Singularity-Free Tracking of Unmanned Underwater Vehicles in 6 DOF

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Abstract

Nonlinear tracking control of unmanned underwater vehicles (UUVs) in 6 degrees of freedom (DOF) is discussed. The 4-parameter unit quaternion is used in a singularity-free representation of attitude. Several control laws based on a generalized Lyapunov function for the attitude dynamics are derived, including feedback from the vector quaternion, the Euler rotation vector and the Rodrigues parameter vector. A new feedback gain matrix for translational motion is also proposed. Lyapunov analysis is used in the tracking error convergence analysis. Extensions to adaptive control are made. The control laws are tested in a simulation study.

1. Introduction

For rigid-bodies in 6 DOF the non-linear dynamic equations of motion have a systematic structure which becomes apparent when applying vector notation. This is exploited in the control literature, particularly in the control of robot manipulators. A nonlinear adaptive tracking control law exploiting the passivity property of robot manipulators was derived by Slotine and Li (Slotine & Li 1987). Later extensions to 3 DOF spacecraft attitude control were made by Slotine and Benedetto (Slotine & Benedetto 1990) in terms of the 3-parameter Gibb's vector (Rodrigues parameter). However, this representation contains a singularity (Stuelpnagel 1964), so only local convergence can be guaranteed.

In the attitude control literature it is common to use Euler parameters, or unit quaternions, to represent attitude. This is a 4-parameter singularity-free representation. The earliest results concentrated on setpoint regulation, see e.g. (Wie, Weiss & Arapostathis 1989) and the references therein, whereas the more general problem of tracking has been discussed more recently. Dwyer III (Dwyer, III 1984) proposed using exact linearization to solve the attitude tracking problem whereas Wen and Kreutz-Delgado (Wen & Kreutz-Delgado 1991) used a scalar gain PD-control law with feed-forward.

The results of Slotine and Benedetto (Slotine & Benedetto 1990) was reformulated in terms of Euler parameters by Egeland and Godhavn (Egeland & Godhavn 1994) in order to prove global convergence. In the latter passivity was used in the convergence analysis, and hence the feedback gain matrix was allowed to be time-varying and positive definite.

In this paper, these two works are extended to tracking control of UUVs in 6 DOF. Nonlinear couplings between the translational and rotational dynamics due to rigid-body and hydrodynamic effects are considered. Lyapunov analysis for non-autonomous systems is applied to prove global convergence. This work is also closely related to (Fossen & Sagatun 1991) where Euler angles are used for attitude representation, (Fjellstad & Fossen 1994b) where setpoint regulation is considered, and (Fjellstad & Fossen 1994a) where similar tracking control laws have been presented without utilizing the unit quaternion group structure in the attitude error representation.

2. Mathematical Modeling

Kinematic Equations of Motion

The kinematic model describes the geometrical relationship between the earth-fixed and the vehicle-fixed motion. The transformation matrix $\boldsymbol{J}(\boldsymbol{q}) \in \mathbb{R}^{7\times 6}$ relates the body-fixed coordinate frame (B-frame) to the inertial coordinate frame (I-frame) according to ($\boldsymbol{\xi} \in \mathbb{R}^3 \times \mathbb{H}$, $\boldsymbol{\nu} \in \mathbb{R}^6$):

$$\dot{\boldsymbol{\xi}} = \boldsymbol{J}(\boldsymbol{q})\boldsymbol{\nu} \Leftrightarrow \begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{q}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{R}(\boldsymbol{q}) & \boldsymbol{0}_{3\times 3} \\ \boldsymbol{0}_{4\times 3} & \frac{1}{2}\boldsymbol{U}(\boldsymbol{q}) \end{bmatrix} \begin{bmatrix} \boldsymbol{v} \\ \boldsymbol{\omega} \end{bmatrix} \tag{1}$$

where $\boldsymbol{x} = [x, y, z]^{\mathrm{T}} \in \mathbb{R}^3$ is the I-frame position of the vehicle and $\boldsymbol{q} = [\eta, \boldsymbol{\epsilon}^{\mathrm{T}}]^{\mathrm{T}} = [\eta, \epsilon_1, \epsilon_2, \epsilon_3]^{\mathrm{T}} \in \mathbb{H}$ is the unit quaternion used to describe attitude. $\boldsymbol{v} = [u, v, w]^{\mathrm{T}} \in \mathbb{R}^3$ and $\boldsymbol{\omega} = [p, q, r]^{\mathrm{T}} \in \mathbb{R}^3$ are the linear and angular velocities of the vehicle in the B-frame. The elements of the unit quaternion $\boldsymbol{q} \in \mathbb{H}$ are called Euler parameters and they satisfy $\eta^2 + \epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2 = 1$. The matrices $\boldsymbol{R}$ and $\boldsymbol{U}$ are defined below.

