, the propulsion system must not compensate for every single wave but only for the low-frequency disturbances. The implementation of such control systems often requires estimates of velocities from measurements of position and heading, while also filtering the oscillatory components of motion due to the waves. This type of filtering, known as wave filtering, is a key aspect in the motion control of marine surface vessels.

An effective way to address both velocity estimation and wave filtering is by augmenting the vessel model with a model of the oscillatory wave-induced motion and using a Kalman filter to estimate the states of the various parts of the model. That is, the model has state variables associated with the low-frequency motion and other state variables associated with wave-frequency motion. Once these states are estimated by the Kalman filter, only the low-frequency states are used as feedback signals in the controller.

In this article, we have described the main components of a ship motion-control system and two particular motion-control problems that require wave filtering, namely, dynamic positioning and heading autopilot. Then, we discussed the models commonly used for vessel response and showed how these models are used for Kalman filter design. We also briefly discussed parameter and noise covariance estimation, which are used for filter tuning. To illustrate the performance, a case study based on numerical simulations for a ship autopilot was considered.

The material discussed in this article conforms to modern commercially available ship motion-control systems. Most of the vessels operating in the offshore industry worldwide use Kalman filters for velocity estimation and wave filtering. Thus, the article provides an up-to-date tutorial and overview of Kalman-filter-based wave filtering.

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