An Output Feedback Controller with Wave Filter for Marine Vehicles

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Abstract

A controller with wave filter for marine vehicles in terms of the nonlinear dynamical model is derived by using a structure based on passive components. Only measurement of the position/attitude is needed to assure asymptotic stability for the position/attitude and the velocity of the vehicle. A simulation study of a ship autopilot illustrates the design procedure.

1. Introduction

One of the main problems in marine vehicle control, is the wave disturbances. To avoid unnecessary use of and wear on actuators we do not want to compensate for 1st-order (oscillatoric) wave disturbances. It is therefore desirable if the effect of these disturbances do not reach the control input. Thus, wave filtering is needed. The following design criteria for the control system are emphasized:

1) The position/attitude of the vehicle should be kept at a desired constant value (1st-order wave disturbances should, however, not be compensated for).
2) Use of actuators due to 1st-order wave disturbances should be avoided (wave filtering).
3) No measurements of the velocity.

Previous work on position/attitude control of marine vehicles include papers on dynamic positioning and autopilot design, see [4]. For wave filtering and state observing, Kalman filters have shown to give good results. They provide a wave filtered estimate of the states which in turn is used for feedback control. However, the robustness of this design to nonlinearities is difficult to analyze, and in the case of parameter uncertainties, stability problems can occur. Also the tuning of the noise covariance matrices can be rather complicated. Another approach to state estimation and wave filtering is to use an observer, and then remove the wave disturbances from the signals by using a notch or dead-band filter. However, the observer and filter are designed separately. This makes it difficult to establish stability properties for nonlinear vehicles.

In [5], a system for dynamic positioning with wave filtering for nonlinear surface vessels using a passive, linear controller structure is derived. The total closed-loop system with controller and wave filter is globally asymptotically stable. In this paper, this approach is modified to avoid measurements of the velocity. Asymptotic stability for the position/attitude about the desired position/attitude, is proven. This paper also shows how wave filtering in a wider frequency range than the one in [5] can be obtained.

2. Dynamical Model of a Marine Vehicle

The dynamical model of a six DOF marine vehicle on which the stability proof and controller design is based, is expressed in the body-fixed reference frame by, [4],

$$\boldsymbol{M}\dot{\boldsymbol{\nu}} + \boldsymbol{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{g}(\boldsymbol{\eta}) = \boldsymbol{\tau} \tag{1}$$

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\boldsymbol{\eta})\boldsymbol{\nu} \tag{2}$$

Here, (1) is the vehicle dynamics and (2) is the kinematics. $\boldsymbol{\nu} = (u\,,\,v\,,\,w\,,\,p\,,\,q\,,\,r)^T$ is a vector of body-fixed linear and angular velocity components, and $\boldsymbol{\eta} = (x\,,\,y\,,\,z\,,\,\phi\,,\,\theta\,,\,\psi)^T$ is a vector of positions and Euler angles. The components of $\boldsymbol{\eta}$ and $\boldsymbol{\nu}$ correspond to the motion variables in surge, sway, heave, roll, pitch, and yaw, respectively. $\boldsymbol{\tau} \in R^6$ is the control vector. $\boldsymbol{g}(\boldsymbol{\eta}) \in R^6$ is a vector of restoring forces and moments. $\boldsymbol{J}(\boldsymbol{\eta})$ is a 6×6 block diagonal transformation matrix relating the body-fixed frame to the inertial reference frame (usually the earth). $\boldsymbol{J}(\boldsymbol{\eta})$ only depends on the Euler angles (ϕ, θ, ψ). It is not defined for a pitch angle at $\theta = \pm 90^o$. $\boldsymbol{M}$ is the inertia matrix, $\boldsymbol{C}(\boldsymbol{\nu})$ is a matrix of Coriolis and centripetal terms, and $\boldsymbol{D}(\boldsymbol{\nu})$ is a matrix of hydrodynamic damping terms. The matrices $\boldsymbol{M}$, $\boldsymbol{C}$, and $\boldsymbol{D}$ have the following properties (assuming wave frequency independence), [4]:

1) $\boldsymbol{M} = \boldsymbol{M}^T > 0 \quad \dot{\boldsymbol{M}} = \boldsymbol{0}$

2) $\boldsymbol{C}(\boldsymbol{\nu}) = -\boldsymbol{C}^T(\boldsymbol{\nu}) \quad \forall\, \boldsymbol{\nu} \in R^6$

