Chapter 5 – Seakeeping Theory

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Chapter 5 - Seakeeping Theory

Equations of Motion

Seakeeping theory is formulated in equilibrium (SEAKEEPING) axes \{s\} but it can be transformed to BODY axes \{b\} by including fluid memory effects represented by impulse response functions.

The transformation is done within a linear framework such that additional nonlinear viscous damping must be added in the time-domain under the assumption of linear superposition.

\[
\begin{align*}
\text{Inertia forces:} & \quad [M_{RB} + M_A] \ddot{v} + C_{RB}(v)v + C_A(v_r)v_r \\
\text{Damping forces:} & \quad + (D_p + D_V)v_r + D_n(v_r)v_r + \mu \\
\text{Restoring forces:} & \quad + g(\eta) + g_o \\
\text{Wind and wave forces:} & \quad = \tau_{wind} + \tau_{wave} \\
\text{Propulsion forces:} & \quad + \tau 
\end{align*}
\]

\(\mu\) is an additional term representing the fluid memory effects.
5.1 Hydrodynamic Concepts and Potential Theory

Strip Theory (2-D Potential Theory)

For slender bodies, the motion of the fluid can be formulated as a 2-D problem. An accurate estimate of the hydrodynamic forces can be obtained by applying strip theory (Newman, 1977; Faltinsen, 1990; Journee and Massie, 2001).

The 2-D theory takes into account that variation of the flow in the cross-directional plane is much larger than the variation in the longitudinal direction of the ship.

The principle of strip theory involves dividing the submerged part of the craft into a finite number of strips. Hence, 2-D hydrodynamic coefficients for added mass can be computed for each strip and then summed over the length of the body to yield the 3-D coefficients.

Commercial Codes: MARINTEK (ShipX-Veres) and Amarcon (Octopus Office)
VERES - VEssel RESponse program is a Strip Theory Program which calculates wave-induced loads on and motions of mono-hulls and barges in deep to very shallow water. The program is based on the famous paper by Salvesen, Tuck and Faltinsen (1970). Ship Motions and Sea Loads. Trans. SNAME.
ShipX (VERES) by MARINTEK
OCTOPUS SEAWAY by Amarcon

and AMARCON cooperate in further development of SEAWAY

The Maritime Research Institute Netherlands (MARIN) and AMARCON agree to cooperate in further development of SEAWAY. MARIN is an internationally recognized authority on hydrodynamics, involved in frontier breaking research programs for the maritime and offshore industries and navies.

SEAWAY is developed by Professor J. M. J. Journée at the Delft University of Technology

SEAWAY is a Strip Theory Program to calculate wave-induced loads on and motions of mono-hulls and barges in deep to very shallow water. When not accounting for interaction effects between the hulls, also catamarans can be analyzed. Work of very acknowledged hydromechanic scientists (such as Ursell, Tasai, Frank, Keil, Newman, Faltinsen, Ikeda, etc.) has been used, when developing this code.

SEAWAY has extensively been verified and validated using other computer codes and experimental data.
5.1 Hydrodynamic Concepts and Potential Theory

Panel Methods (3-D Potential Theory)

For potential flows, the integrals over the fluid domain can be transformed to integrals over the boundaries of the fluid domain. This allows the application of panel or boundary element methods to solve the 3-D potential theory problem.

Panel methods divide the surface of the ship and the surrounding water into discrete elements (panels). On each of these elements, a distribution of sources and sinks is defined which fulfill the Laplace equation.

Commercial code: WAMIT (www.wamit.com)
WAMIT® is the most advanced set of tools available for analyzing wave interactions with offshore platforms and other structures or vessels.

WAMIT® was developed by Professor Newman and coworkers at MIT in 1987, and it has gained widespread recognition for its ability to analyze the complex structures with a high degree of accuracy and efficiency.

Over the past 20 years WAMIT has been licensed to more than 80 industrial and research organizations worldwide.
5.1 Hydrodynamic Concepts and Potential Theory

Potential theory programs typically compute:

- Frequency-dependent added mass, $A(\omega)$
- Potential damping coefficients, $B(\omega)$
- Restoring terms, $C$
- 1st- and 2nd-order wave-induced forces and motions (amplitudes and phases) for given wave directions and frequencies
- ... and much more

One special feature of WAMIT is that the program solves a boundary value problem for zero and infinite added mass. These boundary values are particular useful when computing the retardation functions describing the fluid memory effects.

