Adaptive Control of Nonlinear Systems: A Case Study of Underwater Robotic Systems

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The problem of controlling underwater mobile robots in 6 degrees of freedom (DOF) is addressed. Underwater mobile robots where the number of thrusters and control surfaces exceeds the number of controllable DOF are considered in detail. Unlike robotic manipulators underwater mobile robots should include a velocity dependent thruster configuration matrix $B(\dot{q})$, which modifies the standard manipulator equation to: $M\ddot{q} + C(x, \dot{q})\dot{q} + g(x) = B(\dot{q})u$ where $\dot{x} = J(x)\dot{q}$. Uncertainties in the thruster configuration matrix due to unmodelled nonlinearities and partly known thruster characteristics are modelled as multiplicative input uncertainty. This paper proposes two methods to compensate for the model uncertainties: (1) an adaptive passivity-based control scheme and (2) deriving a hybrid (adaptive and sliding) controller. The hybrid controller combines the adaptive scheme where M , C and g are estimated on-line with a switching term added to the controller to compensate for uncertainties in the input matrix B. Global stability is ensured by applying Barbalat's Lyapunov-like lemma. The hybrid controller is simulated for the horizontal motion of the Norwegian Experimental Remotely Operated Vehicle (NEROV).

INTRODUCTION

Non-destructive testing of underwater structures require high performance manoeuvres of underwater mobile robots within and close to underwater installations. Until recently, remotely operated vehicles (ROVs) have been used as a platform for underwater robot manipulators. Now it is planned to use fully or partially autonomous underwater vehicles (AUVs) in such operations. This imposes stricter requirements on the control system particularly when macro-micro control i.e control of the combined motion between the AUV and robot manipulator is of interest. The schemes presented in this paper are intended for the macro-micro control of such systems. The case study includes a simulation of the macro controller.

In robotics, nonlinear adaptive controllers have given high performance for systems in the form: $M\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau$, Craig et al.[1], Sadegh and Horowitz[2], Slotine and Li[3] and Spong and Ortega[4]. For underwater vehicles means are sought to include the thruster hydrodynamics modelled as: $\tau = B(\dot{q})u$. Letting $\dot{x} = J(x)\dot{q}$ be a nonlinear transformation describing the vehicle's flight path relative an inertial reference frame, the standard manipulator equation is modified to a system in the form: $M\ddot{q}+C(x, \dot{q})\dot{q}+g(x) = B(\dot{q})u$. Usually, the number of inputs is equal or larger than the number of (DOF) to be controlled. For underwater vehicles the input vector may consist of a large number of thrusters and control surface inputs. When designing controllers for high performance underwater robotic systems it is necessary to compensate for model features such as nonlinear dynamics, nonlinear kinematics and nonlinearities due to hysteresis, actuator dead-zones and partly known thruster characteristics. Precise knowledge of all dynamic parameters are required to compensate for nonlinear underwater dynamics. This suggests a robust adaptive control scheme. This paper proposes two globally stable adaptive controllers for underwater robotic systems. Input uncertainties due to imprecise thruster characteristics are discussed in depth.

The paper is outlined as follows. The second section describes the equations of motion for underwater vehicles and considers thruster hydrodynamics. The third section discusses adaptive passivity-based control. Hybrid (adaptive and sliding) control of underwater robotic systems is examined in the fourth section, while results from the case study are presented in the last section. Our conclusions are given at the end of the paper.

NONLINEAR UNDERWATER VEHICLE EQUATIONS OF MOTION

In the following, it is convenient to define the operators:
$\mid x \mid = [\mid x_1 \mid, \mid x_2 \mid, ..., \mid x_n \mid]^T$
$sgn(x) = [sgn(x_1), sgn(x_2), ..., sgn(x_n)]^T$
$x \,.\!\times\, y = [x_1\, y_1, x_2\, y_2, ..., x_n\, y_n]^T$

Underwater Vehicle Dynamics and Kinematics

For underwater vehicles it is necessary to distinguish between vehicle dynamics, thruster dynamics and kinematics. The dynamic behavior of underwater vehicles is described through Newton's laws of linear and angular momentum. The equations of motion of such vehicles are highly nonlinear and coupled due to hydrodynamic added mass, lift and drag forces, which are acting on the vehicle. By applying the results from Fossen and Balchen[5] it is convenient to write the underwater vehicle equations of motion as:

- Vehicle dynamics:

  M $\ddot{q} + C(x,\dot{q})\dot{q} + g(x) + v(t) = \tau$ where $q \in \Re^n$ is the vehicle-fixed coordinates , $x \in \Re^n$ is refered to the earth-fixed reference frame and $\tau \in \Re^n$ is the control force and moment vector. M is an $n \times n$ inertia matrix, $C(x,\dot{q})\dot{q}$ is a nonlinear $n \times 1$ vector including rigid-body kinematic and hydrodynamic forces and moments, $g(x)$ is an $n \times 1$ vector including restoring forces and moments and $v(t)$ is an $n \times 1$ vector of environmental disturbances. These terms are described more closely in Fossen and Balchen[5] and Lewis et al.[6].

