A Quaternion-Based LOS Guidance Scheme for Path Following of AUVs

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Abstract: This paper presents a quaternion version of the well-known Line-of-Sight (LOS) guidance algorithm for marine applications. The transformation from Euler angles is achieved by exploiting the nature of the quaternion structure and using fundamental half-angle formulae from trigonometry. First, the Euler angles version of the LOS guidance algorithm is briefly presented for two uncoupled cases: a) the horizontal $xy$-plane, and b) the vertical $zx$-plane. Then, a coupled case is also considered and the transformation procedure is presented for all three cases. The vehicle considered pertains to a 5-DOF kinematics model where the roll angle is neglected, typical of torpedo-shaped Autonomous Underwater Vehicles (AUVs). Naturally, the Euler angles representation of the system involves singularities which, in the general 3-D space navigation case, should be avoided. The presented method aims at providing a singularity-free and computationally-efficient version of the conventional LOS algorithm.

Keywords: Autonomous underwater vehicles, path following, LOS guidance, quaternions

1. INTRODUCTION

The Line-of-Sight (LOS) algorithm is a well-documented guidance method in both marine and aerial vehicles applications, see for instance Yanushkevskiy (2011), Breivik (2010), Breivik and Fossen (2009). Its main advantages are its simplicity and efficiency in generating appropriate heading reference trajectories such that, when tracked by the heading autopilot, they guarantee the vehicle’s smooth convergence to the desired path. The method is based on a simple geometry which, when it comes to marine applications, refers to the LOS vector starting at the vehicle’s position and passing through a point $p(x_{los}, y_{los})$, which is located on the path-tangential line at a lookahead distance $\Delta h > 0$ ahead of the direct projection of the vehicle’s position $p(x, y)$ on to the path, this can be seen in Fig. 1. The vehicle is then assigned to reach the constantly moving point $p(x_{los}, y_{los})$ and this induces the desired steering behavior.

The performance of the algorithm is affected by factors such as the value of the lookahead distance, as well as the convergence speed of the heading autopilot. Regarding the lookahead distance, large values will result in a smooth convergence, without oscillations around the desired path but also more time-consuming. On the other hand, small values will drive the vehicle faster on the desired path, but this will usually lead to an oscillatory behavior. For this reason, several variable lookahead distance methods have been proposed in the past (Pavlov et al., 2009; Oh and Sun, 2010; Lekkas and Fossen, 2012). The issue of the heading autopilot convergence is usually studied by using cascaded systems theory. In that context, the subsystem consisting of the heading autopilot and the vehicle is considered to be a perturbation to the subsystem consisting of the LOS guidance and the vehicle, for more information the interested reader is referred to Børhaug and Pettersen (2005), Lekkas and Fossen (2012).

It is a well known fact that, in the general case, the Euler angles representation of the 6-DOF kinematics involves singularities for the pitch angles $\theta = \pm 90^\circ$ (Fossen, 2011). Consequently, it is useful to derive a quaternion version of the conventional LOS guidance for AUVs. Moreover, the quaternion representation is more computationally efficient compared to Euler angles since it does not include trigonometric functions. This makes it even more suitable for applications involving unmanned vehicles where the on-board computational power might be more limited. This paper serves as the first step toward this direction. The quaternion representation of the LOS algorithm is derived for two uncoupled 3-DOF cases: a) the horizontal $xy$-plane, and b) the vertical $zx$-plane, and for a coupled case where the sideslip angle is also a function of the vertical motion. For each case, the terms of the guidance law are transformed from Euler angles to quaternion by taking into account the nature of quaternions that correspond to rotations and using simple trigonometric identities.

The rest of this paper is organized as follows: Section 2 presents the vehicle model considered. In Section 3, the Euler representation of the LOS guidance algorithm is presented for the uncoupled horizontal and vertical planes
as well as the coupled case. The quaternion transformation for the uncoupled horizontal plane is given in Section 4, for the uncoupled vertical plane in Section 5 and for the coupled case in Section 6. Some simulation results can be found in Section 7, and Section 8 concludes the paper.

2. VEHICLE MODEL

2.1 Vehicle Dynamics

Following the methodology of Berhaga and Pettersen (2005), for the path-following task we can neglect the roll angle, hence for an underactuated autonomous vehicle the following 5-DOF dynamic model can be used:

\[ \dot{\eta} = J(\eta) \nu, \]

where \( \eta = (x, y, z, \theta, \psi)^T \) and \( \nu = (u, v, w, q, r)^T \) are the vehicle’s inertial position and velocity in Cartesian coordinates, \( \theta \) is the pitch angle and \( \psi \) is the yaw angle. In addition, \( u \) is the surge velocity, \( v \) is the sway velocity, \( w \) is the heave velocity, \( q \) is the pitch rate and \( r \) is the yaw rate.