Processing of Hydrodynamic Data using MSS HYDRO – www.marinecontrol.org
The toolbox reads output data files generated by the hydrodynamic programs:

- ShipX (Veres) by MARINTEK AS
- WAMIT by WAMIT Inc.

and processes the data for use in Matlab/Simulink.
5.2 Seakeeping and Maneuvering Kinematics

Seakeeping Theory (Perturbation Coordinates)
The SEAKEEPING reference frame \{s\} is not fixed to the craft; it is fixed to the equilibrium state:

\[
\zeta = [\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6]^T
\]

\[
\xi = \Delta \eta
\]

\[
\dot{\xi} = \Delta \dot{\eta} = \Delta \nu
\]

\[
\Theta_{sb} = [\xi_4, \xi_5, \xi_6]^T = [\delta \phi, \delta \theta, \delta \psi]^T
\]

- In the absence of wave excitation, \{s\} coincides with \{b\}.
- Under the action of the waves, the hull is disturbed from its equilibrium and \{s\} oscillates, with respect to its equilibrium position.

Transformation between \{b\} and \{s\}

\[
\Delta \nu \approx \nu + U(L \Delta \eta - e_1)
\]

\[
\Delta \nu \approx \dot{\nu} + U L \nu
\]

\[
e_1 = [1, 0, 0, 0, 0, 0]^T
\]
5.3 The Classical Frequency-Domain Model

Seakeeping Analysis
The seakeeping equations of motion are considered to be inertial:

Equations of Motion

\[ \dot{\xi} = \delta \eta = [\delta x, \delta y, \delta z, \delta \phi, \delta \theta, \delta \psi]^T \]

\[ M_{RB} \ddot{\xi} = \tau_{hyd} + \tau_{hs} + \tau_{exc} \]

Cummins (1962) showed that the radiation-induced hydrodynamic forces in an ideal fluid can be related to frequency-dependent added mass \( A(\omega) \) and potential damping \( B(\omega) \) according to:

\[ \tau_{hyd} = -\bar{A} \ddot{\xi} - \int_0^t \bar{K}(t-\tau) \dot{\xi}(\tau)d\tau \]

\[ \bar{A} = A(\infty) \]

\[ \bar{K}(t) = \frac{2}{\pi} \int_0^\infty B(\omega) \cos(\omega t)d\omega \]
5.3 The Classical Frequency-Domain Model

Frequency-dependent added mass $A_{22}(\omega)$ and potential damping $B_{22}(\omega)$ in sway

\[ \bar{A} = A(\infty) \]
5.3 The Classical Frequency-Domain Model

\[ \tau_{\text{hyd}} = -\bar{A}\ddot{\xi} - \int_0^t \bar{K}(t - \tau)\dot{\xi}(\tau)d\tau \]

Matrix of retardation functions given by

\[ \bar{K}(t) = \frac{2}{\pi} \int_0^\infty \text{B}(\omega) \cos(\omega t)d\omega \]

Cummins Model

If linear restoring forces \( \tau_{hs} = -C\ddot{\xi} \) are included in the model, this results in the time-domain model:

\[ (M_{RB} + A(\infty))\ddot{\xi} + \int_0^t \bar{K}(t - \tau)\dot{\xi}(\tau)d\tau + C\ddot{\xi} = \tau_{\text{exc}} \]

The fluid memory effects can be replaced by a state-space model to avoid the integral.
5.3 The Classical Frequency-Domain Model

Longitudinal added mass coefficients as a function of frequency.
5.3 The Classical Frequency-Domain Model

Lateral added mass coefficients as a function of frequency.
5.3 The Classical Frequency-Domain Model

Longitudinal potential damping coefficients as a function of frequency. Exponential decaying viscous damping is included for $B_{11}$. 
5.3 The Classical Frequency-Domain Model

Lateral potential damping coefficients as a function of frequency.
Exponential decaying viscous damping is included for $B_{22}$ and $B_{66}$ while viscous IKEDA damping is included in $B_{44}$.
5.3.1 Potential Coefficients and the Concept of Forced Oscillations

In an experimental setup with a restrained scale model, it is possible to vary the wave excitation frequency $\omega$ and the amplitudes $f_i$ of the excitation force. Hence, by measuring the position and attitude vector $\eta$, the response of the 2nd-order order system can be fitted to a linear model:

\[
[M_{RB} + A(\omega)]\ddot{\xi} + B(\omega)\dot{\xi} + C\xi = f \cos(\omega t)
\]

for each frequency $\omega$.

The matrices $A(\omega), B(\omega)$ and $C$ represents a "hydrodynamic mass-damper-spring system" which varies with the frequency of the forced oscillation.

This model is rooted deeply in the literature of hydrodynamics and the abuse of notation of this false time-domain model has been discussed eloquently in the literature (incorrect mixture of time and frequency in an ODE).