- Thruster hydrodynamics:

  $\tau = B(\dot{q})u$ where $u = \omega \,.\!\times\, |\omega|$ $\omega \in \Re^p$ is a vector of propeller angular velocities and $B(\dot{q})$ is an $n \times p$ velocity dependent thruster configuration matrix which will be given a more detailed interpretation in the next section.

- Actuator dynamics:

  $\dot{\omega}_i = \alpha_i\, \omega_i + \beta_i v_{S_i}$, $i = 1..p$ where the subscript i denotes thruster i , v_{S_i} is the input voltage and the coefficients α_i and β_i are described in the next Sections.

- Kinematics:

  $\dot{x} = J(x)\dot{q}$ where $J(x)$ is an $n \times n$ transformation matrix describing the vehicle's flight path relative to the earth-fixed reference frame.

The state vectors: $\dot{q} = [u, v, w, p, q, r]^T$, where q is a virtual vector and $x = [x, y, z, \phi, \theta, \psi]^T$ are defined according to the Society of Naval Architects and Marine Engineers (SNAME) notation:

Vehicle-fixed velocities:	**Earth-fixed positions/orientations:**
u = surge velocity	x = surge position
v = sway velocity	y = sway position
w = heave velocity	z = heave position
p = roll angular velocity	ϕ = roll angle
q = pitch angular velocity	θ = pitch angle
r = yaw angular velocity	ψ = yaw angle

Thruster Hydrodynamics

Speed-controlled motors require an angular speed sensor e.g. a tachometer which allows us to design a high bandwidth inner loop with feedback from the propeller angular velocity. This is described in the next section.
Small underwater vehicles usually operate over a considerable speed range with no specific speed dominating. For such vehicles the performance of the ducted thrusters will be a function of advance velocity V_A at the propeller, propeller revolutions n and propeller diameter D. The non-dimensional open water characteristics are defined in terms of the open water advance coefficient J_o , Dand and Every[7] :

$$J_o = V_A n D$$

The non-dimensional thrust and torque coefficients K_T and K_Q and thruster open water efficiency η_o are defined as:
$K_T = T\rho n^2 D^4$; $K_Q = Q\rho n^2 D^5$; $\eta_o = J_o 2\pi \cdot K_T K_Q$
where ρ is the water density and T and Q are the propeller thrust and torque, respectively. By carrying out a cavitation tunnel test a unique curve, where J_o is plotted against K_T and K_Q, is obtained for each propeller. Such a curve is shown in Fig. 1.

xxx FIGURE 1 xxx

Figure 1: Non-dimensional thruster characteristics K_T , K_Q and η_o as a function of positive advance coefficient J_o (ahead direction)

A similar curve is obtained if K_T is plotted versus the negative advance coefficient (astern direction). The curves for negative and positive advance coefficients are usually not symmetrical. Based on the curves for the non-dimensional thruster coefficient K_{T_i} it is straightforward to plot the thrust force T_i as a function of speed of advance V_A and propeller angular velocity $\omega_i = 2\pi\, n_i$. This is shown in Fig. 2.

xxx FIGURE 2 xxx

Figure 2: Thruster force T_i as a function of propeller angular velocity ω_i for different speeds of advance V_A.

From Eq. () we obtain:

$$T_i = K_{T_i}(V_A, n)\ \rho D^4 n^2$$

Based on this we propose to approximate the nonlinear characteristics shown in Fig. 2 as:
$T_i \approx b_i(V_A)\ \omega_i|\omega_i| \quad , \quad i = 1..p$
with b_i as a velocity dependent scalar function and ω_i as the propeller angular velocity. The particular choices: $b_i(1) = 0.007$, $b_i(2) = 0.028$ and $b_i(3) = 0.064$ gave the following curves when plotted (solid) together with the experimental data in Fig. 2 (x-marked).

xxx FIGURE 3 xxx

Figure 3: Nonlinear approximations of thruster characteristics

The approximation, Eq. (), shows good agreement with the experimental data. Other experiments based on similar approximations also verify this. The relationship between vehicle speed V and advance velocity at the propeller V_A is:

$$V_A = (1 - w)V$$

where w is the wake fraction number. As a result of this, the thruster forces and moments τ in Eq. () are written as:

$$\tau = B(\dot{q})u \quad \text{where} \quad u = \omega \,.\!\times |\omega|$$

with B as a thruster configuration matrix depending on the vehicle velocity V i.e. the three first components u, v and w of the velocity vector $\dot{q}$. The non-zero elements in B are determined from the functions b_i. The uncertainties in the experimental data and the nonlinear approximation of the experimental data suggest an adaptive control scheme to compensate for the uncertainties in the b_i elements. Cavitation tunnel tests are useful as a priori information for the adaptive control scheme.