2.2 Vehicle Kinematics

The model considers only absolute velocities and is the following, see Berhaga and Pettersen (2005):

\[ \begin{align*}
\dot{x} &= u \cos(\psi) \cos(\theta) - v \sin(\psi) + w \cos(\psi) \sin(\theta), \\
\dot{y} &= u \sin(\psi) \cos(\theta) + v \cos(\psi) + w \sin(\psi) \sin(\theta), \\
\dot{z} &= -u \sin(\theta) + w \cos(\theta), \\
\dot{\theta} &= q, \\
\dot{\psi} &= \frac{1}{\cos(\theta)} r, \quad \cos(\theta) \neq 0.
\end{align*} \]

3. EULER REPRESENTATION OF THE LOS GUIDANCE LAW

3.1 Horizontal Plane LOS Guidance

In the case of decoupled horizontal plane path-following we assume that \( \theta = 0^\circ \), consequently the kinematics equation to be considered is:

\[ \begin{align*}
\dot{x} &= u \cos(\psi) - v \sin(\psi), \\
\dot{y} &= u \sin(\psi) + v \cos(\psi), \\
\dot{\psi} &= r.
\end{align*} \]

The horizontal-plane speed \( U_h \) is given by:

\[ U_h := \sqrt{u^2 + v^2}, \]

and is assumed to be positive and bounded:

\[ U_{h,\min} \leq U_h \leq U_{h,\max}, \quad 0 < U_{h,\min}. \]

From (12)–(13) it is implied that the vessel always has at least a nonzero surge speed. The reason for setting a minimum positive speed \( U_{h,\min} \) is related to the stability proof of the LOS algorithm and a more rigorous approach is outside the scope of this paper and can be found in Lekkas and Fossen (2013). The model (9)–(11) includes only absolute velocities and describes the motion of an underactuated vehicle since only two out of three DOF’s can be controlled independently, namely the yaw angle and the surge velocity.

Path Following Objective: Assuming that the vehicle is assigned to converge to the line connecting the waypoints \( WP_k \rightarrow WP_{k+1} \), the along-track and the cross-track error for a given vehicle position \((x, y)\) are given by:

\[ \begin{bmatrix}
x_e \\
y_e
\end{bmatrix} = \begin{bmatrix}
x - x_k \\
y - y_k
\end{bmatrix}, \]

where \((x_k, y_k)\) is the position of the \( k \)-th waypoint expressed in the NED frame, and the rotation matrix from the inertial frame to the path-fixed reference frame is given by:

\[ R(\gamma_p) = \begin{bmatrix}
\cos(\gamma_p) & -\sin(\gamma_p) \\
\sin(\gamma_p) & \cos(\gamma_p)
\end{bmatrix} \in SO(2). \]

Moreover,

\[ \begin{align*}
x_e &= (x - x_k) \cos(\gamma_p) + (y - y_k) \sin(\gamma_p), \\
y_e &= -(x - x_k) \sin(\gamma_p) + (y - y_k) \cos(\gamma_p),
\end{align*} \]

where \( \gamma_p \) is the horizontal-plane path-tangential angle:

\[ \gamma_p = \arctan \frac{y_{k+1} - y_k}{x_{k+1} - x_k}. \]

Then, the associated control objective for horizontal plane straight-line path-following is:

\[ \lim_{t \to \infty} y_e(t) = 0. \]

Note that the along-track error \( x_e \) does not need to be minimized in a path-following scenario, the contrary is true for applications that impose temporal constraints. The lookahead-based guidance law is given by (see Breivik and Fossen (2009)):

\[ \dot{\psi}_d = \gamma_p + \arctan \frac{y_e}{\Delta x}. \]

In the presence of external disturbances, or during turns, the heading angle \( \psi_d \) and the course angle \( \chi_d \) are not aligned anymore and are related in the following way:

\[ \chi_d = \psi_d + \beta, \]
and therefore the desired heading angle is:
\[ \psi_d = \gamma_p + \arctan \left( \frac{-y_h}{\Delta_h} \right) - \beta. \]  
(22)

where
\[ \beta = \arctan (\gamma, u), \]  
(23)

which is equal to the orientation of the vehicle’s velocity vector \( U_h \) with respect to the body-fixed frame. In other words, (23) is the angle between the vehicle’s velocity orientation and the vehicle’s heading. This is the commonly known as sideslip, or drift, angle.

