Robust output maneuvering for a class of nonlinear systems

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Abstract

The output maneuvering problem involves two tasks. The first, called the geometric task, is to force the system output to converge to a desired path parametrized by a continuous scalar variable \( \theta \). The second task, called the dynamic task, is to satisfy a desired dynamic behavior along the path. This dynamic behavior is further specified via a time, speed, or acceleration assignment. While the main concern is to satisfy the geometric task, the dynamic task ensures that the system output follows the path with the desired speed. A robust recursive design technique is developed for uncertain nonlinear plants in vectorial strict feedback form. First the geometric part of the problem is solved. Then an update law is constructed that bridges the geometric design with the speed assignment. The design procedure is illustrated through several examples.

Keywords: Output maneuvering; Path following; Tracking; Backstepping; Robust nonlinear control; Input-to-state stability

1. Introduction

In many applications it is of primary importance to steer an object (robot arm, ship, vehicle, etc.) along a desired path. The speed assignment along the path may be of secondary interest. Vehicle control applications of this type are described by Micaelli and Samson (1993), Hauser and Hindman (1997), Encarnação and Pascoal (2001), Pettersen and Lefeber (2001), Al-Hiddabi and McClamroch (2002), while a reference for robotics is Song, Tarn, and Xi (2000). Control problems for such applications are usually approached as two separate tasks. The first task is to reach and follow a desired path as a function of a path variable \( \theta \), left as an extra degree of freedom for the second task. In the second task, \( \theta \) is used to satisfy an additional dynamic specification along the path. This setting is more general than the common tracking problem, in which the path variable \( \theta \) is a given function \( \theta(t) \), often just \( \theta = t \).

To determine the path variable \( \theta \), Hauser and Hindman (1995) used a numerical projection from the current state onto the path. An already available tracking controller was then converted into a maneuver regulation controller, and a quadratic Lyapunov function was employed to guarantee that the states converge to and move along the path. Their method applies to feedback linearizable systems, where the path is specified for the full state. Encarnação and Pascoal (2001) proposed a method to solve the output maneuvering problem by backstepping, while Al-Hiddabi and McClamroch (2002) considered nonminimum phase systems.

In this paper, the general maneuvering problem is divided into a geometric task and a dynamic task. The geometric task is to reach the path and then to stay on it, while the dynamic task is to satisfy a time, speed, or acceleration assignment along the path. In Skjetne, Fossen, and Kokotović (2002) and in this paper, a design procedure is developed to solve a robust maneuvering problem for systems in vectorial strict feedback form of any relative degree in presence of bounded disturbances. In \( n \) recursive steps, the design solves the geometric task. It then proceeds to construct an...
update law} that ties together the geometric design with the speed assignment $v_s$, which may depend on the path, $v_s(\theta)$, or may be an exogenous time signal, $v_s(t)$.

**Notation:** In GS, LAS, LES, UGAS, UGES, etc., stands $G$ for global, $L$ for local, $S$ for stable, $U$ for uniform, and $A$ for asymptotic, and $E$ for exponential. Total time derivatives of $x(t)$ are denoted $\dot{x}, \ddot{x}, x^{(3)}, \ldots, x^{(n)}$, while a superscript denotes partial differentiation: $x'(x, \theta, t) := \frac{\partial x}{\partial t}$, $x''(x, \theta, t) := \frac{\partial^2 x}{\partial x^2}$, and $x^n(x, \theta, t) := \frac{\partial^n x}{\partial \theta^n}$, etc. The Euclidean vector norm is $|x| := (x'x)^{1/2}$, the distance to a set $\mathscr{H}$ is $|x|_{\mathscr{H}} := \inf \{|x - y|: y \in \mathscr{H}\}$, while $|x|$ denotes the ess sup $|x(t)|: t \geq 0$. The induced norm of a matrix $A$ is denoted $||A||_x := \inf \{|x - y|: y \in \mathscr{H}\}$. Let $\mathscr{H} := (z, \omega, \theta, t): z = 0, \omega = 0}\}$.

2. **Problem statement and motivating examples**

In maneuvering, the main task is to converge to and follow a desired parametrized path, that is, a geometric curve $Y_d := \{y \in \mathbb{R}^m: \exists \theta \in \mathbb{R} \text{ such that } y = y_d(\theta)\}$, (1)

where $y_d$ is continuously parametrized by the path variable $\theta$. The second task is to satisfy a desired dynamic behavior along the path. This is more general than tracking where time $t$ is used to parametrize the desired motion.

The **output maneuvering problem** is comprised of the two tasks:

1. **Geometric task:** Force the output $y$ to converge to the desired path $y_d(\theta)$,
   \[
   \lim_{t \to \infty} |y(t) - y_d(\theta(t))| = 0 \tag{2}
   \]
   for any continuous function $\theta(t)$.

2. **Speed assignment:** Satisfy one or more of the following assignments:
   2.1. **Time assignment:** Force the path variable $\theta$ to converge to a desired time signal $v_t(t)$,
   \[
   \lim_{t \to \infty} |\theta(t) - v_t(t)| = 0. \tag{3}
   \]
   2.2. **Speed assignment:** Force the path speed $\dot{\theta}$ to converge to a desired speed $v_s(\theta, t)$,
   \[
   \lim_{t \to \infty} |\dot{\theta}(t) - v_s(\theta(t), t)| = 0. \tag{4}
   \]
   2.3. **Acceleration assignment:** Force the path acceleration $\ddot{\theta}$ to converge to a desired acceleration $v_a(\theta, 0, t)$,
   \[
   \lim_{t \to \infty} |\ddot{\theta}(t) - v_a(\dot{\theta}(t), \theta(t), t)| = 0. \tag{5}
   \]

Throughout this paper, the dynamic task is specified as a speed assignment.

**Assumption 2.1.** The path $y_d(\theta)$ and its $n$ partial derivatives are uniformly bounded on $\mathbb{R}^m$. The speed assignment $v_s(\theta, t)$ and its $n - 1$ partial derivatives are uniformly bounded in $\theta$ and $t$.