Linear velocity transformation (rotation matrix)

The rotation matrix $\boldsymbol{R} \in SO(3)$, that is *Special Orthogonal* group of order 3, from I to B in terms of the Euler parameters is written as:

$$\boldsymbol{R}(\boldsymbol{q}) = \boldsymbol{I} + 2\eta\boldsymbol{S}(\boldsymbol{\epsilon}) + 2\boldsymbol{S}^2(\boldsymbol{\epsilon}) \tag{2}$$

where the skew-symmetric matrix $\boldsymbol{S}(\boldsymbol{a}) = -\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{a})$ is defined such that for arbitrary vectors $\boldsymbol{a}, \boldsymbol{b} \in \mathbb{R}^3$ we have $\boldsymbol{a} \times \boldsymbol{b} \equiv \boldsymbol{S}(\boldsymbol{a})\boldsymbol{b}$. There is a two-to-one correspondence between $\mathbb{H}$ and $SO(3)$. From (2) it follows that $\boldsymbol{q}$ and $-\boldsymbol{q}$ represent the same orientation. This double covering of $SO(3)$ is usually solved by choosing the desired quaternion $\boldsymbol{q}_d \in \mathbb{H}$ such that $\eta_d \geq 0$ is non-negative.

The quaternion $\boldsymbol{q} \in \mathbb{H}$ can be interpreted as a complex number with η being the real part and $\boldsymbol{\epsilon}$ the complex part. Hence, the complex conjugate of $\boldsymbol{q} \in \mathbb{H}$ is defined as:

$$\overline{\boldsymbol{q}} \triangleq \begin{bmatrix} \eta \\ -\boldsymbol{\epsilon} \end{bmatrix} \in \mathbb{H} \tag{3}$$

Consequently, the inverse rotation matrix can be written as:

$$\boldsymbol{R}^{-1}(\boldsymbol{q}) = \boldsymbol{R}^{\mathrm{T}}(\boldsymbol{q}) = \boldsymbol{R}(\overline{\boldsymbol{q}}) \in SO(3) \tag{4}$$

Successive rotations involves multiplication between two rotation matrices. It can be shown that:

$$\boldsymbol{R}(\boldsymbol{q}_1)\boldsymbol{R}(\boldsymbol{q}_2) = \boldsymbol{R}(\boldsymbol{q}_1\boldsymbol{q}_2) \in SO(3) \tag{5}$$

where quaternion multiplication is applied.

Angular velocity transformation

The angular velocity transformation matrix $\boldsymbol{U}(\boldsymbol{q})$ can be written as:

$$\boldsymbol{U}(\boldsymbol{q}) = \begin{bmatrix} -\boldsymbol{\epsilon}^{\mathrm{T}} \\ \eta\boldsymbol{I}_{3\times 3} + \boldsymbol{S}(\boldsymbol{\epsilon}) \end{bmatrix} = \begin{bmatrix} -\boldsymbol{\epsilon}^{\mathrm{T}} \\ \boldsymbol{T}(\boldsymbol{q}) \end{bmatrix} \in \mathbb{R}^{4\times 3} \tag{6}$$

Since $\boldsymbol{U}^{\mathrm{T}}(\boldsymbol{q})\boldsymbol{U}(\boldsymbol{q}) = \boldsymbol{I}_{3\times 3}$, the rotational part of (1) yields ($\dot{\boldsymbol{U}}^{\mathrm{T}}\dot{\boldsymbol{q}} = \boldsymbol{0}$):

$$\boldsymbol{\omega} = 2\boldsymbol{U}^{\mathrm{T}}(\boldsymbol{q})\dot{\boldsymbol{q}} \tag{7}$$

$$\dot{\boldsymbol{\omega}} = 2\boldsymbol{U}^{\mathrm{T}}(\boldsymbol{q})\ddot{\boldsymbol{q}} \tag{8}$$

The transformation matrix $\boldsymbol{J}(\boldsymbol{q}) \in \mathbb{R}^{7\times 6}$ has full rank, that is $\mathrm{rank}[\boldsymbol{J}(\boldsymbol{q})] = 6 \,\forall \boldsymbol{q} \in \mathbb{H}$. Hence the kinematic equations contain no singular points. The computation of the kinematic equations involves multiplications and additions only. No function evaluations are needed. Moreover, the rotational kinematic equations are linear.

UUV Dynamic Equations of Motion

The 6 DOF dynamic equations of motion of a vehicle can be expressed in the B-frame as (Fjellstad & Fossen 1994a):

$$\boldsymbol{M}\dot{\boldsymbol{\nu}} + \boldsymbol{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} \tag{9}$$

where

$$\boldsymbol{M} = \begin{bmatrix} \boldsymbol{M}_{11} & \boldsymbol{M}_{12} \\ \boldsymbol{M}_{21} & \boldsymbol{M}_{22} \end{bmatrix} \in \mathbb{R}^{6\times 6}, \ \boldsymbol{M}_{ij} \in \mathbb{R}^{3\times 3} \tag{10}$$

$$\boldsymbol{C}(\boldsymbol{\nu}) = \begin{bmatrix} \boldsymbol{0}_{3\times 3} & -\boldsymbol{S}(\boldsymbol{M}_{11}\boldsymbol{v} + \boldsymbol{M}_{12}\boldsymbol{\omega}) \\ -\boldsymbol{S}(\boldsymbol{M}_{11}\boldsymbol{v} + \boldsymbol{M}_{12}\boldsymbol{\omega}) & -\boldsymbol{S}(\boldsymbol{M}_{21}\boldsymbol{v} + \boldsymbol{M}_{22}\boldsymbol{\omega}) \end{bmatrix} \in \mathbb{R}^{6\times 6} \tag{11}$$