3) $\boldsymbol{D}(\boldsymbol{\nu}) > 0 \quad \forall\, \boldsymbol{\nu} \in R^6\,,\, \boldsymbol{\nu} \neq \boldsymbol{0}$

The dynamical model (1) can be expressed in the earth-fixed reference frame as

$$\boldsymbol{M}_\eta(\boldsymbol{\eta})\ddot{\boldsymbol{\eta}} + \boldsymbol{C}_\eta(\boldsymbol{\nu},\boldsymbol{\eta})\dot{\boldsymbol{\eta}} + \boldsymbol{D}_\eta(\boldsymbol{\nu},\boldsymbol{\eta})\dot{\boldsymbol{\eta}} + \boldsymbol{g}_\eta(\boldsymbol{\eta}) = \boldsymbol{J}^{-T}(\boldsymbol{\eta})\boldsymbol{\tau} \tag{3}$$

where

$$\begin{aligned}
\boldsymbol{M}_\eta(\boldsymbol{\eta}) &= \boldsymbol{J}^{-T}\boldsymbol{M}\boldsymbol{J}^{-1} \\
\boldsymbol{C}_\eta(\boldsymbol{\nu},\boldsymbol{\eta}) &= \boldsymbol{J}^{-T}[\boldsymbol{C} - \boldsymbol{M}\boldsymbol{J}^{-1}\dot{\boldsymbol{J}}]\boldsymbol{J}^{-1} \\
\boldsymbol{D}_\eta(\boldsymbol{\nu},\boldsymbol{\eta}) &= \boldsymbol{J}^{-T}\boldsymbol{D}\boldsymbol{J}^{-1} \\
\boldsymbol{g}_\eta(\boldsymbol{\eta}) &= \boldsymbol{J}^{-T}\boldsymbol{g}
\end{aligned}$$

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The system (3) satisfies the following properties:

4) $\boldsymbol{M}_\eta(\boldsymbol{\eta}) = \boldsymbol{M}_\eta^T(\boldsymbol{\eta}) > 0 \quad \forall \boldsymbol{\eta} \in R^6$

5) $\boldsymbol{D}_\eta(\boldsymbol{\nu},\boldsymbol{\eta}) > 0 \quad \forall \boldsymbol{\eta},\boldsymbol{\nu} \in R^6,\ \boldsymbol{\nu} \neq \boldsymbol{0}$

6) $(\dot{\boldsymbol{M}}_\eta - 2\boldsymbol{C}_\eta)$ is skew-symmetric, i.e.

$\boldsymbol{x}^T[\dot{\boldsymbol{M}}_\eta(\boldsymbol{\eta}) - 2\boldsymbol{C}_\eta(\boldsymbol{\nu},\boldsymbol{\eta})]\boldsymbol{x} = 0 \quad \forall \boldsymbol{\eta},\boldsymbol{\nu},\boldsymbol{x} \in R^6$

3. Main Result

The controller can be thought of as a virtual system of N masses, $N+1$ dampers, and $N+2$ springs. Fig. 1 shows how the vehicle and control system can be interpreted as a mechanical system. In addition, the controller consists of a compensator for the restoring forces and moments $\boldsymbol{g}(\boldsymbol{\eta})$, see (4). We propose the following control law:

$$\boldsymbol{\tau} = \boldsymbol{J}^T(\boldsymbol{\eta})\,\boldsymbol{K}_{P0}(\boldsymbol{\eta}_0 - \boldsymbol{\eta}) + \boldsymbol{g}(\boldsymbol{\eta}) \qquad (4)$$

where $\boldsymbol{K}_{P0} = \boldsymbol{K}_{P0}^T > 0$ is a constant matrix, $\boldsymbol{g}(\boldsymbol{\eta})$ is the vector of restoring forces and moments, $\boldsymbol{\eta}$ is the measured position/attitude, and $\boldsymbol{\eta}_0$ is a virtual reference trajectory given by the force balance equations, which are derived from the system in Fig. 1:

$$\boldsymbol{K}_{P0}(\boldsymbol{\eta}_0 - \boldsymbol{\eta}) = \boldsymbol{K}_{P1}(\boldsymbol{\eta}_1 - \boldsymbol{\eta}_0) + \boldsymbol{K}_{D1}(\dot{\boldsymbol{\eta}}_1 - \dot{\boldsymbol{\eta}}_0) \qquad (5)$$