Actuator Dynamics

The influence of actuator dynamics is usually reduced by designing a high bandwidth inner loop. For underwater vehicles this may be done by measuring the propeller angular velocity vector ω. It is well known that a DC motor can be described as, Fitzgerald et al.[8]:
L d idt $=$ - R i - $K_M\omega + v_S$
$Jd\omega dt = K_M i - Q$

where i is the current, v_S is the input voltage, L is the inductance, R is the resistance and J is the moment of inertia. The field current i_f is assumed to be constant and $K_M = k_f i_f$ where k_f is a motor constant. Q is the propeller torque defined in Eq. (). By assuming $didt = 0$, Eq. (), may be written as Eq. (). The easiest way to obtain the desired angular velocity vector ω_d is to apply a PI-controller:

$$v_S = K_P e + K_I \int_0^t e(\tau)\, d\tau \quad \text{where} \quad e = \omega - \omega_d$$

Here K_P and K_I are diagonal positive definite design matrices of appropriate dimensions. If the PI-controller yields poor performance e.g. as a result of non-linearities and hysteresis a model based controller should be considered. Adaptive sliding controllers for systems with nonlinear actuator dynamics are described in Yoerger et al.[9].

Optimal Distribution of Thruster Forces

For underwater vehicles where $p \geq n$, i.e. equal or more control inputs than controllable DOF, it is possible to find an optimal distribution of thruster forces and also control surface forces, for each DOF. Consider the energy cost function:

$$\text{Min } J = 12u^T W u \qquad \text{subject to} \qquad \tau = Bu$$

where W is positive definite, usually diagonal energy weighting matrix. For underwater vehicles which have both control surfaces and thrusters, the elements in W should be selected such that the use of control surfaces are much more inexpensive than the use of thrusters i.e. providing a means of saving battery energy . Defining the Lagrangian:

$$L(u, \lambda) = 12u^T W u + \lambda^T(\tau - Bu)$$

where the parameter vector λ is the Lagrange multipliers. Differentiating the Lagrangian L with respect to u yields:

$$\nabla_u L = Wu - B^T \lambda = 0$$

From this we obtain:
u = W$^{-1} B^T \lambda$
By using the fact that:

$$\tau = Bu = BW^{-1} B^T \lambda$$

and assuming that $BW^{-1}B^T$ is nonsingular, we find the following optimal solution for the Lagrange multiplicators:

$$\lambda = (BW^{-1}B^T)^{-1}\tau$$

Substituting this result into Eq. () yields the generalized inverse:
B$^+_W = W^{-1} B^T (BW^{-1} B^T)^{-1}$
In the case when all inputs are equality weighted, i.e. $W = I$, Eq. () simplifies to:

$$B^+ = B^T (BB^T)^{-1}$$

Notice that for the square case B^+ is simply equal to B^{-1}.

ADAPTIVE CONTROL OF NONLINEAR SYSTEMS

The results in Slotine and Li[3] and Slotine and Benedetto[10] may be extended from robot manipulator and spacecraft systems to underwater robotic vehicles by assuming multiplicative input uncertainty. We will restrict our treatment to systems with equal or more control inputs than controllable DOF, i.e. $p \geq n$.

Review of Adaptive Passivity-Based Control

Let us again consider the underwater vehicle equations of motion, Eq. () and Eq. (), which may be written as:

$$M^*(x)\ddot{x} + C^*(x,\dot{x})\dot{x} + g^*(x) + v^*(x,t) = J^{-T}(x)\tau$$

where
$\mathrm{M}^*(x) = J^{-T}(x)MJ^{-1}(x)$
$C^*(x,\dot{x}) = J^{-T}(x)\left[C(x,\dot{q}) - MJ^{-1}(x)\dot{J}(x)\right]J^{-1}(x) \quad \text{where} \quad \dot{q} = J^{-1}(x)\,\dot{x}$
$g^*(x) = J^{-T}(x)g(x)$
$v^*(x,t) = J^{-T}(x)v(t)$
Assume the desired trajectory : $\ddot{x}_d$, $\dot{x}_d$ and x_d to be bounded and $v = 0$. Let $\tilde{x} = x - x_d$ be the tracking error. Slotine and Li[3] suggest defining a measure of tracking s as:
$\mathrm{s} = \dot{\tilde{x}} + \lambda\tilde{x}$
where λ is a strictly positive constant which may be interpreted as the control bandwidth. It is convenient to rewrite Eq. () as:

$$s = \dot{x} - \dot{x}_r \quad \text{where} \quad \dot{x}_r = \dot{x}_d - \lambda\tilde{x}$$