For marine vehicles the inertia matrix $\boldsymbol{M}$ and the Coriolis and centrifugal matrix $\boldsymbol{C}(\boldsymbol{\nu})$ include added inertia due to hydrodynamic terms, $\boldsymbol{D}(\boldsymbol{\nu})$ contains hydrodynamic damping terms, $\boldsymbol{g}(\boldsymbol{q})$ is a vector of gravitational and buoyant forces and moments and $\boldsymbol{\tau}$ is the control vector of forces and moments to be specified.

It is assumed that $\boldsymbol{M} = \boldsymbol{M}^{\mathrm{T}}$ is constant and positive definite. In addition, $\boldsymbol{C}(\boldsymbol{\nu})$ defined in (11) is skew-symmetrical and $\boldsymbol{D}(\boldsymbol{\nu})$ is strictly positive, that is:

$$\boldsymbol{y}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{y} > 0 \quad \forall \boldsymbol{y} \neq \boldsymbol{0} \tag{12}$$

$$\boldsymbol{C}(\boldsymbol{\nu}) = -\boldsymbol{C}^{\mathrm{T}}(\boldsymbol{\nu}) \in SS(6) \tag{13}$$

$$\boldsymbol{y}^{\mathrm{T}}\boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{y} > 0 \quad \forall \boldsymbol{y} \neq \boldsymbol{0} \tag{14}$$

Attitude Error Dynamics

The rotation matrix $\boldsymbol{R} \in SO(3)$ from the I-frame to the B-frame represents the actual attitude of the vehicle. Let $\boldsymbol{R}_d$ denote the desired attitude, that is the rotation matrix from the I-frame to a desired coordinate frame, denoted as the D-frame.

The control objective is to make the B-frame coincide with the D-frame such that $\boldsymbol{R} = \boldsymbol{R}_d$. Two alternative error matrices are:

$$\tilde{\boldsymbol{R}}_1 = \boldsymbol{R}_d^{\mathrm{T}}\boldsymbol{R} \in SO(3), \qquad \tilde{\boldsymbol{R}}_2 = \boldsymbol{R}\boldsymbol{R}_d^{\mathrm{T}} \in SO(3) \tag{15}$$

where the control objective is formulated as $\tilde{\boldsymbol{R}}_1 = \boldsymbol{I}_{6\times 6}$ or $\tilde{\boldsymbol{R}}_2 = \boldsymbol{I}_{6\times 6}$, respectively. The matrices $\tilde{\boldsymbol{R}}_1$ and $\tilde{\boldsymbol{R}}_2$ are rotation matrices and therefore they have a structure which can be exploited in the design of the attitude controller. $\tilde{\boldsymbol{R}}_1$ is the rotation matrix from the D-frame to the B-frame. Application of quaternions in the parameterization of $SO(3)$ gives $\boldsymbol{R} = \boldsymbol{R}(\boldsymbol{q})$, $\boldsymbol{R}_d = \boldsymbol{R}(\boldsymbol{q}_d)$ and $\tilde{\boldsymbol{R}}_1 = \boldsymbol{R}_d^{\mathrm{T}}\boldsymbol{R} = \boldsymbol{R}(\tilde{\boldsymbol{q}})$ where $\tilde{\boldsymbol{q}} = \overline{\boldsymbol{q}}_d\boldsymbol{q}$ is obtained by combining (4), (5) and (15).

Perfect tracking in terms of quaternion parameterization is obtained for:

$$\boldsymbol{q} = \pm\boldsymbol{q}_d \Leftrightarrow \tilde{\boldsymbol{q}} = \begin{bmatrix} \pm 1 \\ \boldsymbol{0} \end{bmatrix} \in \mathbb{H} \tag{16}$$

Notice that $\tilde{\boldsymbol{R}}_1$ and $\tilde{\boldsymbol{R}}_2$ are related through a similarity transformation $\tilde{\boldsymbol{R}}_2 = \boldsymbol{R}\tilde{\boldsymbol{R}}_1\boldsymbol{R}^{\mathrm{T}} = \boldsymbol{R}\tilde{\boldsymbol{R}}_1\boldsymbol{R}^{-1}$. Hence, it follows that $\tilde{\boldsymbol{R}}_1$ and $\tilde{\boldsymbol{R}}_2$ have the same eigenvalues, that is eig$(\tilde{\boldsymbol{R}}_1)$, eig$(\tilde{\boldsymbol{R}}_2) \in \left\{1, 2\tilde{\eta}^2 - 1 \pm j2\tilde{\eta}\sqrt{1-\tilde{\eta}^2}\right\}$. Also note that $\tilde{\boldsymbol{R}}_1$ and $\tilde{\boldsymbol{R}}_2$ are strictly positive whenever $\tilde{\eta}^2 > 1/2$, that is:

$$\tilde{\eta}^2 > \frac{1}{2} \Rightarrow \tilde{\boldsymbol{R}}_1 > \boldsymbol{0},\ \tilde{\boldsymbol{R}}_2 > \boldsymbol{0} \tag{17}$$

