Comments on "Hamiltonian Adaptive Control of Spacecraft"

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Abstract – In the adaptive scheme presented by Slotine and Benedetto (1990) for attitude tracking control of rigid spacecraft, the spacecraft is parameterized in terms of the inertial frame. This paper shows how a parameterization in body coordinates considerably simplifies the representation of the adaption scheme. The new symbolic expression for the regressor matrix is easy to find even for 6 degrees of freedom (DOF) Hamiltonian systems with a large number of unknown parameters. If the symbolic expression for the regressor matrix is known in advance, the computational complexity is approximately equal for both representations. In the scheme presented by Slotine and Benedetto (1990) this is not trivial because the transformation matrix between the inertial frame and the body coordinates is included in the expression for the regressor matrix. Hence implementation for higher DOF systems is strongly complicated. An example illustrates the advantage of the new representation when modelling a simple 3 DOF model of the lateral motion of a space shuttle.

I. Introduction

Consider a Hamiltonian systems written as:

$$\begin{aligned} H\ddot{\boldsymbol{q}} + C(\dot{\boldsymbol{q}}, \boldsymbol{x})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{x}) &= \boldsymbol{\tau} \\ \dot{\boldsymbol{x}} &= J(\boldsymbol{x})\dot{\boldsymbol{q}} \end{aligned} \tag{1}$$

where $\boldsymbol{q} \in \Re^n$ is the local coordinates, $\boldsymbol{x} \in \Re^n$ is a vector in the inertial frame and $\boldsymbol{\tau} \in \Re^n$ is the vector of control torques. H is the inertia matrix and $C\dot{\boldsymbol{q}}$ is a nonlinear vector of Coriolis and sentripetal forces, $\boldsymbol{g}(\boldsymbol{x}) \in \Re^n$ is the gravity vector and J is an $n \times n$ transformation matrix. Slotine and Benedetto (1990) write Eq. 1 as:

$$H^*(\boldsymbol{x})\ddot{\boldsymbol{x}} + C^*(\boldsymbol{x}, \dot{\boldsymbol{x}})\dot{\boldsymbol{x}} + \boldsymbol{g}^*(\boldsymbol{x}) = J^{-T}\boldsymbol{\tau}$$

where

$$\begin{aligned} H^*(\boldsymbol{x}) &= J^{-T} H J^{-1} \\ C^*(\boldsymbol{x}, \dot{\boldsymbol{x}}) &= J^{-T}\left[C - HJ^{-1}\dot{J}\right]J^{-1} \\ \boldsymbol{g}^*(\boldsymbol{x}) &= J^{-T}\boldsymbol{g} \end{aligned}$$

Assume the desired trajectory: $\ddot{\boldsymbol{x}}_d$, $\dot{\boldsymbol{x}}_d$ and $\boldsymbol{x}_d$ to be bounded. The tracking error vector $\tilde{\boldsymbol{x}}$ is defined as $\tilde{\boldsymbol{x}} = \boldsymbol{x} - \boldsymbol{x}_d$ while:

$$\boldsymbol{s} = \dot{\tilde{\boldsymbol{x}}} + \lambda\tilde{\boldsymbol{x}} \tag{2}$$

is used as a measure of tracking. λ is a strictly positive constant which may be interpreted as the control bandwidth. It is convenient to rewrite Eq. 2 as:

$$\boldsymbol{s} = \dot{\boldsymbol{x}} - \dot{\boldsymbol{x}}_r \quad \text{where} \quad \dot{\boldsymbol{x}}_r = \dot{\boldsymbol{x}}_d - \lambda\tilde{\boldsymbol{x}}$$