**Stability Result:** It has been shown that choosing the desired heading as in (22) results in a \( \kappa \)-exponentially stable equilibrium point \( y_c = 0 \), see Lekkas and Fossen (2013). This implies that the equilibrium point \( y_c = 0 \) is UGAS and ULES, see Lefeber (2000).

### 3.2 Vertical Plane LOS Guidance

This problem is similar to the one presented in Section 3.1. In the case of decoupled vertical plane path-following of underwater vehicles, it is common to assume that the yaw angle is constant, \( \psi = \psi_0 \). Consequently the kinematic equations to be considered are:
\[ \dot{x} = u \cos (\psi_0) \cos (\theta) - v \sin (\psi_0) + w \cos (\psi_0) \sin (\theta), \]  
(24)
\[ \dot{z} = -u \sin (\theta) + w \cos (\theta), \]  
(25)
\[ \dot{\beta} = q. \]  
(26)

Apparently, for this problem the vertical plane total speed is defined as:
\[ U_v := \sqrt{u^2 + v^2} > 0, \]  
(27)
and is assumed to be positive and bounded:
\[ U_{v, \text{min}} \leq U \leq U_{v, \text{max}}, \quad 0 < U_{v, \text{min}}. \]  
(28)

**Path Following Objective:** Similarly to the case for surface vessels, we assume that the vehicle is supposed to converge to the line connecting the waypoints \( WP_k-WP_{k+1} \), the along-track and the cross-track error for a given vehicle position \((x, z)\) are given by:
\[ \begin{bmatrix} x_e \\ z_e \end{bmatrix} = \mathbf{R} (\alpha_p) \begin{bmatrix} x - x_k \\ z - z_k \end{bmatrix}, \]  
(29)
where \((x_k, z_k)\) is the position of the \( k \)-th waypoint expressed in the NED frame, and the rotation matrix from the inertial frame to the path-tangential frame is given by:
\[ \mathbf{R} (\alpha_p) = \begin{bmatrix} \cos (\alpha_p) & \sin (\alpha_p) \\ -\sin (\alpha_p) & \cos (\alpha_p) \end{bmatrix} \in SO(2). \]  
(30)

Moreover,
\[ x_e = (x - x_k) \cos (\alpha_p) - (z - z_k) \sin (\alpha_p), \]  
(31)
\[ z_e = (x - x_k) \sin (\alpha_p) + (z - z_k) \cos (\alpha_p), \]  
(32)
where \( \alpha_p \) is the vertical path-tangential angle:
\[ \alpha_p = \arctan (- (z_{k+1} - z_k), (x_{k+1} - x_k)). \]  
(33)

Consequently, the associated control objective for vertical plane straight-line path-following is:
\[ \lim_{t \to +\infty} z_e(t) = 0. \]  
(34)

Similarly to the horizontal cross-track error case, we assume that the desired pitch angle is perfectly tracked at all times and choose the desired pitch angle as:
\[ \theta_d = \alpha_p + \alpha + \arctan \left( \frac{x_e}{\Delta_h} \right), \]  
(35)

\[ \beta_{c} = \arctan (\gamma, U_v \cos (\theta - \alpha)). \]  
(36)

The desired yaw angle in this case is:
\[ \psi_d = \gamma_p + \beta_{c} + \arctan \left( \frac{-y_c}{\Delta_h} \right). \]  
(37)

3.3 LOS Guidance for 3-D Coupled Motions

the horizontal LOS guidance is now coupled with the vertical motion of the AUV. Hence 5-DOF’s are considered and the kinematics is given by (4)–(8). The total speed in this case is:
\[ U := \sqrt{u^2 + v^2 + w^2}, \]  
(38)

Due to the system’s structure it is reasonable to consider the two following subsystems in order to solve the problem: a) a depth controller for (6)–(7), and b) a LOS guidance for (4)–(5) and (8). The heave-pitch subsystem is uncoupled with respect to the rest of the states, while on the other hand (4)–(5) and (8) imply that the horizontal motion is coupled with the vertical plane motion via the pitch angle \( \theta \) and the heave velocity \( w \). According to the analysis in Lekkas and Fossen (2013), the solution for the heave-pitch subsystem is the same as in Section 3.2. Regarding the horizontal motion it was shown that the cross-track error is given by:
\[ y_c = \sqrt{U_{h}^2 \cos^2 (\theta - \alpha) + v^2} \sin (\psi - \gamma_p + \beta_{c}) \]  
(39)

where:
\[ \beta_{c} = \arctan (\gamma, (U_v \cos (\theta - \alpha))). \]  
(40)