The kinematic equations for the desired attitude are defined as

$$\dot{\boldsymbol{q}}_d = \frac{1}{2}\boldsymbol{U}(\boldsymbol{q}_d)\,{}^{D}\boldsymbol{\omega}_d = \frac{1}{2}\boldsymbol{U}(\boldsymbol{q}_d)\,\tilde{\boldsymbol{R}}\,\boldsymbol{\omega}_d \tag{18}$$

to be consistent with (1). Here, ${}^{D}\boldsymbol{\omega}_d$ is the angular velocity of the D-frame relative the I-frame decomposed in the D-frame, whereas $\boldsymbol{\omega}_d$ is the same vector decomposed in the B-frame. Hence, the attitude error differential equations can be written:

$$\dot{\tilde{\boldsymbol{q}}} = \frac{1}{2}\boldsymbol{U}(\tilde{\boldsymbol{q}})\tilde{\boldsymbol{\omega}} \Leftrightarrow \begin{cases} \dot{\tilde{\eta}} = -\frac{1}{2}\tilde{\boldsymbol{\epsilon}}^{\mathrm{T}}\tilde{\boldsymbol{\omega}} \\ \dot{\tilde{\boldsymbol{\epsilon}}} = \frac{1}{2}[\tilde{\eta}\boldsymbol{I}_{3\times 3} + \boldsymbol{S}(\tilde{\boldsymbol{\epsilon}})]\tilde{\boldsymbol{\omega}} \end{cases} \tag{19}$$

where the angular velocity error is defined as $\tilde{\boldsymbol{\omega}} = \boldsymbol{\omega} - \boldsymbol{\omega}_d$, all three vectors decomposed in the B-frame.

3. Main Results

In this section we will derive a control law for tracking based on the 6 DOF UUV kinematic and dynamic equations of motion.

Control Law

Define the virtual body-fixed velocity error vector $\boldsymbol{s}$ as:

$$\boldsymbol{s} \triangleq \boldsymbol{\nu} - \boldsymbol{\nu}_r, \qquad \boldsymbol{s} \in \mathbb{R}^6 \tag{20}$$

where $\boldsymbol{\nu}_r \in \mathbb{R}^6$ is a virtual velocity reference signal to be defined later. Consider the positive definite Lyapunov function candidate:

$$V = \frac{1}{2}\boldsymbol{s}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{s} > 0, \qquad \forall\,\boldsymbol{s} \neq \boldsymbol{0} \tag{21}$$

The function V is decrescent, moreover:

$$\lambda_{\min}(\boldsymbol{M})\|\boldsymbol{s}\|^2 \leq 2V \leq \lambda_{\max}(\boldsymbol{M})\|\boldsymbol{s}\|^2 \tag{22}$$

where $\|\boldsymbol{s}\|^2$ belongs to class $\mathcal{K}_\infty$. Differentiation of (21) with respect to time yields:

$$\begin{aligned} \dot{V} &= \boldsymbol{s}^{\mathrm{T}}\boldsymbol{M}\dot{\boldsymbol{s}} \\ &= \boldsymbol{s}^{\mathrm{T}}[\boldsymbol{\tau} - \boldsymbol{M}\dot{\boldsymbol{\nu}}_r - \boldsymbol{C}(\boldsymbol{\nu})\boldsymbol{\nu}_r - \boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{\nu}_r - \boldsymbol{g}] \\ &- \boldsymbol{s}^{\mathrm{T}}\boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{s} \end{aligned} \tag{23}$$

where (12) and (13) have been used. By choosing the control law $\boldsymbol{\tau}$ as:

$$\boldsymbol{\tau} = \boldsymbol{M}\dot{\boldsymbol{\nu}}_r + \boldsymbol{C}(\boldsymbol{\nu})\boldsymbol{\nu}_r + \boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{\nu}_r + \boldsymbol{g} - \boldsymbol{K}_d\boldsymbol{s} \tag{24}$$

where $\boldsymbol{K}_d \geq \boldsymbol{0} \in \mathbb{R}^{6\times 6}$ is positive, $\dot{V}$ becomes:

$$\dot{V} = -\boldsymbol{s}^{\mathrm{T}}[\boldsymbol{K}_d + \boldsymbol{D}(\boldsymbol{\nu})]\boldsymbol{s} < 0, \qquad \forall\,\boldsymbol{s} \neq \boldsymbol{0} \tag{25}$$

$\dot{V}$ is obviously negative definite:

$$\dot{V} \leq -[\lambda_{\min}(\boldsymbol{K}_d) + \lambda_{\min}(\boldsymbol{D}(\boldsymbol{\nu})]\|\boldsymbol{s}\|^2 \tag{26}$$