It is important to notice that the terms H^* and C^* are linear in their parameters. These unknown parameters may be lumped together into a parameter vector $\boldsymbol{\theta}$. Let $\hat{\boldsymbol{\theta}}$ be the time-varying parameter vector estimate and let $\tilde{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}} - \boldsymbol{\theta}$ be the parameter error vector. To prove global stability Slotine and Benedetto (1990) suggest to use a Lyapunov-like function candidate:

$$V(\boldsymbol{s}, \tilde{\boldsymbol{\theta}}, t) = \frac{1}{2}\boldsymbol{s}^T H^* \boldsymbol{s} + \frac{1}{2}\tilde{\boldsymbol{\theta}}^T \Gamma \tilde{\boldsymbol{\theta}}$$

where Γ is a positive definite weighting matrix of appropriate dimension. Differentiating V with respect to time and using the skew-symmetry property: $\dot{\boldsymbol{x}}^T(\dot{H}^* - 2C^*)\dot{\boldsymbol{x}} = 0$ yields the following expression:

$$\dot{V} = \boldsymbol{s}^T(J^{-T}\boldsymbol{\tau} - H^*\ddot{\boldsymbol{x}}_r - C^*\dot{\boldsymbol{x}}_r - \boldsymbol{g}^*) + \dot{\tilde{\boldsymbol{\theta}}}^T \Gamma \tilde{\boldsymbol{\theta}} \tag{3}$$

This suggests that the control law can be selected as:

$$\boldsymbol{\tau} = J^T(\hat{H}^*\ddot{\boldsymbol{x}}_r + \hat{C}^*\dot{\boldsymbol{x}}_r + \hat{\boldsymbol{g}}^* - K_D\boldsymbol{s}) \tag{4}$$

Here the hat denotes the adaptive estimates and K_D is a positive definite design matrix of appropriate dimension. Combining Eq. 3 and 4 yields:

$$\dot{V} = -\boldsymbol{s}^T K_D \boldsymbol{s} + \boldsymbol{s}^T(\tilde{H}^*\ddot{\boldsymbol{x}}_r + \tilde{C}^*\dot{\boldsymbol{x}}_r + \tilde{\boldsymbol{g}}^*) + \dot{\tilde{\boldsymbol{\theta}}}^T \Gamma \tilde{\boldsymbol{\theta}}$$

where $\tilde{H}^* = \hat{H}^* - H^*$, $\tilde{C}^* = \hat{C}^* - C^*$ and $\tilde{\boldsymbol{g}} = \hat{\boldsymbol{g}}^* - \boldsymbol{g}^*$. Using the parameterization of Slotine and Benedetto (1990) implies that:

$$H^* \ddot{\boldsymbol{x}}_r + C^* \dot{\boldsymbol{x}}_r + \tilde{\boldsymbol{g}}^* = \Phi^*(\boldsymbol{x}, \dot{\boldsymbol{x}}, \dot{\boldsymbol{x}}_r, \ddot{\boldsymbol{x}}_r)\, \boldsymbol{\theta} \tag{5}$$

where $\Phi^*(\boldsymbol{x}, \dot{\boldsymbol{x}}, \boldsymbol{x}_r, \dot{\boldsymbol{x}}_r)$ is a known regressor matrix of appropriate dimensions. Hence Eqs. 3 may be written as:

$$\dot{V} = -\boldsymbol{s}^T K_D \boldsymbol{s} + \tilde{\boldsymbol{\theta}}^T (\Gamma \dot{\hat{\boldsymbol{\theta}}} + {\Phi^*}^T \boldsymbol{s})$$

Assuming that $\dot{\boldsymbol{\theta}} = 0$ yields the parameter adaption law:

$$\dot{\hat{\boldsymbol{\theta}}} = -\Gamma^{-1} {\Phi^*}^T (\boldsymbol{x}, \dot{\boldsymbol{x}}, \dot{\boldsymbol{x}}_r, \ddot{\boldsymbol{x}}_r)\, \boldsymbol{s} \tag{6}$$

which implies that: $\dot{V} = -\boldsymbol{s}^T K_D \boldsymbol{s} \leq 0$. This shows that $\boldsymbol{s} \to 0$ and thus that $\tilde{\boldsymbol{x}} \to 0$.