It is important to notice that the terms M^* , C^* and g^* are linear in their parameters. These unknown parameters may be lumped together into a parameter vector θ. Let $\hat{\theta}$ be the time-varying parameter vector estimate and let $\tilde{\theta} = \hat{\theta} - \theta$ be the parameter error vector. To proof global stability Slotine and Benedetto[10] suggest using a Lyapunov-like function:

$$V(s,\tilde{\theta},t) = 12s^TM^*s + 12\tilde{\theta}^T\Gamma\tilde{\theta}$$

where Γ is a symmetric positive definite weighting matrix of appropriate dimension. Differentiating V with respect to time and using the skew symmetry property $\dot{x}^T(\dot{M}^* - 2C^*)\dot{x} = 0$ yields:
$\dot{V} = s^T(J^{-T}\tau - M^*\ddot{x}_r - C^*\dot{x}_r - g^*) + \dot{\tilde{\theta}}^T\Gamma\tilde{\theta}$
Let the control law be:
$\tau = J^T(\hat{M}^*\ddot{x}_r + \hat{C}^*\dot{x}_r + \hat{g}^* - K_Ds)$
where the hat denotes the adaptive estimates and K_D is a symmetric positive definite design matrix of appropriate dimension. Combining Eqs. () and () yields:
$\dot{V} = -s^TK_Ds + s^T(\tilde{M}^*\ddot{x}_r + \tilde{C}^*\dot{x}_r + \tilde{g}^*) + \dot{\tilde{\theta}}^T\Gamma\tilde{\theta}$
where $\tilde{M}^* = \hat{M}^* - M^*$, $\tilde{C}^* = \hat{C}^* - C^*$ and $\tilde{g}^* = \hat{g}^* - g^*$. Using the parameterization of Slotine and Benedetto[10], namely:
$\mathrm{M}^*\ddot{x}_r + C^*\dot{x}_r + g^* = \Phi^*(x,\dot{x},\dot{x}_r,\ddot{x}_r)\,\theta$

where $\Phi^*(x, \dot{x}, x_r, \dot{x}_r)$ is a known regressor matrix of appropriate dimensions, Eq. () is written as:

$$\dot{V} = -s^T K_D s + \tilde{\theta}^T (\Gamma \dot{\tilde{\theta}} + \Phi^{*\,T} s)$$

Assuming $\dot{\theta} = 0$, yields the parameter adaption law:
$\dot{\hat{\theta}} = -\Gamma^{-1} \Phi^{*\,T}(x, \dot{x}, x_r, \dot{x}_r) s$
which implies that: $\dot{V} = -s^T K_D s \leq 0$. Applying Barbalat's lemma shows that $s \rightarrow 0$ and thus $\tilde{x} \rightarrow 0$.

Reparameterization of the Adaptive Scheme

Before we discuss an extension to underwater robotic systems, it is worth noticing that the parameterization of Slotine and Benedetto[10], Eq. (), may be simplified in terms of M^* , C^* and g^*, Fossen[11]. Define a virtual vector $\dot{q}_r$ which satisfies the transformation:
$\dot{x}_r = J(x) \dot{q}_r$
implies that the virtual reference vectors $\dot{q}_r$ and $\ddot{q}_r$ are calculated as:
$\dot{q}_r = J^{-1}(x) \dot{x}_r$
$\ddot{q}_r = J^{-1}(x) \ddot{x}_r + J^{-1}(x) \dot{J}(x) J^{-1}(x) \dot{x}_r$
We now notice that Eq. () may be rewritten as:

$$J^T \left[M^* \ddot{x}_r + C^* \dot{x}_r + g^* \right] = M \ddot{q}_r + C \dot{q}_r + g = \Phi(x, \dot{q}, \dot{q}_r, \ddot{q}_r)\, \theta$$