Assume that the body-fixed virtual velocity reference vector is continuously differentiable, that is $\boldsymbol{\nu}_r \in C^1[\mathbb{R}_+] \times \mathbb{R}^6$. Then application of Lyapunov's direct method theorem for non-autonomous systems guarantees globally uniformly asymptotically stability of the equilibrium point $\boldsymbol{s} = \boldsymbol{0}$ (Khalil 1992). Let us define:

$$\boldsymbol{\nu}_r \triangleq \boldsymbol{\nu}_d - \boldsymbol{\Lambda}\boldsymbol{e} \tag{27}$$

where

$$\boldsymbol{\nu}_d \triangleq \begin{bmatrix} \boldsymbol{v}_d \\ \boldsymbol{\omega}_d \end{bmatrix} \in C^1[\mathbb{R}_+] \times \mathbb{R}^6 \quad (28)$$

$$\boldsymbol{\Lambda} \triangleq \begin{bmatrix} \boldsymbol{K}_P & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & -2c\frac{\partial W}{\partial\tilde{\eta}}\boldsymbol{I}_{3\times3} \end{bmatrix} \in C^1[\mathbb{R}_+] \times \mathbb{R}^{6\times6}, \quad c > 0 \quad (29)$$

$$\boldsymbol{e} \triangleq \begin{bmatrix} \tilde{\boldsymbol{x}} \\ \tilde{\boldsymbol{\epsilon}} \end{bmatrix} \in C^1[\mathbb{R}_+] \times \mathbb{R}^6, \quad \tilde{\boldsymbol{x}} \triangleq \boldsymbol{x} - \boldsymbol{x}_d \quad (30)$$

The scalar function $W(\tilde{\eta})$ is non-negative on the interval $\tilde{\eta} \in [-1, 1]$ and it vanishes only at $\tilde{\eta} = -1$ and/or $\tilde{\eta} = 1$. $W(\tilde{\eta})$ also satisfies the Lipschitz condition on the interval $\tilde{\eta} \in [-1, 1]$.

We obtain the following expressions for the translational and rotational error dynamics:

$$\tilde{\boldsymbol{v}} + \boldsymbol{K}_P\tilde{\boldsymbol{x}} = \mathbf{0} \quad (31)$$

$$\tilde{\boldsymbol{\omega}} - 2c\frac{\partial W}{\partial\tilde{\eta}}\tilde{\boldsymbol{\epsilon}} = \mathbf{0} \quad (32)$$

where $\tilde{\boldsymbol{v}} \triangleq \boldsymbol{v} - \boldsymbol{v}_d$ and $\boldsymbol{K}_P$ and c must be chosen such that $\tilde{\boldsymbol{v}}$, $\tilde{\boldsymbol{x}}$, $\tilde{\boldsymbol{\omega}}$ and $\tilde{\boldsymbol{\epsilon}}$ converge to $\mathbf{0}$ (perfect tracking). This is discussed in the next two sections.

Convergence Analysis of the Translational Error Dynamics

Let $\dot{\boldsymbol{x}}_d = \boldsymbol{R}(\boldsymbol{q})\boldsymbol{v}_d$. This definition combined with the kinematic equation (1) gives:

$$\tilde{\boldsymbol{v}} = \boldsymbol{R}^{\mathrm{T}}(\boldsymbol{q})\dot{\tilde{\boldsymbol{x}}} \quad (33)$$

Substituting (33) into (31) yields the following translational error dynamics:

$$\dot{\tilde{\boldsymbol{x}}} + \boldsymbol{R}(\boldsymbol{q})\boldsymbol{K}_P\tilde{\boldsymbol{x}} = \mathbf{0} \quad (34)$$

Convergence of $\tilde{\boldsymbol{x}}$ to zero is obtained by choosing $\boldsymbol{K}_P$ such that $-\boldsymbol{R}(\boldsymbol{q})\boldsymbol{K}_P$ is Hurwitz. Sufficient conditions are that $\boldsymbol{R}(\boldsymbol{q})\boldsymbol{K}_P$ is positive definite or strictly positive. In (Fossen & Sagatun 1991) $\boldsymbol{K}_P(\boldsymbol{q}) = \lambda\boldsymbol{R}^{\mathrm{T}}(\boldsymbol{q})$, $\lambda > 0$ was chosen, which yields the stable translational error dynamics $\dot{\tilde{\boldsymbol{x}}} + \lambda\tilde{\boldsymbol{x}} = \mathbf{0}$.

We propose the following candidate for $\boldsymbol{K}_P$:

$$\boldsymbol{K}_P(\boldsymbol{q}_d) = \lambda\boldsymbol{R}^{\mathrm{T}}(\boldsymbol{q}_d) \in C^1[\mathbb{R}_+] \times \mathbb{R}^{6\times6}, \ \lambda > 0 \quad (35)$$

Then $\dot{\boldsymbol{K}}_P(\boldsymbol{q}_d, \boldsymbol{\omega}_d) = -\lambda\boldsymbol{S}(\boldsymbol{\omega}_d)\boldsymbol{R}^{\mathrm{T}}(\boldsymbol{q}_d)$. Substituting (35) into (34) gives the translational error dynamics:

$$\dot{\tilde{\boldsymbol{x}}} + \lambda\tilde{\boldsymbol{R}}_2\tilde{\boldsymbol{x}} = \mathbf{0} \quad (36)$$

where $\tilde{\boldsymbol{R}}_2$ is strictly positive for $\tilde{\eta}^2 > 1/2$ (see Section). Consequently, the position error converges to zero if $\tilde{\eta}^2 > 1/2$. The Euler parameters are usually defined from the angle-axis parameterization of $SO(3)$. For $\tilde{\boldsymbol{R}}_1$ we obtain:

$$\tilde{\eta} = \cos\frac{\tilde{\alpha}}{2}, \qquad \tilde{\boldsymbol{\epsilon}} = \tilde{\boldsymbol{n}}\sin\frac{\tilde{\alpha}}{2} \quad (37)$$