II. Reparameterization

The representation of the adaptive scheme presented in Section I may be simplified considerably by defining a vector $\dot{\boldsymbol{q}}_r$ such that (Niemeyer and Slotine (1991)):

$$\dot{\boldsymbol{x}}_r = J(\boldsymbol{x}) \dot{\boldsymbol{q}}_r \tag{7}$$

This implies that $\dot{\boldsymbol{q}}_r$ and $\ddot{\boldsymbol{q}}_r$ may be calculated as:

$$\begin{aligned} \dot{\boldsymbol{q}}_r &= J^{-1}(\boldsymbol{x}) \dot{\boldsymbol{x}}_r \\ \ddot{\boldsymbol{q}}_r &= J^{-1}(\boldsymbol{x}) \ddot{\boldsymbol{x}}_r - J^{-1}(\boldsymbol{x}) \dot{J}(\boldsymbol{x}) J^{-1}(\boldsymbol{x}) \dot{\boldsymbol{x}}_r \end{aligned} \tag{8}$$

We now notice that Eq. 5 may be rewritten as:

$$\begin{aligned} & J^T \left[H^* \ddot{\boldsymbol{x}}_r + C^* \dot{\boldsymbol{x}}_r + \boldsymbol{g}^* \right] = \\ & \qquad H \ddot{\boldsymbol{q}}_r + C \dot{\boldsymbol{q}}_r + \boldsymbol{g} = \Phi(\boldsymbol{x}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}_r, \ddot{\boldsymbol{q}}_r)\, \boldsymbol{\theta} \end{aligned} \tag{9}$$

By using the vector $\boldsymbol{q}_r$ instead of $\boldsymbol{x}_r$, the known transformation matrix $J(\boldsymbol{x})$ is eliminated from the new regressor matrix $\Phi(\boldsymbol{x}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}_r, \ddot{\boldsymbol{q}}_r)$. Using the parameterization of Eq. 9, the following expression for $\dot{V}$ is found:

$$\dot{V} = \boldsymbol{s}^T J^{-T} (\boldsymbol{\tau} - H \ddot{\boldsymbol{q}}_r - C \dot{\boldsymbol{q}}_r - \boldsymbol{g}) + \dot{\hat{\boldsymbol{\theta}}}^T \Gamma \tilde{\boldsymbol{\theta}}$$

The control law Eq. 4 then simplifies to:

$$\boldsymbol{\tau} = \hat{H} \ddot{\boldsymbol{q}}_r + \hat{C} \dot{\boldsymbol{q}}_r + \hat{\boldsymbol{g}} - J^T K_D \boldsymbol{s}$$

while the adaption law Eq. 6 is modified to:

$$\dot{\hat{\boldsymbol{\theta}}} = -\Gamma^{-1} \Phi^T (\boldsymbol{x}, \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}_r, \ddot{\boldsymbol{q}}_r) J^{-1}\, \boldsymbol{s}$$

which again implies that: $\dot{V} = -\boldsymbol{s}^T K_D \boldsymbol{s} \leq 0$. Notice that the new control law is written in terms of $\hat{H}$, $\hat{C}$ and $\hat{\boldsymbol{g}}$ instead of $\hat{H}^*$, $\hat{C}^*$ and $\hat{\boldsymbol{g}}^*$. Hence Φ is given in body coordinates which suggests that the elements can be found by inspection or recursive methods. Since the kinematics enters Φ^* this expression will be quite complex for larger systems.

III. Implementation Considerations

To illustrate the advantageous representation of the new scheme, the symbolic expressions for the regressor matrices Φ^* and Φ were calculated for the lateral motion of a space shuttle. Let m be the vehicle's mass, I_x and I_z the vehicle's moments of inertia about the x- and z-axis respectively, I_{xy}, I_{xz} and I_{yz} the vehicle's products of inertia and (x_G, y_G, z_G) the center of gravity. Hence Newton's 2nd laws of angular and linear momentum yield:

$$H = \begin{bmatrix} m & -m z_G & m x_G \\ -m z_G & I_x & -I_{xz} \\ m x_G & -I_{xz} & I_z \end{bmatrix}$$

and

$$C(\dot{\boldsymbol{q}}) = \begin{bmatrix} 0 & -m y_G p & -m y_G r \\ m y_G p & 0 & I_{xy} p + I_{yz} r \\ m y_G r & -I_{xy} p - I_{yz} r & 0 \end{bmatrix}$$