where $\boldsymbol{\eta}_i$, $i \in \{1,..,N\}$, are given by the equations

$$\begin{gathered}\boldsymbol{M}_i\ddot{\boldsymbol{\eta}}_i + \boldsymbol{K}_{Pi}(\boldsymbol{\eta}_i - \boldsymbol{\eta}_{i-1}) + \boldsymbol{K}_{Di}(\dot{\boldsymbol{\eta}}_i - \dot{\boldsymbol{\eta}}_{i-1}) \\ + \boldsymbol{K}_{P(i+1)}(\boldsymbol{\eta}_i - \boldsymbol{\eta}_{i+1}) + \boldsymbol{K}_{D(i+1)}(\dot{\boldsymbol{\eta}}_i - \dot{\boldsymbol{\eta}}_{i+1}) = \boldsymbol{0} \\ \vdots \\ \dot{\boldsymbol{\eta}}_{N+1} = \boldsymbol{0}\end{gathered} \qquad (6)$$

Here $\boldsymbol{\eta}_{N+1} = \boldsymbol{\eta}_d$ (constant) is the desired position/attitude. $\boldsymbol{K}_{Pi} = \boldsymbol{K}_{Pi}^T > 0$, $\boldsymbol{K}_{Di} = \boldsymbol{K}_{Di}^T > 0$, $i \in \{1,..,N+1\}$, and $\boldsymbol{M}_i = \boldsymbol{M}_i^T > 0$, $i \in \{1,..,N\}$, are constant regulator design matrices. We select the state vector as

$$\begin{aligned}\boldsymbol{x} &= [(\boldsymbol{\eta} - \boldsymbol{\eta}_0)^T, (\boldsymbol{\eta}_0 - \boldsymbol{\eta}_1)^T, \cdots, (\boldsymbol{\eta}_i - \boldsymbol{\eta}_{i+1})^T, \\ &\quad \cdots, (\boldsymbol{\eta}_N - \boldsymbol{\eta}_{N+1})^T, \dot{\boldsymbol{\eta}}^T, \cdots, \dot{\boldsymbol{\eta}}_i^T, \cdots, \dot{\boldsymbol{\eta}}_N^T]^T\end{aligned} \qquad (7)$$

for $i \in \{1,..,N\}$. Note that $\boldsymbol{x} \in R^{6(3+2N)}$.

Theorem 1 Consider the marine vehicle (3) with controller (4), (5), and (6). This system is asymptotically stable about $\boldsymbol{x} = \boldsymbol{0}$, where $\boldsymbol{x}$ is given in (7). Consequently, $\boldsymbol{\eta}$ is asymptotically stable about $\boldsymbol{\eta}_{N+1} = \boldsymbol{\eta}_d$.

Proof: See Appendix A.

Remark 1: The system is globally asymptotically stable for all states exept for a pitch angle $\theta = \pm 90^o$. This is because the transformation matrix $\boldsymbol{J}(\boldsymbol{\eta})$ is not defined at $\theta = \pm 90^o$.

Remark 2: In the case of no wave disturbances, no filtering is required. Thus, the number of virtual masses can be chosen to be zero. In this case, our controller reduces to:

$$\begin{aligned}\boldsymbol{\tau} &= \boldsymbol{J}^T[\boldsymbol{K}_{P1}(\boldsymbol{\eta}_d - \boldsymbol{\eta}_0) - \boldsymbol{K}_{D1}\dot{\boldsymbol{\eta}}_0] + \boldsymbol{g} \\ \dot{\boldsymbol{\eta}}_0 &= \boldsymbol{K}_{D1}^{-1}[-(\boldsymbol{K}_{P0} + \boldsymbol{K}_{P1})\boldsymbol{\eta}_0 + \boldsymbol{K}_{P0}\boldsymbol{\eta} + \boldsymbol{K}_{P1}\boldsymbol{\eta}_d]\end{aligned}$$

We see that in this case the controller has a PD structure, and a linear observer provides estimates of the position/attitude $\boldsymbol{\eta}_0$ and velocity $\dot{\boldsymbol{\eta}}_0$. Except for the transformation matrix $\boldsymbol{J}^T$, this controller has many similarities to robot controllers presented in [1], [2] and [6].

4. Analysis of Wave Influence on the Control Input

As mentioned earlier, one of the control problems of marine vehicles is the disturbances caused by waves. In this section, the frequency response properties of a SISO linear model of a marine vehicle is analyzed to show that by proper selection of the controller parameters, the wave modulation on the control input can be reduced.