Consider the nonlinear system
   \[
   \dot{x} = f(x, \theta, t), \quad x \in \mathbb{R}^m, \\
   \dot{\theta} = w(x, \theta, t), \quad \theta \in \mathbb{R}, \\
   y = h(x), \quad y \in \mathbb{R}^m. \tag{6}
   \]

For each initial condition $(x_0, \theta_0) = (x(0), \theta(0))$, let $(x(t), \theta(t))$ denote the solution defined on its maximal interval of existence $[0, T)$. The system is said to be **forward complete** if the solution exists for all $t \geq 0$ so that $T = +\infty$.

Suppose (6) solves the output maneuvering problem where $\theta$ is the path variable. Moreover, suppose $x = \xi(\theta, t)$ is uniquely determined by $h(x) = y_d(\theta)$ and the equations obtained by differentiating $h(x) = y_d(\theta)$ $n - 1$ times along the solutions of (6) with $\dot{\theta} = v_s(\theta, t)$. Then, $\xi(\theta, t)$ is the state path corresponding to the output path $y_d(\theta)$ and speed assignment $v_s(\theta, t)$. Let $\gamma: \mathbb{R}^m \to \mathbb{R}^m$ be such that $z := \gamma(x - \xi(\theta, t))$, $\omega := v_s(\theta, t) - \dot{\theta}$ is a global change of coordinates, and take $\tau$ as a state with $\tau(0) = 0$.

Then the Output Maneuvering Problem with a speed assignment is solved if the noncompact ‘error’ set
   \[
   \mathscr{H} := \{z, \omega, \theta, t): z = 0, \omega = 0\} \tag{7}
   \]

is rendered **uniformly globally asymptotically stable**. Two examples are given to illustrate important aspects of the maneuvering problem. First, in the absence of disturbances, its objective is met by an invariant manifold of the closed-loop system. Second, $\theta$-parametrizations can be designed for smooth tracing of non-smooth shapes, such as triangles.

**Example 1. Manifold interpretation of a maneuvering objective:** For the nonlinear system
   \[
   x_1 = x_2 + x_1^2, \\
   x_2 = u, \tag{8}
   \]

where $u$ is the control, it is required that

1. the output $y = x_1$ converges to the path $y_d(\theta) = \sin(\theta)$.
2. the path angle $\theta$ converges to $t - \phi$, where the constant phase $\phi$ is left free (this corresponds to $v_s = 1$).

A dynamic state feedback controller designed using the backstepping methodology of Section 3 results in the closed-loop system
   \[
   x_1 = x_2 + x_1^2, \\
   x_2 = g(x_1, x_2, \omega, \theta), \\
   \dot{\theta} = 1 - \omega, \\
   \omega = g(x_1, x_2, \omega, \theta) \tag{9}
   \]
A typical trajectory in the space of \((1)\). A control law that achieves this is \(\dot{z}_{1}(t) = \sin(\theta(t))\), and the speed assignment error \(\omega_{s}\) to zero. Thus, the desired manifold is in \(\mathbb{R}^4\) is

\[
\begin{align*}
x_1 &= \xi_1(\theta) = \sin(\theta), \\
(x_1, x_2, \omega_s, \theta): x_2 &= \xi_2(\theta) = \cos(\theta) - \sin(\theta)^2, \\
\omega_s &= 0,
\end{align*}
\]

where \(\xi_1\) and \(\xi_2\) defines the state path. This one-dimensional manifold in \(\mathbb{R}^4\) is to be made a globally attractive invariant manifold of \(\mathcal{O}\). A control law that achieves this is

\[
u = -z_1 - z_2 - (2x_1 + 1)x_1 + (y_\theta' - \theta')(1 - \sin(\theta)),
\]

where \(z_1 := x_1 - y_\theta(\theta)\) and \(z_2 := z_1 + x_2 + x_1^2 - y_\theta^2(\theta)\). On the designed manifold, the motion of the closed-loop system is of that a harmonic oscillator, \((1)\) possesses a globally attractive invariant manifold which meets the above requirements. To derive the expression of the desired invariant manifold we differentiate \(y = x_1 = y_\theta(\theta) = \sin(\theta)\), get \(\dot{x}_1 = x_2 + x_1^2 = \cos(\theta)\theta = \cos(\theta)(1 - \omega_s)\), and set the speed assignment error \(\omega_s\) to zero. Thus, the desired manifold in \(\mathbb{R}^4\) is

\[
\begin{align*}
\dot{y}_{\theta} &= \frac{\pi}{2|y_{\theta}'|} \arctan \left( \frac{\theta - \theta - a_1}{a_2} \right) + \frac{m_s}{2|y_{\theta}'|}, \\
\theta &\in [\theta_k, \theta_k + \frac{\theta_{k+1} - \theta_k}{2}], \\
\vdots & \vdots \\
\theta &\in [\theta_k + \frac{\theta_{k+1} - \theta_k}{2}, \theta_{k+1}],
\end{align*}
\]

where \(\theta\) is the path angle \(\theta\) and the speed assignment error \(\omega_s\). The functions \(x\) and \(\chi\) are designed to guarantee that in the state space \(\mathbb{R}^4\) the closed-loop system \((1)\) possesses a globally attractive invariant manifold which meets the above requirements. To derive the expression of the desired invariant manifold we differentiate \(y = x_1 = y_\theta(\theta) = \sin(\theta)\), get \(\dot{x}_1 = x_2 + x_1^2 = \cos(\theta)\theta = \cos(\theta)(1 - \omega_s)\), and set the speed assignment error \(\omega_s\) to zero. Thus, the desired manifold in \(\mathbb{R}^4\) is

An application of Matrosov’s Theorem (Loria, Panteley, Popović, & Teel, 2002) implies that the equilibrium \((\bar{z}_1, \bar{z}_2, \omega_s, \omega_a) = 0\) is UGAS, so that \(\phi(t) \to 0\); see Section 3.3.