Here v is the sway velocity and, p and r is the angular velocity in roll and yaw respectively. A star-fixed inertial frame where y is the vehicle's sway position, ϕ is the roll angle and ψ is the heading angle is related to the body-fixed reference frame by the transformation matrix:

$$J(\boldsymbol{x}) = \begin{bmatrix} \cos\phi \cos\psi & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \cos\phi \end{bmatrix}$$

This corresponds to $\boldsymbol{x} = (y, \phi, \psi)$ and $\dot{q} = (v, p, r)$. Let $\boldsymbol{\theta} = (m, m x_G, m y_G, m z_G, I_x, I_z, I_{xy}, I_{xz}, I_{yz})$ be the unknown parameter vector. It is straightforward to write down the new regressor matrix Φ i.e.:

$$\Phi = \begin{bmatrix} \dot{v}_r & \dot{r}_r & -p p_r - r r_r & -\dot{p}_r & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & p v_r & -\dot{v}_r & \dot{p}_r & 0 & p r_r & -\dot{r}_r & r r_r \\ 0 & \dot{v}_r & r v_r & 0 & 0 & \dot{r}_r & -p p_r & -\dot{p}_r & -r p_r \end{bmatrix}$$

which is used in the modified adaption law. Here (v_r, p_r, r_r) and $(\dot{v}_r, \dot{p}_r, \dot{r}_r)$ must be calculated *numerically* from Eq. 8. In the scheme presented by Slotine and Benedetto we have to obtain the symbolic expression for Φ^* which is quite tedious already for this simple 3 DOF example. After some calculations the following symbolic expression was found for Φ^*:

$$\Phi^* = \left[\begin{matrix} \frac{\ddot{y}_r}{c^2\phi c^2\psi} + \left(\frac{s\phi\, p}{c^5\phi c^4\psi} + \frac{s\psi\, r}{c^3\phi c^5\psi}\right)\dot{y}_r & \frac{\ddot{\psi}_r}{c^2\phi c\psi} + \frac{s\phi\, p\dot{\psi}_r}{c^5\phi c^2\psi} & -\frac{p\dot{\phi}_r}{c\phi c\psi} - \frac{r\dot{\psi}_r}{c^2\phi c\psi} & -\frac{\ddot{\phi}_r}{c\phi c\psi} \\ 0 & 0 & \frac{p\dot{y}_r}{c\phi c\psi} & -\frac{\ddot{y}_r}{c\phi c\psi} - \left(\frac{s\phi\, p}{c^3\phi c^2\psi} + \frac{s\psi\, r}{c\phi c^3\psi}\right)\dot{y}_r \\ 0 & \frac{\ddot{y}_r}{c^2\phi c\psi} + \left(\frac{s\phi\, p}{c^5\phi c^2\psi} + \frac{s\psi\, r}{c^3\phi c^3\psi}\right)\dot{y}_r & \frac{r\dot{y}_r}{c^2\phi c\psi} & 0 \end{matrix} \right.$$

$$\left. \begin{matrix} 0 & 0 & 0 & 0 & 0 \\ \ddot{\phi}_r & 0 & \frac{p\dot{\psi}_r}{c\phi} & -\frac{\ddot{\psi}_r}{c\phi} - \frac{s\phi\, p\dot{\psi}_r}{c^3\phi} & \frac{r\dot{\psi}_r}{c\phi} \\ 0 & \frac{\ddot{\psi}_r}{c^2\phi} + \frac{s\phi\, p\dot{\psi}_r}{c^5\phi} & -\frac{p\dot{\phi}_r}{c\phi} & -\frac{\ddot{\phi}_r}{c\phi} & -\frac{r\dot{\phi}_r}{c\phi} \end{matrix} \right]$$

Here c and s denote the cosine and sine function respectively. For a general vehicle in 6 DOF the expression for Φ^* will be extremely complicated while Φ can be seen directly from the model i.e. H, C and $\boldsymbol{g}$. The complexity of Φ^* depends on the terms associated with the transformation matrix J and its time derivative $\dot{J}$. Including these terms in H^*, C^* and $\boldsymbol{g}^*$ strongly complicates the regressor matrix. As the example shows, this is easily avoided in the new scheme.

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