By using q_r instead of x_r , the known coordinate transformation matrix $J(x)$ which strongly complicates the regressor matrix $\Phi^*(x, \dot{x}, \dot{x}_r, \ddot{x}_r)$ is eliminated in the new regressor matrix $\Phi(x, \dot{q}, \dot{q}_r, \ddot{q}_r)$. This implies that Eq. simply is written as:
$\dot{V} = s^T J^{-T} (\tau - M \ddot{q}_r - C \dot{q}_r - g) + \dot{\tilde{\theta}}^T \Gamma \tilde{\theta}$
The control law Eq. () then simplifies to:
$\tau = \hat{M} \ddot{q}_r + \hat{C} \dot{q}_r + \hat{g} - J^T K_D s$
while the adaption law Eq. () is modified to:

$$\dot{\hat{\theta}} = -\Gamma^{-1} \Phi^T (x, \dot{q}, \dot{q}_r, \ddot{q}_r) J^{-1}(x)\, s$$

which again implies that: $\dot{V} = -s^T K_D s \leq 0$. Notice that the new control law is written in terms of $\hat{M}$, $\hat{C}$ and $\hat{g}$ instead of $\hat{M}^*$, $\hat{C}^*$ and $\hat{g}^*$. This implies that it is easy to switch between a position and velocity scheme because the structure of the new regressor matrix will be equal in both cases.

Adaptive Control of Underwater Robotic Systems

To extend the results for spacecrafts to underwater robotic systems where $B(\dot{q})$ is an unknown $n \times p$ input matrix, we introduce the following multiplicative input uncertainty description:
$\tau = B(\dot{q}) u \quad , \quad B(\dot{q}) = (I + \Delta) B_o(\dot{q}) \quad , \quad \Delta \in \{\Delta : \sigma(\Delta) < 1\}$
where $u \in \Re^p$, $\tau \in \Re^n$ and $p \geq n$. Δ is an unknown $n \times n$ perturbation matrix, $\sigma(\Delta)$ is the maximum singular value of Δ and $B_o(\dot{q})$ is a known $n \times p$ matrix representing the thruster and control surface hydrodynamics. For underwater vehicles $B_o(\dot{q})$ should be based on

experimental results from cavitation tunnel tests. It is sought to control the propeller angular velocity vector ω which is related through:

$$u_i = \omega_i|\omega_i| \quad , \quad \omega_i = sgn(u_i)\ (|u_i|)^{1/2} \quad , \quad i = 1..p$$

Substituting Eq. () into Eq. () yields:

$$\dot{V} = s^T J^{-T}\left[(I+\Delta)B_o u - M\ddot{q}_r - C\dot{q}_r - g\right] + \dot{\tilde{\theta}}^T \Gamma\tilde{\theta}$$

Defining:
$M_\Delta = (I+\Delta)^{-1}M$
$C_\Delta = (I+\Delta)^{-1}C$
$g_\Delta = (I+\Delta)^{-1}g$
$\dot{V}$ may be written as:

$$\dot{V} = s^T J^{-T}(I+\Delta)\left[B_o u - M_\Delta\ddot{q}_r - C_\Delta\dot{q}_r - g_\Delta\right] + \dot{\tilde{\theta}}^T \Gamma\tilde{\theta}$$

Taking the control law to be:
$u = B_o^+\left[\hat{M}_\Delta\ddot{q}_r + \hat{C}_\Delta\dot{q}_r + \hat{g}_\Delta - J^T K_D s\right] = B_o^+\left[\Phi_\Delta\hat{\theta} - J^T K_D s\right]$
where B_o^+ is a generalized inverse satisfying $B_o B_o^+ = I$ and where the regressor matrix $\Phi_\Delta(x, \dot{q}, \dot{q}_r, \ddot{q}_r)$ is found from:

$$M_\Delta\ddot{q}_r + C_\Delta\dot{q}_r + g_\Delta = \Phi_\Delta(x, \dot{q}, \dot{q}_r, \ddot{q}_r)\,\theta$$

The adaption law:

$$\dot{\hat{\theta}} = -\Gamma^{-1}\Phi_\Delta^T(x, \dot{q}, \dot{q}_r, \ddot{q}_r)J^{-1}(x)\,s$$

yields:

$$\dot{V} = -s^T J^{-T}(I+\Delta)J^T K_D s \leq 0$$

where we have used the fact that $\sigma(\Delta) < 1$ implies that $(I+\Delta) > 0$ and thus:

$$J^{-T}(I+\Delta)J^T > 0$$

i.e. positiveness of a matrix is invariant of scaling. As in the previous case $\dot{V} \leq 0$ implies that $s \to 0$ and thus $\tilde{x} \to 0$. Not surprisingly if $\Delta = 0$ and $B_o = I$, the control law, Eq. (), is equal to the control law Eq. (). If the uncertainty matrix Δ is chosen as a full matrix, the structure of Eq. () may be quite complicated. In such cases the scheme presented in the next section is advantageous.