The desired yaw angle in this case is:
\[ \psi_d = \gamma_p + \beta_{c} + \arctan \left( \frac{-y_c}{\Delta_h} \right). \]  
(41)

Consequently, the only difference w.r.t. the solution of Section 3.1 is the coupled sideslip angle \( \beta_{c} \).
4. HORIZONTAL LOS GUIDANCE IN QUATERNION FORM

This section deals with transforming the horizontal LOS guidance (22) in quaternion form. A quaternion \( q_\psi \) representing a rotation of an angle \( \psi \) around the z-axis can be expressed as follows (Kuipers, 1999):

\[
q_\psi = \cos \left( \frac{\psi}{2} \right) + k \sin \left( \frac{\psi}{2} \right).
\]

4.1 Transforming \( \psi_r \) to the quaternion \( q_\psi \).

First we define:

\[
y_r^2 + \Delta_h^2 := R_h^2
\]

\[
\sin (\psi_r) = -\frac{y_h}{R_h}, \quad \cos (\psi_r) = \frac{\Delta_h}{R_h}.
\]

Then, since

\[
\sin (\psi_r) = 2 \sin \left( \frac{\psi_r}{2} \right) \cos \left( \frac{\psi_r}{2} \right),
\]

\[
\cos (\psi_r) = 2 \cos^2 \left( \frac{\psi_r}{2} \right) - 1,
\]

we get:

\[
\sin (\psi_r) = 2 \varepsilon_3 \eta_r,
\]

\[
\cos (\psi_r) = 2 \eta_r^2 - 1.
\]

Combining (42)–(43) with (44)–(45) yields:

\[
2\varepsilon_3 \eta_r = -\frac{y_h}{\sqrt{\Delta_h^2 + y_h^2}},
\]

\[
2\eta_r^2 - 1 = \frac{\Delta_h}{\sqrt{\Delta_h^2 + y_h^2}},
\]

and solving w.r.t. \( \eta_r \) and \( \varepsilon_3 \), gives:

\[
\eta_r = \pm \sqrt{\frac{\Delta_h + R_h}{2R_h}},
\]

\[
\varepsilon_3 = -\frac{y_h}{2R_h \eta_r}
\]

\[
= \frac{-y_h}{\sqrt{2R_h (\Delta_h + R_h)}}.
\]

It is important to note that either the positive or negative value for \( \eta_r \) can be chosen, hence affecting the sign of \( \varepsilon_r \) accordingly. Both cases correspond to the same rotation, as it is shown by (A.5). Therefore, the corresponding quaternion, \( q_\psi \), is written:

\[
q_\psi_r = \sqrt{\frac{\Delta_h + R_h}{2R_h}} + k \frac{-y_h}{\sqrt{2R (\Delta_h + R)}}.
\]

It can be confirmed that (53) corresponds to a rotation quaternion by computing the norm:

\[
||q|| = \eta_r^2 + \varepsilon_3^2 = \frac{\Delta_h + R_h}{2R_h} + \frac{y_h^2}{2R_h (\Delta_h + R_h)} = 1.
\]

4.2 Transforming \( \gamma_p \) to the quaternion \( q_\gamma \).

Similarly to Section 5.1 we have:

\[
q_\gamma = \cos \left( \frac{\gamma_p}{2} \right) + k \sin \left( \frac{\gamma_p}{2} \right),
\]

and, for the sake of notational brevity, we define:

\[
R_{xy} = \sqrt{\Delta x^2 + \Delta y^2},
\]

consequently the quaternion transformation gives:

\[
\eta_{\gamma_p} = \pm \frac{\Delta y}{\sqrt{2R_{xy} (\Delta x + R_{xy})}},
\]

\[
\varepsilon_{\gamma_p} = \frac{\Delta y}{\sqrt{2R_{xy} (\Delta x + R_{xy})}}.
\]

4.3 Transforming \( \beta \)

Implementing the same procedure for the sideslip angle \( \beta \) yields:

\[
q_{\beta} = \cos \left( \frac{\beta}{2} \right) + k \sin \left( \frac{\beta}{2} \right),
\]

\[
\beta = \arctan2 \left( \frac{\nu}{u} \right).
\]

which gives:

\[
\eta_{\beta} = \pm \frac{u + U}{\sqrt{2U (u + U)}},
\]

\[
\varepsilon_{\beta} = \frac{u + U}{\sqrt{2U (u + U)}}.
\]