We want to determine the transfer function between the control input and the external disturbances $\frac{\delta(s)}{w(s)}$ to show that the control action may be attenuated in some frequency range around the dominant wave frequency.

Suppose that some external disturbances w, perhaps caused by waves, are acting on the vehicle. Consider the linear ship model in [7]

$$m\ddot{\psi} + d\dot{\psi} = \delta + w \qquad (8)$$

where δ is the control input (rudder), m is the constant moment of inertia of the ship, d is a constant damping term, and ψ is the yaw angle (heading) of the ship which is measured.

The controller (4), (5), and (6) with only one virtual mass ($N = 1$) and the ship model (8) is considered and given in the s-plane as

$$\begin{aligned}(ms^2 + ds)\psi(s) &= \delta(s) + w(s) \\ \delta(s) &= K_{P0}(\psi_0(s) - \psi(s)) \\ K_{P0}(\psi(s) - \psi_0(s)) &= (K_{D1}s + K_{P1})\tilde{\psi}(s) \\ (m_1s^2 + K_{D2}s + K_{P2})\psi_1(s) &= (K_{D1}s + K_{P1})\tilde{\psi}(s)\end{aligned}$$

where $\tilde{\psi} = \psi_0 - \psi_1$. For simplicity, the desired yaw angle is chosen to be $\psi_d = 0$.

The transfer function between δ and w becomes

$$\frac{\delta}{w}(s) = \frac{-(K_{D1}s + K_{P1})h_2(s)}{(ms^2 + d_1s)h_4(s) + h_1(s)h_2(s)} \qquad (9)$$

where $h_1(s)$, $h_2(s)$, $h_3(s)$, and $h_4(s)$ are defined as

$$\begin{aligned}h_1(s) &= K_{D1}s + K_{P1} \\ h_2(s) &= m_1s^2 + K_{D2}s + K_{P2} \\ h_3(s) &= K_{D1}s + K_{P1} + K_{P0} \\ h_4(s) &= h_1(s) + K_{P0}^{-1}h_2(s)h_3(s)\end{aligned} \qquad (10)$$

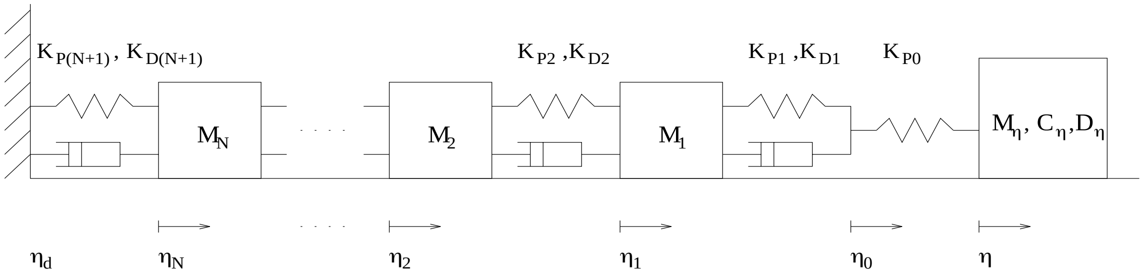

Figure 1: Vessel with virtual control system

From (9) it can be seen that by proper choices of the transfer functions $h_i(s)$, $i \in \{1, .., 4\}$, the wave influenced motion on the rudder can be reduced. This is possible by letting the transfer function $h_2(s)$ have complex conjugated zeros with small damping and by ensuring that the denominator of $\frac{\delta}{w}(s)$ is not too small.

Moreover, we rewrite (10) as

$$h_2(s) = m_1(s^2 + 2\xi\omega_n s + \omega_n^2)$$

where ξ and ω_n are constants which can be interpreted as the relative damping factor and the natural frequency, respectively. If ω_n is chosen equal to the dominant wave frequency, which is assumed to be known with some accuracy, suppression of the wave influence on the control input will be acheived.

For simplicity, we have only analyzed the case with one virtual mass in this section. In Section 5, however, a comparison in the frequency domain between a controller with one ($N = 1$) and with four virtual masses ($N = 4$) is given. This is to investigate how the filtering properties of the controller depend on N.