**Example 2. A typical maneuvering problem:** The motion of a cutting tool is represented by

\[
M \ddot{x} + D(\dot{x}) \dot{x} + K(x) \dot{x} = u,
\]

where \(x \in \mathbb{R}^2\) is the position in the plane, the force \(u \in \mathbb{R}^2\) is the control, \(M = M^T > 0\) is the system inertia matrix, and \(D(\dot{x}) = D_0 + D_1(\dot{x}) > 0\) and \(K(x) = K_0 + K_1(x)\) are the respective linear and nonlinear damping and spring matrices.

The control objective is for the tip of the cutting tool \(y = x\) to trace the triangular path \(y_\theta(\theta)\) in Fig. 2 parametrized by \(\theta\) as follows:

\[
y_{\theta}(\theta) = \begin{cases} 
[\theta, \theta]^T & : \theta \in [0, 1) \\
[\theta, 2 - \theta]^T & : \theta \in [1, 2) \\
[4 - \theta, 0]^T & : \theta \in [2, 4].
\end{cases}
\]

To illustrate that many feasible parametrizations are possible, \(\theta\) is not the distance travelled by the cutting tool; while \(\theta\) progresses from 0 to 1, the distance travelled is 1.41 m, and the total distance around the path is 4.83 m.

The task is to trace the path as fast as possible under a maximum speed constraint of about \(m_s \approx 0.1\) m s\(^{-1}\), and with the deviation from the triangular path less than \(10^{-3}\) m. Since the triangular path is not smooth, the cutting tool is to trace each edge, and stop and restart at each corner. To avoid large transients, the desired speed \(v_{\theta}(\theta)\) should be small near the corners. Thus, between the corners \(k\) and \(k+1\), \(k = 1, 2, 3\), we assign the following speed profile:

\[
v_{\theta}(\theta) = \begin{cases} 
\frac{m_s}{\pi |y_{\theta}'|} \arctan \left( \frac{\theta - \theta - a_1}{a_2} \right) + \frac{m_s}{2|y_{\theta}'|}, \\
\theta &\in [\theta_k, \theta_k + \frac{\theta_{k+1} - \theta_k}{2}], \\
\vdots & \vdots \\
\theta &\in [\theta_k + \frac{\theta_{k+1} - \theta_k}{2}, \theta_{k+1}],
\end{cases}
\]

where \(\theta\) is the path angle \(\theta\) and the speed assignment error \(\omega_s\).
where $\theta_1 = 0$, $\theta_2 = 1$, $\theta_3 = 2$, and $\theta_4 = 4$, as shown in Fig. 3 for $\theta \in [0, 4)$. The parameter $a_1$ sets the width of the low speed regions around the corners, while $a_2$ smoothens the square wave. A complete controller design is given in Section 4.

3. Robust output maneuvering

Consider the nonlinear plant in strict feedback form of vector relative degree $n$

$$\begin{align*}
\dot{x}_1 &= G_1(x_1)x_2 + f_1(x_1) + W_1(x_1)\delta_1(t), \\
\dot{x}_2 &= G_2(x_2)x_3 + f_2(x_2) + W_2(x_2)\delta_2(t), \\
& \vdots \\
\dot{x}_n &= G_n(x_n)u + f_n(x_n) + W_n(x_n)\delta_n(t), \\
y &= h(x_1),
\end{align*}$$

(18)

where $x_i \in \mathbb{R}^m$, $i = 1, \ldots, n$, are the states, $y \in \mathbb{R}^m$ is the output, $u \in \mathbb{R}^m$ is the control, and $\delta_i$ are unknown bounded disturbances. The matrices $G_i(\bar{x}_i)$ and $h^i(x_1) : = (\partial h/\partial \bar{x}_1)(x_1)$ are invertible for all $\bar{x}_i$, $h(x_1)$ is a diffeomorphism, and $G_i$, $f_i$, and $W_i$ are smooth.

The control objective is to design a maneuvering controller that solves the output maneuvering problem for a desired parametrized output path $y_d(\theta)$ and speed assignment $u_\circ(\theta, t)$ where Assumption 2.1 is satisfied.

Due to the disturbances $\delta_i$ in (18), the goal is to render the closed-loop system input-to-state stable (ISS) with respect to the set $\mathcal{M}$ in (7), see (Lin, 1992; Lin, Sontag, & Wang, 1995). Such a set is said to be $0$-invariant for a closed-loop system if it is invariant for the disturbance-free, $0 \in \mathcal{M}$, closed-loop system. Recall that if a forward complete system admits an ISS-Lyapunov function with respect to a closed, $0$-invariant set $\mathcal{M}$, then it is ISS with respect to $\mathcal{M}$. This will be the main tool to solve the maneuvering problem for (18).

3.1. Design procedure

The first step of a backstepping design is given to show how to deal with $\dot{\theta}$. The $i$th step for $i = 2, \ldots, n$ is given in Table 1. The design procedure borrows much from adaptive tracking by Krstić, Kanellakopoulos, and Kokotović (1995) including the notion of a tuning function.

Step 1: The new variables

$$\begin{align*}
z_1(x_1, \theta) &:= y - y_d(\theta) = h(x_1) - y_d(\theta), \\
z_i(\bar{x}_i, \theta, t) &:= x_i - x_{i-1}(\bar{x}_{i-1}, \theta, t), \quad i = 2, \ldots, n, \\
\omega_\circ(\dot{\theta}, \theta, t) &:= u_\circ(\theta, t) - \dot{\theta}
\end{align*}$$