$\tilde{\boldsymbol{R}}_1$, and hence $\tilde{\boldsymbol{R}}_2$ are strictly positive if $|\tilde{\alpha}| < \pi/2$ which is consistent with (17) and (37). In practice, the desired reference attitude will be specified within the bandwidth of the actuators. Since the attitude error dynamics converges uniformly to zero, the condition will be satisfied at least after a transient period. The advantage of this choice for $\boldsymbol{K}_P$ over the first alternative is that the actual angular velocity $\boldsymbol{\omega}$ is substituted by the desired angular velocity $\boldsymbol{\omega}_d$ in the translational part of the tracking controller. This improves the robustness in case of noisy measurements.

Convergence Analysis of the Rotational Error Dynamics

Several choices for rotational feedback are discussed.

Let $W(\tilde{\eta})$ be a Lyapunov function candidate. Differentiation of $W(\tilde{\eta})$ and substituting (32) yields:

$$\begin{aligned}\dot{W}(\tilde{\eta}) &= \frac{\partial W}{\partial\tilde{\eta}}\dot{\tilde{\eta}} = -\frac{1}{2}\frac{\partial W}{\partial\tilde{\eta}}\tilde{\boldsymbol{\epsilon}}^{\mathrm{T}}\tilde{\boldsymbol{\omega}} \\ &= -c(\frac{\partial W}{\partial\tilde{\eta}})^2\tilde{\boldsymbol{\epsilon}}^{\mathrm{T}}\tilde{\boldsymbol{\epsilon}} < 0, \forall \frac{\partial W}{\partial\tilde{\eta}} \neq 0, \tilde{\boldsymbol{\epsilon}} \neq \mathbf{0}\end{aligned} \quad (38)$$

Feedback from the vector quaternion $\boldsymbol{\epsilon}$ will first be discussed. Defining $W(\tilde{\eta})$ as:

$$W(\tilde{\eta}) = 1 - |\tilde{\eta}| \Rightarrow \dot{W}(\tilde{\eta}) = -c\tilde{\boldsymbol{\epsilon}}^{\mathrm{T}}\tilde{\boldsymbol{\epsilon}} \quad (39)$$

yields

$$\boldsymbol{\nu}_r = \begin{bmatrix} \boldsymbol{\nu}_{r1} \\ \boldsymbol{\nu}_{r2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{v}_d - \lambda\boldsymbol{R}^{\mathrm{T}}(\boldsymbol{q}_d)\tilde{\boldsymbol{x}} \\ \boldsymbol{\omega}_d - 2c\,\mathrm{sgn}(\tilde{\eta})\tilde{\boldsymbol{\epsilon}} \end{bmatrix} \quad (40)$$

in the feedback control law. The signum function is defined as

$$\mathrm{sgn}(x) = \begin{cases} -1, & x < 0 \\ 1, & x \geq 0 \end{cases} \quad (41)$$

The function $W(\tilde{\eta})$ vanishes at $\tilde{\eta} = \pm1$. Hence, $\tilde{\boldsymbol{x}} = \mathbf{0}$, $\tilde{\eta} = \pm1$ are both asymptotically stable equilibrium points ($\tilde{\boldsymbol{\epsilon}} = \mathbf{0}$). Notice that the signum function is non-zero by definition in order to avoid an extra (unstable) equilibrium point at $\tilde{\eta} = 0$.

$\tilde{\eta} = 1$ is unstable. From the previous sections it follows that asymptotically convergence is obtained for the feedback laws given by (24) where $\boldsymbol{\nu}_r$ is computed from the class of functions $W(\tilde{\eta})$. The properties and performance of the closed loop system is changed by simply shaping $W(\tilde{\eta})$. Two classical approaches are the Euler rotation feedback and the Rodrigues parameter feedback. The former is obtained by choosing:

$$W(\tilde{\eta}) = 1 - \tilde{\eta}^2 \;\Rightarrow\; \boldsymbol{\nu}_{r2} = \boldsymbol{\omega}_d - 4c\tilde{\eta}\tilde{\boldsymbol{\epsilon}} \tag{42}$$

whereas the latter comes from:

$$W(\tilde{\eta}) = -\ln(|\tilde{\eta}|) \;\Rightarrow\; \boldsymbol{\nu}_{r2} = \boldsymbol{\omega}_d - \frac{2c}{\tilde{\eta}}\tilde{\boldsymbol{\epsilon}} \tag{43}$$

A summary of the "rotational part" of the presented feedback control laws and also some alternatives to them, are given in Table 1. In the table it is distinguished between asymptotic stable equilibrium points (a.s.e.p.), unstable equilibrium points (u.e.p.) and singular points (s.p.).