5. Ship Autopilot

The following nonlinear ship steering model was proposed in [8]

$$m\ddot{\psi} + d(\dot{\psi})\dot{\psi} = \delta + w \quad (11)$$

where $d(\dot{\psi}) = d_1 + d_3\dot{\psi}^2$, and d_1 and d_3 are damping terms which are assumed to be positive (course-stable ship). Similar to Section 4, m is a positive constant, ψ is the yaw angle which is measured, $\dot{\psi}$ is the yaw rate, δ is the rudder angle (control input), and w is the wave disturbance.

The model parameters used in this example is adopted from [9]. In this reference, "the R.O.V. Zeefakkel" is described by the following set of parameters: $m = 62$, $d_1 = 2$, and $d_3 = 0.8$.

5.1. Frequency response analysis

In this section we want to establish what influence N has on the frequency range where good filtering properties can be achieved.

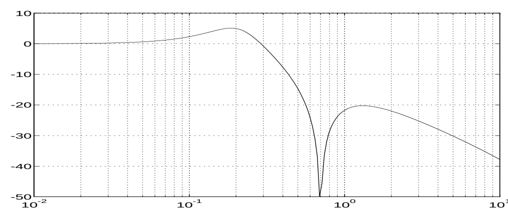

Figure 2: Amplitude $|\frac{\delta}{w}(s)|$ in dB as a function of frequency, ($N = 1$).

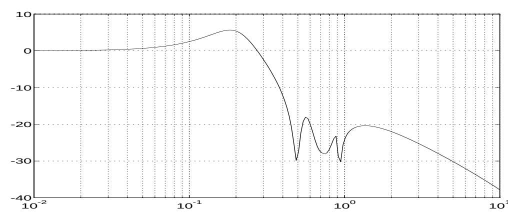

Figure 3: Amplitude $|\frac{\delta}{w}(s)|$ in dB as a function of frequency, ($N = 4$).

We study frequency plots of the transfer function $\frac{\delta}{w}(s)$ for two controllers given by ($N = 1$) and ($N = 4$) applied to the ship model (11) with $d_3 = 0$. The parameters for both controllers are chosen as

$$\begin{array}{lcl}
m_1 & = & 0.25\,m \\
m_i & = & 6\,m_{i-1}, \quad (2 \le i \le 4) \\
\omega_i & = & 0.7, \quad (1 \le i \le 4) \\
\xi_i & = & 0.0001, \quad (1 \le i \le 3) \\
\xi_4 & = & 0.26 \\
K_{P(i+1)} & = & m_i\omega_i^2, \quad (1 \le i \le 4) \\
K_{D(i+1)} & = & 2\xi_i\omega_i m_i, \quad (1 \le i \le 4) \\
K_{D1} & = & 7000 \cdot K_{D2} \\
K_{P1} & = & 0.2 \cdot K_{D1} \\
K_{P0} & = & 100 \cdot K_{P1}
\end{array}$$

The amplitudes of the transfer functions, $\frac{\delta}{w}(s)$, for $N = 1$ and $N = 4$ are shown in Fig. 2 and Fig. 3, respectively. The wave disturbances are suppressed around $\omega_n = 0.7$rad/s (notch effect). Note that we obtain filtering properties in a wider frequency range by increasing the number of virtual masses. However, the damping amplitude at the dominating wave frequency is smaller for the controller with $N = 4$.

5.2. Simulation Study

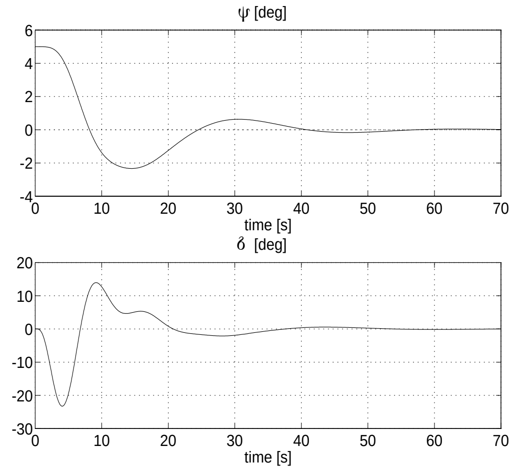

Figure 4: Yaw angle ψ and rudder angle δ versus time, $(w = 0)$.

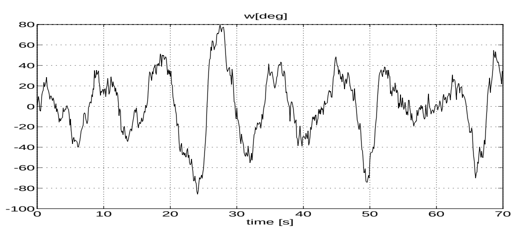

Figure 5: Wave disturbance w versus time.