4.4 Horizontal Rotation Quaternion

A quaternion-based control law system will make use of the horizontal rotation quaternion, \( q_h \), which is given by the quaternion products of \( q_\psi \), \( q_\beta \), \( q_\gamma \). Therefore:

\[
q_h = q_\psi \odot q_\beta \odot q_\gamma.
\]

Due to the fact that all the rotations considered in (66) are around the z-axis, the order of the multiplication is not important. Note, however that this is a special case.

5. VERTICAL LOS GUIDANCE IN QUATERNION FORM

The transformation procedure is similar to the one in Section 4. A quaternion \( q_\theta \) representing a rotation of an
Similarly to Section 5.1 we have:

\[ q_{\beta} = \cos \left( \frac{\alpha}{2} \right) + j \sin \left( \frac{\alpha}{2} \right). \]  

**5.2 Transforming \( \beta_{c} \) to the quaternion \( q_{\beta_{c}} \)**

\[ \zeta_{c}^{2} + \Delta^{2} = R_{c}^{2} \]  
\[ \sin (\theta_{c}) = \frac{z_{c}}{R_{c}} \cos (\theta_{c}) = \frac{\Delta_{c}}{R_{c}}, \]

Then, since

\[ \sin (\theta_{c}) = 2 \sin \left( \frac{\theta_{c}}{2} \right) \cos \left( \frac{\theta_{c}}{2} \right), \]
\[ \cos (\theta_{c}) = 2 \cos^{2} \left( \frac{\theta_{c}}{2} \right) - 1, \]

we get:

\[ \sin (\theta_{c}) = 2 \varepsilon_{3} \eta_{c}, \]
\[ \cos (\theta_{c}) = 2 \eta_{c}^{2} - 1. \]

Combining (68)–(69) with (70)–(71) yields:

\[ 2 \varepsilon_{3} \eta_{c} = \frac{z_{c}}{\sqrt{\Delta_{c}^{2} + z_{c}^{2}}}, \]
\[ 2 \eta_{c}^{2} - 1 = \frac{\Delta_{h}}{\sqrt{\Delta_{h}^{2} + z_{c}^{2}}}, \]

and solving w.r.t. \( \eta_{c} \) and \( \varepsilon_{3} \), gives:

\[ \eta_{c} = \pm \frac{\Delta_{c} + R_{c}}{2R_{c}}, \]
\[ \varepsilon_{3} = \frac{z_{c}}{2R_{c} \eta_{c}}, \]
\[ = \frac{z_{c}}{\sqrt{2R_{c}(\Delta_{c} + R_{c})}}. \]

Therefore, the corresponding quaternion, \( q_{\beta_{c}} \), is written:

\[ q_{\beta_{c}} = \sqrt{\frac{\Delta_{c} + R_{c}}{2R_{c}}} + j \frac{z_{c}}{\sqrt{2R_{c}(\Delta_{c} + R_{c})}}. \]

**5.3 Transforming \( \alpha \) to the quaternion \( q_{\alpha} \)**

Implementing the same procedure for the angle of attack \( \alpha \) yields:

\[ q_{\alpha} = \cos \left( \frac{\alpha}{2} \right) + j \sin \left( \frac{\alpha}{2} \right), \]
\[ \alpha = \arctan \left( \frac{w}{u} \right), \]

which gives:

\[ \eta_{\alpha} = \pm \frac{u + U_{c}}{2U_{c} \nu}, \]
\[ \varepsilon_{3\alpha} = \frac{\nu}{\sqrt{2U_{c}(u + U_{c})}}. \]

**6. LOS GUIDANCE FOR 3-D COUPLED MOTIONS IN QUATERNION FORM**

As discussed earlier, the only difference in this case is the coupled sideslip angle \( \beta_{c} \). Assuming perfect pitch tracking \( \alpha = \theta_{d} \), we have from (35):

\[ \theta - \alpha = \arctan (\frac{-\Delta_{z}}{\Delta_{x}}) + \arctan (\frac{z_{c}}{\Delta_{c}}), \]
\[ = \arctan (\frac{z_{c} \Delta_{x} - \Delta_{z} \Delta_{x}}{\Delta_{x} \Delta_{x} + z_{c} \Delta_{z}}), \]

then we can calculate the quantity:

\[ \lambda_{2} := \cos (\theta - \alpha) = 1/(1 + \lambda_{2}^{2}). \]