HYBRID ADAPTIVE CONTROL OF NONLINEAR SYSTEMS

Previous work on sliding mode control of underwater vehicles, Yoerger and Slotine[12] does not compensate for the time-varying behaviour of the control input matrix due to the thruster hydrodynamics, i.e. $\tau = B(\dot{q})u$. In this section we will derive a hybrid (adaptive and sliding) control scheme which compensates for the uncertainty in the B matrix by adding a discontinuous term to the existing adaptive control law. We will consider both position control x as well as the velocity control $\dot{q}$ of underwater vehicles.

Position Control

Let us again consider the 6 DOF equations of motion for an underwater vehicle i.e.

$$M^*(x)\ddot{x} + C^*(x,\dot{x})\dot{x} + g^*(x) + v^*(x,t) = J^{-T}(x)\tau \quad , \quad \tau = B(\dot{q})u \quad , \quad u = \omega \,.\!\times |\omega|$$

Assume that the thruster configuration matrix B satisfies a multiplicative uncertainty:
B $(\dot{q}) = (I + \Delta)B_o(\dot{q}) \quad , \quad |\Delta_{ij}| \leq D_{ij} \quad , \quad i = 1..n \;\; , \;\; j = 1..n$
and that the environmental disturbances e.g. sea currents are bounded as:
— v$_i| < \delta_i \quad , \quad \delta_i > 0 \quad i = 1..n$
A Lyapunov-like function candidate:

$$V(s,\tilde{\theta},t) = 12 s^T M^* s + 12 \tilde{\theta}^T \Gamma \tilde{\theta}$$

yields the following expression for $\dot{V}$:

$$\dot{V} = s^T J^{-T}\left[(I+\Delta)B_o u - M\ddot{q}_r - C\dot{q}_r - g - v\right] + \dot{\tilde{\theta}}^T \Gamma \tilde{\theta}$$

Letting the control law be: $\omega_i = sgn(u_i)\,(|u_i|)^{1/2}$, where:
u $=$ B$_o^+ \left[\hat{M}\ddot{q}_r + \hat{C}\dot{q}_r + \hat{g} - J^T K_D s - k \,.\!\times sgn(J^{-1}s)\right]$
Here we have included a switching term $k \,.\!\times sgn(J^{-1}s)$ to compensate for the uncertainty in the B matrix. Conditions on the non-negative switching gain vector k are found by using the linear parameterization:

$$M\ddot{q}_r + C\dot{q}_r + g = \Phi(x,\dot{q},\dot{q}_r,\ddot{q}_r)\,\theta$$

where $\Phi(x,\dot{q},\dot{q}_r,\ddot{q}_r)$ is the regressor matrix. We further select the adaption law as:

$$\dot{\hat{\theta}} = -\Gamma^{-1}\Phi^T(x,\dot{q},\dot{q}_r,\ddot{q}_r)J^{-1}(x)\,s$$

which implies that $\dot{V}$ may be written as:

$$\dot{V} = -s^T K_D s + (J^{-1}s)^T\left[\Delta(\Phi\hat{\theta} - J^T K_D s) - (I+\Delta)\;k \,.\!\times sgn(J^{-1}s) - v\right]$$

The particular choice $k_i \geq k'_i \quad \forall\, i$ where k' satisfies:
(I-D) $k' = D\,|\Phi\hat{\theta} - J^T K_D s| + \delta + \eta \quad , \quad \eta_i > 0 \quad , \quad i = 1..n$
where the elements D_{ij} and δ_i are defined in Eqs. () and () while the matrix D is defined as:

$$\mathrm{D} = \begin{bmatrix} D_{11} & -D_{12} & \ldots & -D_{1n} \\ -D_{21} & D_{22} & & -D_{2n} \\ \vdots & & \ddots & \vdots \\ -D_{n1} & -D_{n2} & \ldots & D_{nn} \end{bmatrix}$$

yields:

$$\dot{V} \leq -s^T K_D s - \eta^T |J^{-1}s| \leq 0$$

Applying Barbalat's lemma implies that $s \to 0$ and thus $x \to 0$. According to the Frobenius-Perron lemma, see e.g. Slotine and Li[13], the existence of a unique k vector is guaranteed, namely:

$$k' = (I-D)^{-1}\left[D\,|\Phi\hat{\theta} - J^T K_D s| + \delta + \eta\right]$$

Notice, that the design matrix K_D directly accelerates the convergence rate. Slotine and Li suggest smoothing out the control law within boundary layers to avoid chattering, which is imposed by the discontinuous term $k \,.\!\times sgn(J^{-1}s)$. This is described in detail in Slotine and Li[13].