Consequently, we will use Cummins time-domain model and transform this model to the frequency domain – no mixture of time and frequency!
5.3.2 Frequency-Domain Seakeeping Models

\[
(M_{RB} + A(\infty))\ddot{\xi} + \int_0^t \bar{K}(t - \tau)\dot{\xi}(\tau)d\tau + C\dot{\xi} = \tau_{\text{exc}}
\]

Cummins equation can be transformed to the frequency domain (Newman, 1977; Faltinsen 1990) by assuming that the vessel carries out harmonic oscillations in 6 DOF (see Section 5.4.1):

\[
\dot{\xi} = \cos(\omega t)i, \quad i = [1, 1, 1, 1, 1, 1]^T
\]

\[
-\omega^2 \left\{ [M_{RB} + A(\omega)] \cos(\omega t) - \omega \left\{ B_{\text{total}}(\omega) \sin(\omega \tau) d\tau \right\} \sin(\omega t)
\right. \\
\left. + C \cos(\omega t) = \tau_{\text{wind}} + \tau_{\text{wave}} + \delta \tau
\]

The potential coefficients \(A(\omega)\) and \(B(\omega)\) are usually computed using a seakeeping program but the frequency response will not be accurate unless viscous damping is included.

The optional viscous damping matrix \(B_\nu(\omega)\) can be used to model viscous damping such as skin friction, surge resistance and viscous roll damping (for instance IKEDA roll damping).
5.3.2 Frequency-Domain Seakeeping Models

Viscous frequency-dependent damping:

\[
-\omega^2 \{ [M_{RB} + A(\omega)] \cos(\omega t) - \omega \{ B_{total}(\omega) \sin(\omega \tau) d\tau \} \} \sin(\omega t) \\
+ C \cos(\omega t) = \tau_{\text{wind}} + \tau_{\text{wave}} + \delta \tau
\]

\[
B_{total}(\omega) = B(\omega) + B_{\nu}(\omega)
\]

\[
B_{\nu}(\omega) = \begin{bmatrix}
\beta_1 e^{-\alpha \omega} + N_{\text{ITTC}}(A_1) & 0 & 0 & 0 & 0 & 0 \\
0 & \beta_2 e^{-\alpha \omega} & 0 & 0 & 0 & 0 \\
0 & 0 & \beta_3 e^{-\alpha \omega} & 0 & 0 & 0 \\
0 & 0 & 0 & \beta_{\text{IKEDA}}(\omega) & 0 & 0 \\
0 & 0 & 0 & 0 & \beta_6 e^{-\alpha \omega} & 0 \\
0 & 0 & 0 & 0 & 0 & \beta_8 e^{-\alpha \omega}
\end{bmatrix}
\]

Viscous skin friction:

\[
\beta_i e^{-\alpha \omega}
\]

Quadratic ITTC drag:

\[
X = -X_{|u|u} |u|u \\
\approx N_{\text{ITTC}}(A_1)u
\]

Quadratic damping is approximated using describing functions (similar to the equivalent linearization method):

\[
u = A \sin(\omega t)
\]

\[
y = c_1 x + c_2 x |x| + c_3 x^3
\]

\[
y = N(A) u
\]

\[
N(A) = c_1 + \frac{8A}{3\pi} c_2 + \frac{3A^2}{4} c_3
\]
5.4 Time-Domain Models including Fluid Memory Effects

Cummins equation in SEAKEEPING coordinates
(linear theory which includes fluid memory effects)

Transform from SEAKEEPING to BODY coordinates
(linearized kinematic transformation)

Linear seakeeping equations in BODY coordinates
(fluid memory effects are approximated as state-space models)

Unified maneuvering and seakeeping model
(nonlinear viscous damping/maneuvering coefficients
are added manually)
5.4.1 Cummins Equation in SEAKEEPING Coordinates

Cummins (1962) Equation

\[(M_{RB} + \tilde{A})\ddot{\xi} + \int_{-\infty}^{t} \tilde{K}(t - \tau)\dot{\xi}(\tau) d\tau + \tilde{C}\xi = \tau_{\text{wind}} + \tau_{\text{wave}} + \delta \tau\]

The Ogilvie (1964) Transformation gives

\[\tilde{A} = A(\infty)\]

\[\tilde{K}(t) = \frac{2}{\pi} \int_{0}^{\infty} B_{\text{total}}(\omega) \cos(\omega t) d\omega\]