(19) (20) (21)

are introduced, where $x_{i-1}$ are virtual controls to be specified later. Differentiating (19) with respect to time results in

$$\dot{z}_1 = \dot{y} - y_d^\circ(\theta)\dot{\theta} = h^i G_1 z_2 + h^i G_1 x_1 + h^i f_1 + h^i W_1 \delta_1 - y_d^\circ(\theta)\dot{\theta}.$$
\[ z_i = x_i - x_{i-1} = G_1 z_{i+1} + G_2 x_i + f_i + W_i \delta_i \]
\[ \dot{V}_i = V_i - z_i^T P_i z_i + 2z_i^T P_i z_{i+1} + \tau_i \omega_k \]
\[ \dot{V}_2 \leq -\sum_{j=1}^{i} z_j^T Q_j z_j + \frac{1}{\kappa_i} \sum_{j=1}^{i} \sigma_j \delta_j + 2z_i^T P_2 [W_2 \delta_{1} - \omega_{1,1} \delta_1] \]
\[ + 2z_i^T \{ G_3^T [h_i^0]^T P_3 z_1 \]
\[ + P_2(G_2 z_2 + f_2 - \sigma_i - x_i^0 \nu_1) \} + \tau_i \omega_k + 2z_i^T P_2 z_i \sigma_i + 2z_i^T P_2 z_2 G_2 z_3 \]
\[ \dot{V}_i \leq -\sum_{j=1}^{i} z_j^T Q_j z_j + \frac{1}{\kappa_i} \sum_{j=1}^{i} \sigma_j \delta_j \]
\[ + 2z_i^T \{ G_3^T [P_{i-1} z_{i-1}] \]
\[ + P_i [G_2 z_i + f_i - \sigma_{i-1} - x_i^0 \nu_1] \} + \tau_i \omega_k + 2z_i^T P_i \sigma_i + 2z_i^T P_i G_3 z_{i+1} \]
\[ \tau_i := \tau_{i-1} + 2z_i^T P_i \sigma_i \]

### Tuning Function
\[ z_2 = z_2(\tilde{x}_2, \tilde{\theta}, t) = G_2^{-1} [A_2 z_2 + P_2 G_1 (h_i^0)^T P_1 z_1 \]
\[ - f_2 + \sigma_2 - x_2^0 \nu_2 + 2 \nu_2] \]
\[ z_i = z_i(\tilde{x}_i, \tilde{\theta}, t) = G_i^{-1} [A_i z_i + P_i G_1 (h_i^0)^T P_{i-1} - z_{i-1} \]
\[ - f_i + \sigma_{i-1} - x_i^0 \nu_2 + \nu_2] \]
\[ z_0 = z_0(\tilde{x}_0, \tilde{\theta}, t) \]
\[ \rho_i = - \frac{1}{\kappa_i} [W_i W_i^T + \sum_{j=1}^{i} \sigma_j \sigma_j^T] P_i \]
\[ P_i A_i + A_i^T P_i = -Q_i < 0 \quad \text{and} \quad \kappa_i > 0 \]

\[ \dot{z}_2 = - P_2^{-1} G_1 (h_i^0)^T P_2 z_2 + A_2 z_2 + G_2 z_3 - x_2^0 \omega_k \]
\[ - \frac{1}{\kappa_i} [W_i W_i^T + \sum_{j=1}^{i} \sigma_j \sigma_j^T] P_i z_2 - \omega_{1,1} \delta_1 + W_i \delta_1 \]
\[ \dot{z}_i = - P_i^{-1} G_1 (h_i^0)^T P_{i-1} - z_{i-1} - A_i z_i + G_i z_{i+1} + x_i^0 \omega_k \]
\[ - \frac{1}{\kappa_i} [W_i W_i^T + \sum_{j=1}^{i} \sigma_j \sigma_j^T] P_i z_i + W_i \delta_i - \sum_{j=1}^{i-1} \sigma_j \delta_j \]
\[ \dot{V}_i \leq - \sum_{j=1}^{i} z_j^T Q_j z_j + 2z_i^T P_i G_3 z_{i+1} + \tau_i \omega_k + A_i^T K_i A_i \]
\[ A_i := [\delta_i \delta_j \cdots] \]
\[ K_i := \text{diag}(\frac{1}{\kappa_i} + \frac{1}{\kappa_2} + \cdots + \frac{1}{\kappa_i}) \]

The result of Step 1 is
\[ \dot{z}_1 = A_1 z_1 - \frac{1}{2} \kappa_1 h_1^0 W_1 W_1^T (h_1^0)^T P_1 z_1 + h_1^0 z_2 \]
\[ + y_1^0 \omega_k + h_1^0 W_1 \delta_1 \]

and the result of Step 3 is
\[ \dot{V}_1 \leq -z_1^T Q_1 z_1 + 2z_1^T P_1 h_1^0 G_1 z_2 + \tau_1 \omega_k \]
\[ + A_1^T K_1 A_1, \]

where \( A_1 := \delta_1 \) and \( K_1 := 1/\kappa_1 \). If this was the last step, then \( z_2 = 0 \) and \( \omega_k = 0 \) would reduce (27) to
\[ \dot{V}_1 \leq - q_1 |z_1|^2 + k_1 |A_1|^2 < 0, \quad \forall |z_1| > \sqrt{\frac{k_1}{q_1}} |A_1|, \]

where \( q_1 = \lambda_{\text{min}}(Q_1) \), \( k_1 = 1/\kappa_1 \), which implies ISS from the disturbance \( \delta_1 \) to the state \( z_1 \). To aid next step, let
\[ \dot{z}_1 = \sigma_1 + x_1^0 \dot{\theta} + \sigma_{1,1} \delta_1, \]

where \( \sigma_1 \) collects the terms in \( z_1 \) not containing \( \dot{\theta} \) and \( \delta_1 \), and \( \sigma_{1,1} \) collects the terms multiplying the disturbance \( \delta_1 \)
\[ \sigma_1(x_1, \dot{\theta}, t) = x_1^0 (x_1, \dot{\theta}, t) [G_1(x_1) x_2 + f_1(x_1)] \]
\[ + x_1^1 (x_1, \dot{\theta}, t), \]

\[ \sigma_{1,1}(x_1, \dot{\theta}, t) = x_1^0 (x_1, \dot{\theta}, t) W_1(x_1). \]

Steps \( i = 2, \ldots, n \) are summarized in Table 1. Upon the completion of Step \( n \), the choice
\[ u = \sigma_n(\tilde{x}_n, \dot{\theta}, t) \]
\[ = G_n^{-1} [A_n \sigma_n - P_n^{-1} G_n^T P_{n-1} z_{n-1} - f_n + \sigma_{n-1} + x_1^0 \nu_2 + u_0] \]

with \( \kappa_n > 0 \), results in
\[ z_n = - P_n^{-1} G_n^{-1} P_{n-1} z_{n-1} + A_n \sigma_n + x_1^0 \omega_k \]
\[ - \frac{1}{2} \kappa_n [W_n W_n^T + \sum_{j=1}^{n-1} \sigma_{n-1,j} \sigma_{n-1,j}^T] P_n \sigma_n \]
\[ + W_n \delta_n - \sum_{j=1}^{n-1} \sigma_{n-1,j} \delta_j. \]