$W(\tilde{\eta})$	a.s.e.p.	u.e.p.	s.p.
$1 - \|\tilde{\eta}\|$	$\tilde{\eta} = \pm 1$		
$1 - \tilde{\eta}$	$\tilde{\eta} = 1$	$\tilde{\eta} = -1$	
$1 + \tilde{\eta}$	$\tilde{\eta} = -1$	$\tilde{\eta} = 1$	
$1 - \|\tilde{\eta}\|^{p+1}$	$\tilde{\eta} = \pm 1$	$\tilde{\eta} = 0$	
$\cos^p(\frac{\pi\tilde{\eta}}{2})$	$\tilde{\eta} = \pm 1$	$\tilde{\eta} = 0$	
$-\ln(\|\tilde{\eta}\|)$	$\tilde{\eta} = \pm 1$		$\tilde{\eta} = 0$
$\frac{1}{\|\tilde{\eta}\|^p} - 1$	$\tilde{\eta} = \pm 1$		$\tilde{\eta} = 0$
$(\frac{2}{1+\tilde{\eta}})^p - 1$	$\tilde{\eta} = 1$		$\tilde{\eta} = -1$
$(\frac{2}{1-\tilde{\eta}})^p - 1$	$\tilde{\eta} = -1$		$\tilde{\eta} = 1$

Table 1: Alternative choices of $W(\tilde{\eta})$. p is a positive integer.

Applications to Adaptive Control

For most vehicles the parameters of the dynamic model (9) are unknown or impeded by uncertainty. Besides, they can be slowly time varying. The model based control law (24) is easily extended to an adaptive version by utilizing that the model is linear in all parameters. Moreover, we can write:

$$\boldsymbol{M}\dot{\boldsymbol{\nu}} + \boldsymbol{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{g} = \boldsymbol{\Phi}(\boldsymbol{q},\boldsymbol{\nu},\dot{\boldsymbol{\nu}})\boldsymbol{\theta} \tag{44}$$

where $\boldsymbol{\Phi}(\boldsymbol{q},\boldsymbol{\nu},\dot{\boldsymbol{\nu}})$ is the regressor matrix which consists of known parameters and signals, and $\boldsymbol{\theta}$ is a vector of unknown parameters. Let $\hat{\boldsymbol{\theta}}$ denote the parameter estimate vector, and let the parameter error vector be denoted by $\tilde{\boldsymbol{\theta}} = \boldsymbol{\theta} - \hat{\boldsymbol{\theta}}$. The parameter error vector is included in the Lyapunov function as follows:

$$V = \frac{1}{2}\boldsymbol{s}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{s} + \frac{1}{2}\tilde{\boldsymbol{\theta}}^{\mathrm{T}}\boldsymbol{\Gamma}\tilde{\boldsymbol{\theta}} \tag{45}$$

Using the parameter estimates in the model based control law (24) yields:

$$\boldsymbol{\tau} = \boldsymbol{\Phi}(\boldsymbol{q},\boldsymbol{\nu},\boldsymbol{\nu}_r,\dot{\boldsymbol{\nu}}_r)\hat{\boldsymbol{\theta}} - \boldsymbol{K}_d\boldsymbol{s} \tag{46}$$

By choosing the parameter update law as $\dot{\hat{\boldsymbol{\theta}}} = \dot{\tilde{\boldsymbol{\theta}}} = -\boldsymbol{\Gamma}^{-1}\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{q},\boldsymbol{\nu},\boldsymbol{\nu}_r,\dot{\boldsymbol{\nu}}_r)\boldsymbol{s}$, $\dot{V}$ becomes negative definite. Hence, globally uniformly asymptotically stability of the equilibrium point $\boldsymbol{s} = \boldsymbol{0}$ is obtained. Notice that convergence of the parameter estimates to their true values is not guaranteed.

4. Simulation study

The control laws have been simulated for an underwater vehicle in 6 DOF with $m = 185$ kg given by the following set of parameters:

$$\begin{aligned} \boldsymbol{M} &= \mathrm{diag}\{215, 265, 265, 40, 80, 80\} \\ \boldsymbol{D}(\boldsymbol{\nu}) &= \mathrm{diag}\{70, 100, 100, 30, 50, 50\} \\ &+ \mathrm{diag}\{100|u|, 200|v|, 200|w|, 50|p|, 100|q|, 100|r|\} \end{aligned}$$

with $\boldsymbol{C}(\boldsymbol{\nu})$ given by (11). The vehicle is assumed to be neutrally buoyant. The control law parameters were chosen as $c = 1$, $\boldsymbol{K}_P = \boldsymbol{R}^{\mathrm{T}}(\boldsymbol{q}_d)$ and $\boldsymbol{K}_d = \boldsymbol{I}_{6\times 6}$. The desired position $\boldsymbol{x}_d$ was generated from 3rd-order filtering of a square wave shifting between 0 and 5 (m) with period 20 (sec). The desired attitude signal was generated from an angle-axis parameterisation of $SO(3)$, that is $\boldsymbol{R}_d = \boldsymbol{R}(\alpha_d, \boldsymbol{n}_d)$. The rotation axis was chosen constant $\boldsymbol{n}_d = (1/\sqrt{3})\,[1\,1\,-1]^{\mathrm{T}}$ whereas the desired angle α_d was generated from 3rd-order filtering of a square wave shifting between 0 and $2\pi/3$ (rad) with period 20 (sec). The desired attitude $\boldsymbol{q}_d$ and its derivatives were computed from, see (37):

$$\eta_d = \cos\frac{\alpha_d}{2}, \qquad \boldsymbol{\epsilon}_d = \sin\frac{\alpha_d}{2}\boldsymbol{n}_d$$