From a numerical point of view is it better to integrate the difference:

\[K(t) = \frac{2}{\pi} \int_{0}^{\infty} [B_{\text{total}}(\omega) - B_{\text{total}}(\infty)] \cos(\omega t) d\omega\]

This can be done by rewriting Cummins equation as:

\[[M_{RB} + A(\infty)]\ddot{\xi} + B_{\text{total}}(\infty)\dot{\xi} + \int_{0}^{t} K(t - \tau)\dot{\xi}(\tau) d\tau + C\xi = \tau_{\text{wind}} + \tau_{\text{wave}} + \delta \tau\]
5.4.2 Linear Time-Domain Seakeeping Equations in BODY Coordinates

\[
[M_{RB} + A(\infty)]\ddot{\xi} + B_{\text{total}}(\infty)\dot{\xi} + \int_0^t K(t - \tau)\dot{\xi}(\tau)d\tau + C\xi = \tau_{\text{wind}} + \tau_{\text{wave}} + \delta\tau
\]

It is possible to transform the time-domain representation of Cummins equation from \(\{s\}\) to \(\{b\}\) using the kinematic relationships:

\[
\begin{align*}
\delta v & \approx v + U(L\delta\eta - e_1) \\
\delta \dot{v} & \approx \dot{v} + ULv \\
\xi & = \delta\eta \\
\dot{\xi} & = \delta \dot{v}
\end{align*}
\]

This gives:

\[
[M_{RB} + A(\infty)][\dot{v} + UL\dot{v}] + B_{\text{total}}(\infty)[v + U(L\delta\eta - e_1)] + \int_0^t K(t - \tau)\delta\dot{v}(\tau)d\tau + C\delta\eta = \tau_{\text{wind}} + \tau_{\text{wave}} + (\tau - \bar{\tau})
\]

The steady-state control force \(\tau\) needed to obtain the forward speed \(U\) when \(\tau_{\text{wind}} = \tau_{\text{wave}} = 0\) and \(\delta\eta = 0\) is:

\[
\bar{\tau} = B_{\text{total}}(\infty)Ue_1
\]

Hence,

\[
[M_{RB} + A(\infty)][\dot{v} + UL\dot{v}] + B_{\text{total}}(\infty)[v + UL\delta\eta] + \int_0^t K(t - \tau)\delta\dot{v}(\tau)d\tau + C\delta\eta = \tau_{\text{wind}} + \tau_{\text{wave}} + \tau
\]
5.4.2 Linear Time-Domain Seakeeping Equations in BODY Coordinates

\[
[M_{RB} + A(\infty)][\ddot{v} + U L \dot{v}] + B_{total}(\infty)[v + U L \delta \eta] + \int_{0}^{t} K(t - \tau) \delta v(\tau) \, d\tau + C \delta \eta = \tau_{\text{wind}} + \tau_{\text{wave}} + \tau
\]

When computing the damping and retardation functions, it is common to neglect the influence of \( \delta \eta \) on the forward speed such that:

\[
\delta v \approx v + U(L \delta \eta - e_1) \approx v - U e_1
\]

Finally, let use replace \( v \) by the relative velocity \( v_r \) to include ocean currents and define:

\[
M = M_{RB} + M_A
\]
such that:

\[
M \ddot{v} + C_{RB}^* v + C_A^* v_r + D v_r + \int_{0}^{t} K(t - \tau)[v(\tau) - U e_1] \, d\tau + G \delta \eta = \tau_{\text{wind}} + \tau_{\text{wave}} + \tau
\]

where

\[
M_A = A(\infty),
C_A^* = U A(\infty) L,
C_{RB}^* = U M_{RB} L,
D = B_{total}(\infty),
G = C
\]

Linear Coriolis and centripetal forces due to a rotation of \{b\} about \{s\}.

Lecture Notes TTK 4190 Guidance and Control of Vehicles (T. I. Fossen)
5.4.2 Linear Time-Domain Seakeeping Equations in BODY Coordinates

Fluid Memory Effects
The integral in the following equation represents the fluid memory effects:

\[
M\ddot{v} + C_{RB}^*v + C_A^*v_r + Dv_r + \int_0^t K(t - \tau)[v(\tau) - Ue_1]d\tau + G\eta = \tau_{\text{wind}} + \tau_{\text{wave}} + \tau
\]

\[
\mu := \int_0^t K(t - \tau)[v(\tau) - Ue_1]d\tau
\]

Approximated by a state-space model

\[
\mu = H(s)[v - Ue_1]
\]

\[
\dot{x} = A_r x + B_r \delta v
\]

\[
\mu = C_r x
\]