Let \( z := [z_1^T, \ldots, z_n^T]^T \) and \( Q := \text{diag}(Q_1, Q_2, \ldots, Q_n) \). Then the derivative of the Step \( n \) CLF is bounded by
\[ \dot{V}_n \leq - z^T Q z + \tau_n \omega_k A_n \]

With \( P := \text{diag}(P_1, P_2, \ldots, P_n) \) we rewrite the \( n \)th tuning function as
\[ \tau_n(\tilde{x}_n, \dot{\theta}, t) = 2b(\tilde{x}_{n-1}, \dot{\theta}, t)^T P z(\tilde{x}_n, \dot{\theta}, t), \]

and the \( z \)-system as
\[ \dot{z} = A_2(\tilde{x}_n, \dot{\theta}, t) z + b(\tilde{x}_{n-1}, \dot{\theta}, t) \omega_k \]
\[ + W(\tilde{x}_n, \dot{\theta}, t) A_n(t), \]
where $A_\epsilon(x, \theta, t)$, $W(x, \theta, t) \in \mathbb{R}^{m \times m}$, and $b(x_{n-1}, \theta, t) \in \mathbb{R}^m$ are defined by (A.1)–(A.3) in Appendix A.

### 3.2. Closing the loop by speed assignment designs

While expressions (31) and (32) define the static part of the control law, the dynamic part, specifying either $\dot{x}$ or $(\theta, \dot{\omega}_n)$, is to render the term $\tau_n \omega_n$ in (34) nonpositive. When $\tau_n \omega_n \leq 0$ is achieved, then (34) guarantees that the closed-loop system is ISS where the damping gains $\kappa_i$ can be adjusted for a desired level of disturbance attenuation. Following Jiang, Teel, Praly (1994), one can assign the gain from the disturbances $\delta_i$ to the output error $z_1 = y - y_d(\theta)$ to ensure any desired output maneuvering accuracy.

For the dynamic part of the control law, the design variable is $\omega_n$. Three different choices, referred to as the Tracking, the Gradient, or the Filtered-Gradient update laws, respectively, are proposed to render $\tau_n \omega_n$ nonpositive in (34).

1. **Tracking update law**: Setting $\dot{\omega}_n = 0$ satisfies the speed assignment (4) identically. The dynamic part of the control law becomes

$$\dot{\theta} = v_\theta(\theta, t), \quad (37)$$

which can be used to achieve tracking of a desired output $y_d(t) = y_d(\theta(t))$ where (37) is integrated to get $\theta(t) = \theta(0) + \int_0^t v_\theta(\theta(t), \tau) \, d\tau$.

2. **Gradient update law**: Setting $\dot{\omega}_n = -\mu_1 \tau_n(x, 0, t)$, $\mu_1 \geq 0$, satisfies the speed assignment (4) asymptotically since $\tau_n \to 0$ as $z \to 0$. We call this a **Gradient** update law because

$$\tau_n(x, 0, t) = -\frac{\partial V_n}{\partial x} (\tilde{x}, 0, t) = -V_n^\theta(\tilde{x}, 0, t) \quad (38)$$

and the dynamic part of the control law becomes

$$\dot{\theta} = v_\theta(\theta, t) + \mu_1 \tau_n(x, 0, t)$$
$$= v_\theta(\theta, t) - \mu_1 V_n^\theta(\tilde{x}, 0, t). \quad (39)$$

For $\mu_1 = 0$ it reduces to (37).

**Theorem 3.1.** The closed-loop system resulting from the Gradient update law:

$$\dot{z} = A_z \dot{z} - 2\mu_1 b b^\top Pz + W A_n,$$

$$\dot{\theta} = v_\theta(\theta, t) - \omega_n,$$

$$(which \ equals \ the \ tracking \ system \ for \ \mu_1 = 0) \ is, \ under \ the \ stated \ assumptions, \ forward \ complete \ and \ solves \ the \ maneuvering \ problem, \ that \ is, \ system \ (40) \ is \ ISS \ with \ respect \ to \ the \ closed, \ 0-invariant \ set$$

$$\mathcal{M} = \{ (z, \omega_n, 0, t) \in \mathbb{R}^{m} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}^+: (z, \omega_n) = 0 \}. \quad (41)$$

**Proof.** A direct application of Lemma 6.1 gives the result by setting $x_1 := z$ and $x_2 := (\theta, t)$, and using Assumption 2.1 for boundedness and $V_n$ as the ISS-Lyapunov function.

By the achieved ISS property, $z$ and $\omega_n$ are only guaranteed to enter a small residual set $\Omega$ due to the disturbances. This may cause $\dot{\theta}$ in (39) or (44), and therefore $y_d(\theta)$, to stop, which happens if $\omega_n(t) = v_\theta(\theta(t), t)$ for some $t$. Reducing $\Omega$, with larger nonlinear damping gains $\kappa_i$, will alleviate this problem.

For comparison to the above three designs, Hauser and Hindman (1995) defined a desired path $\zeta(\theta)$ for the full state $x$ and used a numerical projection algorithm from the current state $x(t)$ onto the path to ensure that $\tau_n = -V_n^\theta \equiv 0$.
so that the speed assignment \( v_\delta = 1 \) is asymptotically satisfied by \( \lim_{\tau \to \infty} [\theta(\tau) - 1] = 0 \). However, in addition to a more complicated implementation (hybrid), they also make stronger assumptions on the path. For example, they preclude self-intersecting paths to avoid multiple global minima, a problem not encountered for the dynamic gradient algorithms (see Skjetne, Teel, and Kokotović, 2002a, b).

### 3.3. Including phase assignment

In some guidance applications, the time specification along the path, or specifically the arrival time \( T \) at the destination, is important. A maneuvering based control design results in a phase shift \( \phi \) with respect to such a time-specification. It is of interest to control this phase, perhaps to \( \phi = 0 \), while retaining the gradient properties of the maneuvering system.