The desired linear and angular velocities were specified according to:

$$\begin{aligned} \boldsymbol{v}_d &= \boldsymbol{R}^{\mathrm{T}}(\boldsymbol{q})\dot{\boldsymbol{x}}_d \\ \dot{\boldsymbol{v}}_d &= \boldsymbol{R}^{\mathrm{T}}(\boldsymbol{q})\ddot{\boldsymbol{x}}_d - \boldsymbol{S}(\boldsymbol{\omega})\boldsymbol{R}^{\mathrm{T}}(\boldsymbol{q})\dot{\boldsymbol{x}}_d \\ \boldsymbol{\omega}_d &= 2\boldsymbol{R}^{\mathrm{T}}(\tilde{\boldsymbol{q}})\boldsymbol{U}^{\mathrm{T}}(\boldsymbol{q}_d)\dot{\boldsymbol{q}}_d \\ \dot{\boldsymbol{\omega}}_d &= 2\boldsymbol{R}^{\mathrm{T}}(\tilde{\boldsymbol{q}})\boldsymbol{U}^{\mathrm{T}}(\boldsymbol{q}_d)\ddot{\boldsymbol{q}}_d - 2\boldsymbol{S}(\tilde{\boldsymbol{\omega}})\boldsymbol{R}^{\mathrm{T}}(\tilde{\boldsymbol{q}})\boldsymbol{U}^{\mathrm{T}}(\boldsymbol{q}_d)\dot{\boldsymbol{q}}_d \end{aligned}$$

which are consistent with (7), (8), (18) and (33). The initial values were chosen as $\tilde{\boldsymbol{\xi}}(0) = [\mathbf{0}^{\mathrm{T}}\, 1\, \mathbf{0}^{\mathrm{T}}]^{\mathrm{T}}$ and $\dot{\tilde{\boldsymbol{\xi}}}(0) = \mathbf{0}$. All simulations were performed by applying Runge-Kutta's 4th-order method with sampling time 0.1 (sec). Figure 1 shows the simulation results for the adaptive control law (46) in terms of vector quaternion feedback. The parameter update gains were chosen as $\boldsymbol{\Gamma} = 0.02\boldsymbol{I}_{6\times 6}$. The parameter estimate vector were set to $\hat{\boldsymbol{\theta}}(0) = \mathbf{0}$ initially. The tracking performance is very good after one cycle.

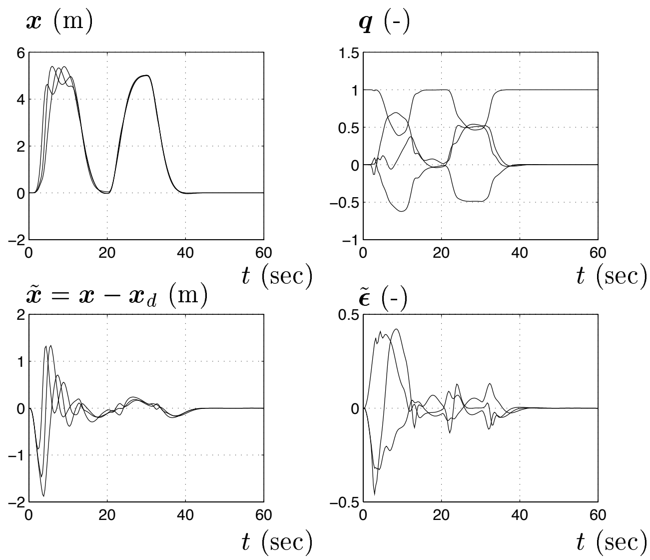

Figure 1: Vector quaternion feedback: Adaptive case.

5. Conclusions

An extension of the 3 DOF passivity based adaptive attitude controller of Egeland and Godhavn (Egeland & Godhavn 1994) to 6 DOF vehicle control has been made. Lyapunov analysis was used to prove convergence of position and attitude.

For the rotational dynamics perfect tracking has been shown for a controller based on a generalized Lyapunov function. This includes feedback from the vector quaternion, the Euler rotation and the Rodrigues parameters among others.

For the translational motion a new feedback gain matrix has been proposed. The position error dynamics is affected by the rotational dynamics. Moreover, the position error converges to zero only if the attitude error satisfies $\tilde{\eta}^2 > 1/2$ (or $|\tilde{\alpha}| < \pi/2$). However, for practical implementations this is not a problem.

The adaptive version of the control scheme has also been presented and simulated with excellent overall system performance.

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