Velocity Control

Instead of controlling the earth-fixed position and orientation x, it is often sufficient to control the vehicle-fixed linear and angular velocities $\dot{q}$. The position scheme derived in previous section is quite easy to modify to a velocity scheme. Consider the velocity model:

$$M\ddot{q} + C(q,\dot{q})\dot{q} + g(x) + v(t) = B(\dot{q})u \quad , \quad u = \omega \;.\!\times |\omega|$$

where the states $\dot{q}$ and x are assumed as measured. Further, assume the desired velocity vector $\dot{q}_d$ to be bounded and define the velocity error vector $\dot{\tilde{q}} = \dot{q} - \dot{q}_d$. Define a Lyapunov-like function candidate:

$$V(\dot{\tilde{q}}, \tilde{\theta}, t) = 12\dot{\tilde{q}}^T M \dot{\tilde{q}} + 12\tilde{\theta}^T \Gamma \tilde{\theta}$$

and let the control law be chosen as: $\omega_i = sgn(u_i)\,(|u_i|)^{1/2}$, where:

$$u = B_o^+ \left[\hat{M}\ddot{q}_d + \hat{C}\dot{q}_d + \hat{g} - K_D\dot{\tilde{q}} - k \;.\!\times sgn(\dot{\tilde{q}})\right]$$

and the unknown parameters are updated as:

$$\dot{\hat{\theta}} = -\Gamma^{-1}\Phi^T(x, \dot{q}, \dot{q}_d, \ddot{q}_d)\,\dot{\tilde{q}}$$

The structure of the regressor matrix $\Phi(x, \dot{q}, \dot{q}_d, \ddot{q}_d)$ is unchanged from the position scheme but q_r has been replaced with q_d. Letting $k_i \geq k_i' \;\; \forall\, i$, where k' satisfies:

$$k' = (I - D)^{-1}\left[D\,|\Phi\hat{\theta} - K_D\dot{\tilde{q}}| + \delta + \eta\right] \quad , \quad \eta_i > 0 \quad , \quad i = 1..n$$

with D defined as in Eq. () yields:

$$\dot{V} \leq -\dot{\tilde{q}}^T K_D \dot{\tilde{q}} - \eta^T|\dot{\tilde{q}}| \leq 0$$

which by applying Barbalat's lemma implies that $\dot{\tilde{q}} \to 0$.

SIMULATION STUDY

Simplified Model of the Horizontal Motion of the NEROV Vehicle

The simulation study is based on a simplified model of the Norwegian Experimental Remotely Operated Vehicle (NEROV), Fig. 4. The NEROV vehicle is an experimental energy autonomous underwater vehicle which is designed at the Division of Engineering Cybernetics at the Norwegian Institute of Technology. A brief sketch of the vehicle's general arrangement is shown in Fig. 4.

Figure 4: General arrangement of the NEROV vehicle

The vehicle will be controllable in all 6 DOF. The propulsion system is based on 6 DC permanent magnet motors with propeller angular velocity measurements. To demonstrate the adaptive controller, we will consider a simplified model describing the horizontal motion

of the NEROV vehicle i.e. the coupled motion in surge, sway and yaw. Consider the NEROV equations of motion, Sagatun and Fossen[14] :
M $\ddot{q} + C(\dot{q})\dot{q} = B(\dot{q})u$
$\dot{x} = J(x)\dot{q}$
where $\dot{q} = [u, v, r]^T$, $x = [x, y, \psi]^T$ and $u_i = \omega_i|\omega_i|$. The subscript i corresponds to thruster $T_1 - T_4$ in Fig. 4 . M and C were approximated as:

$$M = \begin{bmatrix} 186 & 0 & 0 \\ 0 & 268 & 0 \\ 0 & 0 & 29 \end{bmatrix} \quad ; \quad C\dot{q} = \begin{bmatrix} 119u|u| - 268vr \\ 208v|v| + 213ur \\ 15r|r| \end{bmatrix}$$

All thrusters are of the same type in the NEROV vehicle. In the computer simulations we used the following nonlinear thruster characteristic for the 4 actual thrusters:

$$T_i(V_A) = b(V_A)\,\omega_i|\omega_i| \quad \text{where} \quad b(V_A) = 0.03V_A - 0.025 \quad , \quad i = 1..4$$

The wake fraction number was chosen as: $w = 0.2$ for each thruster, which implies that: $V_A = 0.8V$. This suggests the following thruster configuration matrix:

$$B_o = \begin{bmatrix} b(u) & b(u) & 0 & 0 \\ 0 & 0 & -b(v) & b(v) \\ -\alpha\, b(u) & \alpha\, b(u) & 0 & 0 \end{bmatrix} = \begin{bmatrix} b(u) & 0 & 0 \\ 0 & b(v) & 0 \\ 0 & 0 & b(u) \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ -\alpha & \alpha & 0 & 0 \end{bmatrix}$$

where $\alpha = 0.4$ is the moment arm. The transformation matrix for surge, sway and yaw is simply:

$$J = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Simulation Results

In the computer simulations the uncertainties in the thruster characteristics were chosen as a diagonal matrix Δ with diagonal elements $[0.3\ -0.5\ 0.4]$ due to the diagonal structure of the uncertainties in B_o. The initial values for the parameter estimates were chosen as zero. The sampling rate was set at 10 Hz. Figure 5 shows the time responses for the commanded inputs x_d, y_d and ψ_d. The propeller angular velocities ω_{1-4}, Figure 6, are calculated from the hybrid control law, Eq. (). The simulations show that the tracking errors $e_x = x - x_d$, $e_y = y - y_d$ and $e_\psi = \psi - \psi_d$ are less than 0.05 for all DOF, which is quite satisfactory. All tracking errors converge to zero.

xxx FIGURE 5-6 xxx

CONCLUSIONS

Two adaptive controllers for nonlinear robotic systems have been presented in this paper. The first controller is an extension of an adaptive passivity-based controller for robot

manipulators and spacecrafts to nonlinear underwater robotic systems. The second scheme is a hybrid controller utilizing both the results from the adaptive controller and the theory of sliding mode control. Systems where the number of inputs is equal or larger than the number of (DOF) to be controlled are discussed in depth.
Macro-micro control of the combined motion between an underwater vehicle and a robot manipulator is mentioned as an application for nonlinear adaptive control. Such systems are useful e.g. in non-destructive testing of underwater structures. The macro controller i.e. the control of the underwater vehicle is described in detail. Uncertainties in the vehicle thruster configuration matrix are particularly considered. The paper shows how an adaptive and hybrid (adaptive and sliding) controller can exploit the nonlinear thruster characteristics found from open water tests. The hybrid controller is demonstrated in the simulation study of the NEROV vehicle.

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References

[1] J. J. Craig, P. Hsu. and S. Sastry, "Adaptive Control of Mechanical Manipulators," *IEEE Int. Conf. on Robotics and Automation*, San Francisco, April, (1986).

[2] N. Sadegh, and R. Horowitch, "Stability Analysis of an Adaptive Controller for Robotic Manipulators," *in proceedings IEEE Int. Conf. on Robotics and Automation*, Raleigh, North Carolina, 1987.

[3] J. J. E. Slotine and W. Li, "Adaptive Manipulator Control: A Case Study," *in proceedings IEEE Int. Conference on Robotics and Automation*, 1987, pp. 1392-1401.

[4] M. W. Spong and R. Ortega, "On Adaptive Inverse Dynamics Control of Rigid Robots," *IEEE Trans. Automatic Control*, **AC-35**, No.1, 92-95 (1990).

[5] T. I. Fossen and J. G. Balchen, "Modelling and Nonlinear Self-Tuning Robust Trajectory Control of an Autonomous Underwater Vehicle," *Modelling, Identification and Control*, **MIC-9**, No. 4, 165-177 (1988).

[6] D. J. Lewis, J. M. Lipscombe and P. C. Thomasson, "The Simulation of Remotely Operated Vehicles," *in proceedings ROV'84*, 1984, pp. 245-251.

[7] I. Dand and M. J. Every, "An Overview of the Hydrodynamics of Umbilical Cables and Vehicles", *SUBTECH' 83*, Paper No. 10.4., 1983.

[8] A. E. Fitzgerald, C. Kingsley Jr. and S. D. Umans, *Electric Machinery*, McGraw-Hill International, 1983.

[9] D. R. Yoerger, J. G. Cooke and J. J. E. Slotine, “The Influence on Thruster Dynamics on Underwater Vehicle Behavior and Their Incorporation Into Control Systems Design,” *IEEE Journal of Oceanic Engineering*, **OE-15**, No. 3, 167-179 (1990).

[10] J. J. E. Slotine and M. D. Di Benedetto, “Hamiltonian Adaptive Control of Spacecraft,” *IEEE Trans. Automatic Control*, **AC-35**, No. 7, 848-852, 1990.

[11] T. I. Fossen, “Comments on: Hamiltonian Adaptive Control of Spacecraft”. *Submitted to IEEE Trans. Automatic Control* (1991).

[12] D. R. Yoerger and J. J. E. Slotine, “Robust Trajectory Control of Underwater Vehicles,” *IEEE Journal of Oceanic Engineering*, **OE-10**, No. 4, 462-470 (1985).

[13] J. J. E. Slotine and W. Li, *Applied Nonlinear Control*, Prentice-Hall, 1990.

[14] S. I. Sagatun and T. I. Fossen, “Design Study of The Norwegian Experimental Remotely Operated Vehicle,” *Division of Engineering Cybernetics, Norwegian Institute of Technology*, Report No. 90-27-U, August 1990.