For simplicity, let \( v_\nu(\theta, t) = v_0 \neq 0 \) be a constant speed assignment, and assume the plant is disturbance-free, \( A_\nu = 0 \). Then \( \phi(t) = v_0 t - \theta(t) \) converges to a limit. Define \( \omega_\nu := \phi - \phi_d = v_0 t - \theta - \phi_d \) where \( \phi_d \) is a desired constant phase. This has the derivative

\[
\dot{\omega}_\nu = v_0 - \dot{\theta} = \omega_\delta, \tag{47}
\]

The Step \( n \) CLF is now augmented to \( V_n := z^T P z + (\mu_0/2) \omega_\nu^2 \) so that, instead of (34), the new design inequality is

\[
\dot{V}_n \leq -z^T Q z + (\tau_n + \mu_0 \omega_\nu) \omega_\nu.
\]

It is easily verified for the new CLF that \( \tau_n + \mu_0 \omega_\nu = -V_n^\theta \). Hence, treating \( \tau = \tau_n + \mu_0 \omega_\nu \) as the new tuning function, both the Gradient or Filtered–Gradient update laws can be constructed as before, with the new feature that \( \omega_\nu(t) \to 0 \) as \( t \to \infty \). This is stated next for the Filtered–Gradient update law.

**Theorem 3.3.** The closed-loop system resulting from the Filtered–Gradient update law with phase assignment is

\[
\dot{z} = A(z, \theta, t) z + b(z_{n-1}, \theta, t) \omega_\nu,
\]

\[
\dot{\omega}_\nu = \omega_\nu,
\]

\[
\dot{\omega}_\delta = -[\omega_\delta + \mu_0 \mu_1 \omega_\nu + 2 \mu_1 b(z_{n-1}, \theta, t)^T P z], \tag{48}
\]

and has, under Assumption 2.1, a UGAS equilibrium \( (z_0, \omega_0, \omega_\nu) = 0 \), which means that the maneuvering problem with phase assignment is solved: \( \dot{y}(t) = y_\nu(\theta(t)) \to 0 \), \( \theta(t) \to \theta_0 \), and \( \phi(t) \to \phi_d \) as \( t \to \infty \).

**Proof.** See Appendix A. \( \square \)

The designed closed-loop system can be divided into four parts, the **plant**, the **measurement system**, the **maneuvering controller**, and the **guidance system**, as in Fig. 4. The controller incorporates the dynamic control law, and provides the control signal \( u \) in (31) to the plant and the state \( \theta \) to the guidance system. The guidance system incorporates the path definition (1), the speed assignment \( v_\delta(\theta, 0) \), and their partial derivatives. The path definition must be specified a priori, while the speed assignment along the path can be modified online (which accounts for the \( t \)-dependence in \( v_\delta(s, t) \)). The guidance system is therefore, tightly interconnected with the controller such that for each \( \theta \) (and exogenous user input), it returns the necessary path and speed assignment signals.

### 4. Design example: the cutting tool

We return to Example 2 and design a robust maneuvering system for the cutting tool application. For the plant (15), let \( D_1 \) and \( K_1 \) be unknown state-dependent matrices, bounded by

\[
\sup_{\theta} \| K_1(\theta) \| \leq m_K, \sup_{\theta} \| D_1(\theta) \| \leq m_D,
\]

where \( m_K, m_D \) need not be known. Let \( x_1 := x \) and \( x_2 := \dot{x} \), and define \( W := [W_1, W_2] \in \mathbb{R}^{2 \times 8} \), where \( W_1 := -\text{diag}(x_1, x_1) \in \mathbb{R}^{2 \times 4} \) and \( A := [\delta_1^T, \delta_2^T] \in \mathbb{R}^8 \) where \( \delta_1, \delta_2 \in \mathbb{R}^4 \) contains the two rows of \( K_1 \) and \( D_1 \) stacked in one column vector. The plant

\[
\dot{x}_1 = x_2,
\]

\[
M \dot{x}_2 + D_0 x_2 + K_0 x_1 = u + WA
\]

is in the form of (18). The design procedure then yields

\[
z_1 = x_1 - y_\nu(\theta),
\]

\[
z_2 = x_2 - \alpha_1(x_1, y_\nu(\theta), y_\nu^2(\theta), v_\nu(\theta)),
\]

\[
\alpha_1 = -K_p z_1 + y_\nu^1(\theta) v_\nu(\theta),
\]

\[
\sigma_1 = -K_p z_2,
\]

\[
y_\nu^0 = K_p y_\nu^0(\theta) + y_\nu^2(\theta) v_\nu(\theta) + y_\nu^0(\theta) v_\nu^2(\theta),
\]

where \( A_1 = -K_p, K_p = K_p^T > 0 \), and \( A_2 = -K_d, K_d = K_d^T > 0 \). This closed-loop guidance and control system is divided into...
the following modules:

**Plant:**

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix}
x_2 \\
M^{-1}(u - D(x)x_2 - K(x)x_1)
\end{bmatrix},
\]

\[y = x_1,\]

input : \{u\},

output : \{x_1, x_2\}.

**Control:**

\[
\begin{bmatrix}
\dot{\theta} \\
\dot{\omega}_k
\end{bmatrix} = \begin{bmatrix}
v_d(\theta) - \omega_k \\
-\lambda \omega_k - 2\mu_1(p_1 z_1^T y_d^\theta(\theta) + p_2 z_2^T Mx_1^\theta)
\end{bmatrix},
\]

\[u = -\frac{p_1}{p_2} z_1 - K_d z_2 + K_0 x_1 + D_0 x_1 + Mx_1 + \frac{1}{2} \kappa WW^T p_2 z_2,
\]

input : \{x_1, x_2, y_d(\theta), y_d^\theta(\theta), y_d^{\theta^2}(\theta), v_d(\theta), v_k(\theta), \omega_k(\theta)\},

output : \{u, \theta\}.