Impulse response function

\[
K(t) = \frac{2}{\pi} \int_0^{\infty} [B(\omega) - B(\infty)] \cos(\omega t)d\omega
\]
5.4.3 Nonlinear Unified Seakeeping and Maneuvering Model with Fluid Memory Effects

Linear Seakeeping Equations (BODY coordinates)

\[ M \ddot{\mathbf{v}} + C_{RB}^* \mathbf{v} + C_A^* \mathbf{v}_r + D \mathbf{v}_r + \mu + G \eta = \tau_{\text{wind}} + \tau_{\text{wave}} + \tau \]

Unified Nonlinear Seakeeping and Maneuvering Model

- Use nonlinear kinematics
- Replace linear Coriolis and centripetal forces with their nonlinear counterparts
- Include maneuvering coefficients in a nonlinear damping matrix (linear superposition)

\[ \dot{\eta} = J_\Theta(\eta) \mathbf{v} \]

\[ M \ddot{\mathbf{v}}_r + C_{RB}(\mathbf{v}) \mathbf{v} + C_A(\mathbf{v}_r) \mathbf{v}_r + D(\mathbf{v}_r) \mathbf{v}_r + \mu + G \eta = \tau_{\text{wind}} + \tau_{\text{wave}} + \tau \]
5.5 Case Study: Identification of Fluid Memory Effects

The fluid memory effects can be approximated using frequency-domain identification. The main tool for this is the MSS FDI toolbox (Perez and Fossen 2009) - www.marinecontrol.org

When using the frequency-domain approach, the property that the mapping: $\tilde{\delta}v \rightarrow \mu$ has relative degree one is exploited. Hence, the fluid memory effects $\mu$ can be approximated by a matrix $H(s)$ containing relative degree one transfer functions:

$$\mu = H(s)\tilde{\delta}v$$

$$h_{ij}(s) = \frac{p_rs^r+p_{r-1}s^{r-1}+...+p_0}{s^n+q_{n-1}s^{n-1}+...+q_0}$$

$$r = n - 1, \quad n \geq 2$$

State-space model:

$$H(s) = C_r(sI - A_r)^{-1}B_r$$

$$\dot{x} = A_rx + B_r\tilde{\delta}v$$

$$\mu = C_rx$$
5.5.1 Frequency-Domain Identification using the MSS FDI Toolbox

Consider the FPSO data set in the MSS toolbox (FDI tool) and assumes that the infinite added mass matrix is unknown. Hence, we can estimate the fluid transfer function $h_{33}(s)$ by using the following Matlab code:

```matlab
load fpso
Dof = [3,3]; %Use coupling 3-3 heave-heave
Nf = length(vessel.freqs);
W = vessel.freqs(1:Nf-1)';
Ainf = vessel.A(Dof(1),Dof(2),Nf); % Ainf computed by WAMIT

A = reshape(vessel.A(Dof(1),Dof(2),1:Nf-1),1,length(W))';
B = reshape(vessel.B(Dof(1),Dof(2),1:Nf-1),1,length(W))';

FDIopt.OrdMax = 20;
FDIopt.AinfFlag = 0;
FDIopt.Method = 2;
FDIopt.Iterations = 20;
FDIopt.PlotFlag = 0;
FDIopt.LogLin = 1;
FDIopt.wsFactor = 0.1;
FDIopt.wminFactor = 0.1;
FDIopt.wmaxFactor = 5;

[KradNum,KradDen,Ainf] = FDIRadMod(W,A,0,B,FDIopt,Dof)
```
5.5.1 Frequency-Domain Identification using the MSS FDI Toolbox

FPSO identification results for $h_{33}(s)$ without using the infinite added mass $A_{33}(\infty)$. The left-hand-side plots show the complex coefficient and its estimate while added mass and damping are plotted on the right-hand-side.
5.5.1 Frequency-Domain Identification using the MSS FDI Toolbox

\[
h_{33}(s) = \frac{1.672e007 s^3 + 2.286e007 s^2 + 2.06e006 s}{s^4 + 1.233 s^3 + 0.7295 s^2 + 0.1955 s + 0.01639}
\]

\[
\mu = H(s)[\nu - U e_1]
\]

\[
\dot{x} = A_r x + B_r \delta \nu
\]

\[
\mu = C_r x
\]

\[
A_r = \begin{bmatrix}
-1.2335 & -0.7295 & -0.1955 & -0.0164 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\end{bmatrix}
\]

\[
B_r = \begin{bmatrix}
1 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

\[
C_r = \begin{bmatrix}
1.672e007 & 2.286e007 & 2.06e006 & 0 \\
\end{bmatrix}
\]

\[
D_r = 0
\]