The transformation in this case gives:

\[ \eta_{\lambda_{2}} = \pm \frac{U_{c} \lambda_{2} + \zeta}{2 \zeta}, \]
\[ \varepsilon_{3\lambda_{2}} = \frac{\nu}{\sqrt{2 \zeta (U_{c} \lambda_{2} + \zeta)}}. \]

**7. SIMULATIONS**

In order to test the validity of the transformation methodology presented in the previous sections, a simple numerical example was constructed and implemented. The lookahead distance was chosen as \( \Delta_{h} = 4 \) m and the cross-track error increased linearly from −10 m to +10 m with a step of .01 m, hence generating 2001 samples. In Fig. 5 the quaternion computed by (53) is presented and Fig. 6 shows the corresponding Euler angles. It can be observed that the
In this paper, the quaternion attitude representation is used to transform the 6-DOF motions into corresponding to the coupled 6-DOF motions. The transformation leading to (53) can be expressed in quaternion form:

$$\mathbf{w}_q = \mathbf{q} \mathbf{v} \mathbf{q}^*.$$  \hspace{1cm} (A.4)

The rotation corresponding to the negative of a quaternion $\mathbf{q}$ is the same as the rotation due to the quaternion $\mathbf{q}$:

$$\mathbf{w}_q = \mathbf{w}_{-\mathbf{q}}.$$  \hspace{1cm} (A.5)

Moreover, the product of two quaternions $\mathbf{q}_1$, $\mathbf{q}_2$ is defined as:

$$\mathbf{q}_1 \otimes \mathbf{q}_2 := \begin{bmatrix} \eta_1 \eta_2 - \varepsilon_1^T \varepsilon_2 \\ \eta_2 \varepsilon_1 + \eta_1 \varepsilon_2 + \varepsilon_1 \times \varepsilon_2 \end{bmatrix}.$$  \hspace{1cm} (A.6)

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ACKNOWLEDGEMENTS

This work was supported by the Norwegian Research Council through the Centre for Autonomous Marine Operations and Systems at NTNU.

Appendix A. QUATERNION FUNDAMENTALS

In this paper, the quaternion attitude representation is used, where the unit quaternion $\mathbf{q}$ corresponding to a rotation in $\mathbb{R}^3$ is defined as:

$$\mathbf{q} = \eta + \varepsilon_1 \mathbf{i} + \varepsilon_2 \mathbf{j} + \varepsilon_3 \mathbf{k},$$  \hspace{1cm} (A.1)

with $||\mathbf{q}|| = 1$. The complex conjugate of $\mathbf{q}$ is given by:

$$\mathbf{q}^* = \eta - \varepsilon_1 \mathbf{i} - \varepsilon_2 \mathbf{j} - \varepsilon_3 \mathbf{k},$$  \hspace{1cm} (A.2)

and the inverse quaternion is given by:

$$\mathbf{q}^{-1} = \mathbf{q}^*.$$  \hspace{1cm} (A.3)

Following (Kuipers, 1999, Ch. 5.9), let us associate an angle $\lambda$, with the quaternion $\mathbf{q}$. Then, the image $\mathbf{w} \in \mathbb{R}^3$ of a vector $\mathbf{v} \in \mathbb{R}^3$ due to the rotation by an angle $\lambda$ can be expressed in quaternion form:

$$\mathbf{w}_q = \mathbf{q} \mathbf{v} \mathbf{q}^*.$$  \hspace{1cm} (A.4)

The computed quaternion corresponds to a rotation around the z-axis. Moreover, Fig. 7 shows the values computed by the term $\arctan(-y_c/\Delta h)$ and the result is that the plot is the same as the one in Fig. 6, hence confirming the validity of the transformations leading to (53).

8. CONCLUSIONS

This paper proposed a simple technique in order to transform the conventional LOS guidance algorithm in quaternion form. The main motivation is to avoid singularities in the more general 6-DOF case and also get a more computationally-efficient algorithm. The transformation methodology was presented here for the uncoupled motions in the horizontal and vertical planes. Future work includes the transformation of the LOS guidance corresponding to the coupled 6-DOF motions.

Fig. 5. Quaternion elements computed by the transformation algorithm.

Fig. 6. Euler angles corresponding to the quaternion computed by the transformation algorithm.

Fig. 7. Euler angles computed by $\arctan(-y_c/\Delta h)$.