**Guidance:**

\[
\begin{bmatrix}
\dot{\theta} \\
\dot{\gamma}(\theta)
\end{bmatrix} = \begin{bmatrix}
y_d(\theta), y_d^\theta(\theta), y_d^{\theta^2}(\theta), v_d(\theta), v_k(\theta), \omega_k(\theta)
\end{bmatrix}.
\]

Stability is verified with \(V = p_1 z_1^T z_1 + p_2 z_2^T Mz_2 + (1/2\mu_1) \omega_k^2\) and its derivative \(\dot{V} \leq -z_1^T Q_1 z_1 - z_2^T Q_2 z_2 - (\lambda/\mu_1) \omega_k^2 + 1/2 A^T A\).

To satisfy the performance specifications in Example 2, we constructed the speed assignment (17), which has the derivative

\[
v_\theta^\theta(\theta) = \begin{cases}
\frac{m_k}{\pi |y_d^\theta|} \frac{a_2}{a_2^2 + (\theta - \theta_k - a_1)^2}, \\
\theta \in \left[\theta_k, \theta_k + \frac{\theta_{k+1} - \theta_k}{2}\right], \\
\frac{-m_k}{\pi |y_d^\theta|} \frac{a_2}{a_2^2 + (\theta_k + 1 - a_1 - \theta)^2}, \\
\theta \in \left[\theta_0 + \frac{\theta_k + 1 - \theta_k}{2}, \theta_{k+1}\right].
\end{cases}
\]

Since the assigned speed is very slow at the nodes, there is a danger that the tracing stops if the disturbances are not attenuated enough. It is therefore important to choose \(\kappa\) large enough. In the simulation shown next, \(v_\theta(\theta) \geq 0.0013\) for all \(\theta\) and \(\kappa = 150\) ensures that the residual set for \(\omega_k\) is smaller than 0.0013.

In the simulation, we let the matrices \(K\) and \(D\), including the ‘unknown’ trigonometric disturbance terms, be

\[
K(x_1) = \begin{bmatrix}
10 + 0.15 \sin(7.5x_{11}) \cos(7.5x_{12}) & 0 \\
0 & 10 + 0.15 \sin(7.5x_{12}) \cos(7.5x_{11})
\end{bmatrix},
\]

and further:

\[
D(x_2) = \begin{bmatrix}
2 & 0.1 \sin(x_{21}) \\
0.1 \sin(x_{22}) & 2
\end{bmatrix},
\]

and \(M = I\). The controller gains were set to \(K_p = K_d = 50I\), \(p_1 = 5\), \(p_2 = 1\), \(\kappa = 150\), \(\lambda = 40\), \(\mu_1 = 1\), and the speed assignment parameters were set to \(a_1 = 0.005\), \(a_2 = 0.001\), and \(m_k = 0.1\). The position of the tool started from rest at \((0, 0)\) and \((\theta(0), \omega_k(0)) = (0, 0)\). The output, shown in Fig. 5, accurately traces the path. From Fig. 6 the output error \(|z_1(t)| = |x_1(t) - y_d(\theta(t))|\) is observed to be in the order of \(10^{-5}\) which is well below the specified limit. The speed along the path is seen from Fig. 7 to be approximately \(0.1\) m s\(^{-1}\) with a small ripple due to the ‘disturbances’ in \(K(x_1)\) and \(D(x_2)\). In Fig. 8 we also verify that the control effort is far from being excessive, despite the large nonlinear damping gain. This illustrates the gradient-based maneuvering system’s capability to always keep the error signals small. Time spent tracing the path is 51.1 s. If one were able to trace the entire path with speed \(m_k\), then the total time would be 48.3 s so that the time loss is only 5.8%. We conclude that our maneuvering design satisfies the problem specifications well.
5. Conclusion

The Maneuvering Problem was defined as solving a Geometric Task and a Dynamic Task. The geometric task was to converge to, and stay on, a desired parametrized path, and the dynamic task was to satisfy a desired dynamic behavior along the path, here specified as a speed assignment. It was shown that this problem implied the existence of a globally attractive, forward invariant manifold in the state space, given by a state path, to which the solutions had to converge.

A robust design procedure was proposed in Section 3, that solved the maneuvering problem and made the closed-loop ISS with respect to the desired set. This was achieved by constructing a dynamic control law as a Tracking, Gradient, or Filtered-Gradient update law. In the cutting tool design example, the flexibility of picking a suitable path parametrization and constructing the speed assignment were shown to be powerful tools to satisfy the design specifications. Section 3 ended with a procedure for also controlling the resulting phase between a \( \theta \)-parametrized path and the corresponding \( t \)-parametrized path.

Maneuvering is a control design procedure with great flexibility. Choosing the path parametrization and constructing the speed assignment makes it suitable for a range of applications. In addition, the gradient-based update laws provide robustness and performance improvements in the closed-loop.

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Appendix A.

Finalizing the design in Section 3, the functions in (35) and (36) are

\[
b(\tilde{x}_{n-1}, \theta, t) = \begin{bmatrix} y_1^\theta(\theta)^\top, x_1^\theta(x_1, \theta, t)^\top, \cdots, x_{n-1}^\theta(x_{n-1}, \theta, t)^\top \end{bmatrix}^\top,
\]

(A.1)

\[
W(\tilde{x}_n, \theta, t) := \begin{bmatrix} h^1W_1 & 0 & 0 & \cdots & 0 \\ -\sigma_{1,1} & W_2 & 0 & \cdots & 0 \\ -\sigma_{2,1} & -\sigma_{2,2} & W_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\sigma_{n-1,1} & -\sigma_{n-1,2} & -\sigma_{n-1,3} & \cdots & W_n \end{bmatrix},
\]

(A.2)

\[
A_2(\tilde{x}_n, \theta, t) := \begin{bmatrix} A_1 + \rho_1 & h^1G_1 & 0 \\ -P_2^{-1}G_1^\top (h^1)^\top P_1 & A_2 + \rho_2 & G_2 \\ 0 & -P_3^{-1}G_3^\top P_2 & A_3 + \rho_3 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix},
\]

(A.3)
To check feasibility of the output path together with the speed assignment with respect to state constraints, the state path $\xi(t)$ is derived by setting $z(x,0,0) = 0$, giving
\[ z_1 = 0 \Rightarrow x_1 = h^{-1}(y_2(0)) = \xi_1(0), \]
\[ z_2 = 0 \Rightarrow x_2 = x_1(\xi_1(0),0,0) = \xi_2(0,t). \]
\[ \vdots \]
\[ z_n = 0 \Rightarrow x_n = x_{n-1}(\xi_{n-1}(0),0,0) = \xi_n(0,t). \] (A.4)
The next lemma will help prove ISS of the noncompact sets under consideration.

**Lemma A.1.** Consider the noncompact set
\[ \mathcal{A} := \{(\xi_1, \xi_2) \in \mathbb{R}^n \times \mathbb{R}^n : \xi_1 = 0\} \] (A.5)
and the system
\[ \dot{x}_1 = f_1(x_1, x_2, \delta_1), \]
\[ \dot{x}_2 = f_2(x_1, x_2, \delta_2), \] (A.6)
where $(x_1, x_2) \in \mathbb{R}^n \times \mathbb{R}^n$ are the states, $(\delta_1, \delta_2) \in \mathcal{D}_1 \times \mathcal{D}_2 \subset \mathbb{R}^m \times \mathbb{R}^m$ are inputs with values in compact sets $\mathcal{D}_1, \mathcal{D}_2$, the vector fields $f_1$ and $f_2$ are smooth, and $f_1$ satisfies $f_1(0,\xi_2,0) = 0$ for all $\xi_2 \in \mathbb{R}^n$. If the following hold:

1. For each compact set $\mathcal{X} \subset \mathbb{R}^n$ and $\mathcal{Y} \subset \mathbb{R}^m$ there exist $L \geq 0$ and $\varepsilon \geq 0$ such that for all $\xi_1 \in \mathcal{X}$ and all $\mu \in \mathcal{Y}$,
\[ |f_2(\xi_1, x_2, \mu)| \leq L|x_2| + \varepsilon, \] (A.7)
2. There exist a smooth function $V : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}_{\geq 0}$ and $\mathcal{X}_\infty$-functions $x_i, i = 1, \ldots, 4$, such that
\[ x_i(|x_1|) \leq V(x_1, x_2) \leq x_2(|x_1|), \] (A.8)
\[ V(\xi_1, x_2)f_1(x_1, x_2, \delta_1) + V(\xi_2, x_2)f_2(x_1, x_2, \delta_2) \leq -x_3(|x_1|) + x_4(|\delta|) \] (A.9)
hold, where $\delta = [\delta_1^\top, \delta_2^\top]^\top$,

then system (A.6) is input-to-state stable with respect to the closed, 0-invariant set (A.5).

**Proof (Sketch).** Since
\[ \frac{d}{dt}V(x_1(t), x_2(t)) \leq -x_1(V(x_1(t), x_2(t))) + x_4(|\delta(t)|) \]
\[ \leq -\frac{1}{2}x_1(V(x_1(t), x_2(t))), \]
for all $V(x_1(t), x_2(t)) \geq V^{-1}(2x_4(|\delta(t)|))$
where $x_1 = x_3 \circ x_2^{-1} \in \mathcal{X}_\infty$ and $\delta$ is bounded, then $V(x_1(t), x_2(t))$, and consequently $x_1(t)$ from (A.8), is bounded on a maximal interval of existence $t \in [0, T)$. As a result, there exist some $L$ and $c$ such that $\forall t \in [0, T)$, $f_2(x_1(t), x_2(t), \delta_2(t))$ satisfies the linear growth condition (A.7), independently of $T$. This implies that for each $x_{20} = x_2(t) \in \mathbb{R}^m$, then $x_2(t)$ is bounded on $[0, T)$, independently of $T$. It follows that $T = +\infty$ and the system (A.6) is forward complete. By the assumption on $f_1$, it is seen that $\mathcal{A}$ is 0-invariant for (A.6). Recall the definition of an ISS-Lyapunov function and the fact that if a system admits an ISS-Lyapunov function with respect to the closed, 0-invariant set $\mathcal{A}$, then the system is ISS with respect to $\mathcal{A}$ (see e.g. Lin et al., 1995, Definition 3.2 and Proposition A.2). Since $\{(x_1, x_2) \in \mathcal{A} = |x_1|$, the function $V$ is an ISS-Lyapunov function for (A.6) with respect to (A.5), and the conclusion of the lemma follows. \hfill $\square$

**Proof of Theorem 3.3.** The Lyapunov function
\[ V = z^TPz + \frac{b_0}{2} \alpha^2 + \frac{1}{2\lambda} \alpha^2, \]
has a time-derivative
\[ \dot{V} \leq -z^TPz - \frac{1}{\mu} \alpha^2 =: Y_1(z, \omega_0, \omega_t) \leq 0, \] (A.10)
which implies UGS of $(z, \omega_0, \omega_t) = 0$, and there are no finite escape times. Define $W(z, \omega_0, \omega_t) := \omega_0 \omega_t$. Differentiating $W$ along the trajectories of (48) gives $\dot{W} \leq Y_2(z, \omega_0, \omega_t, t)$ where
\[ Y_2 := \alpha_0^2 - \lambda \omega_0 \omega_t - \lambda \mu_0 \omega_t \alpha^2, \]
\[ + 2\lambda \mu_1 \|P\| b(\xi_{n-1}, 0, t) \|z\| |\omega_t|. \]
With an abuse of notation, define $\phi(z, \omega_0, \omega_t) := b(\xi_{n-1}, 0, t)$ where the diffeomorphisms $z = \gamma(x - \xi(0,t))$ and $\omega_t = \omega - \delta - \phi(t)$ must be used. From Assumptions 2.1, $\phi$ is bounded since $|z|$ is bounded. We get that $Y_1 = 0 \Rightarrow Y_2 \leq 0$ for all bounded $(z, \omega_0, \omega_t)$ and further that $Y_1 = Y_2 = 0 \Rightarrow (z, \omega_0, \omega_t) = 0$. All the prerequisites of Matrosov’s Theorem as stated by Loria et al. (2002, Theorem 1) are then satisfied. \hfill $\square$

**References**


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