In this section we study the properties of our controller applied to the ship model (11) in the time domain. A controller with two virtual masses ($N = 2$) is chosen. The 1st-order wave disturbance, w, is given as

$$\dot{\boldsymbol{x}}_w = \boldsymbol{A}_w \boldsymbol{x}_w + \boldsymbol{b}_w \eta\,, \quad w = \boldsymbol{c}_w^T \boldsymbol{x}_w \tag{12}$$

where

$$\boldsymbol{A}_w = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\xi\omega_n \end{bmatrix}$$

$$\boldsymbol{b}_w = [0 \;\; K_w]^T,\; \boldsymbol{c}_w = [0 \;\; 1]^T$$

η is a zero mean Gaussian white noise sequence, ω_n is the dominating wave frequency, ξ is the relative damping ratio of the waves, and K_w is a gain that is dependent on the wave energy. This model is motivated by the Pierson-Moskowitz wave spectrum [3]. The parameters of the wave model are chosen as: $\omega_n = 0.7$, $\xi = 0.1$, and $K_w = 1.0$.

In the simulation study a sampling frequency of 10 Hz is used. The desired yaw angle $\psi_d = 0$. The controller

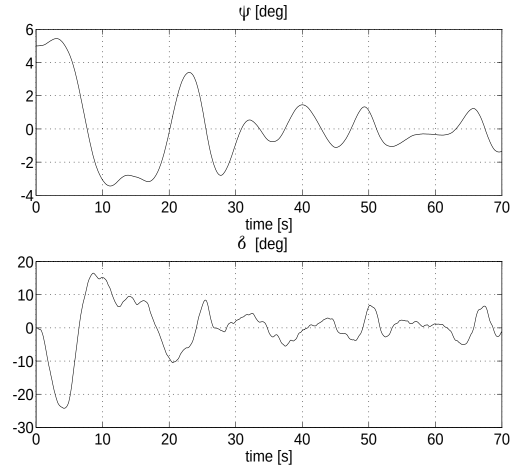

Figure 6: Yaw angle ψ and rudder angle δ versus time using the control strategy in this paper, with wave disturbance w.

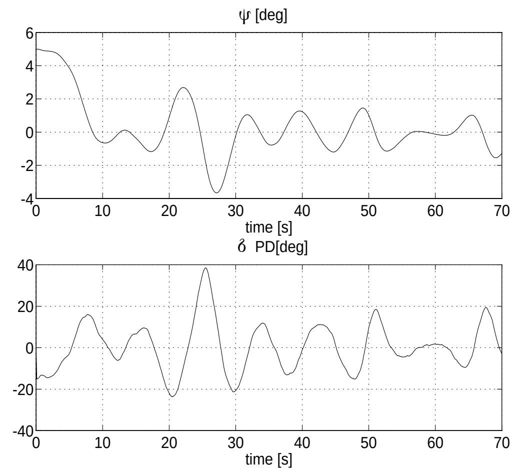

Figure 7: Yaw angle ψ and rudder angle δ_{PD} versus time using a conventional PD-controller, with wave disturbance w.

parameters are chosen to be

$$\begin{array}{lllllll} m_1 & = & 0.25\,m & \quad & K_{P3} & = & m_2\omega_2^2 \\ m_2 & = & 6\,m_1 & & K_{D3} & = & 2\xi_2\omega_2 m_2 \\ \omega_1 & = & \omega_n & & K_{P2} & = & m_1\omega_1^2 \\ \omega_2 & = & \omega_n & & K_{D2} & = & 2\xi_1\omega_1 m_1 \\ \xi_1 & = & 0.0001 & & K_{D1} & = & K_{D2}\cdot 7000 \\ \xi_2 & = & 0.26 & & K_{P1} & = & K_{D1}\cdot 0.2 \\ & & & & K_{P0} & = & K_{P1}\cdot 100 \end{array}$$

At first we assume no disturbances, that is $w = 0$. The yaw and rudder angle are shown in Fig. 4. We see that the yaw angle converges to the desired yaw angle in finite time.

We now add the disturbance given in (12), see Fig. 5. We want to compare the yaw and rudder angle that are obtained by using the control strategy in this paper and by using a conventional PD -controller given as

$$\delta = \delta_{PD} = -K_{P1}\psi - K_{D1}\dot{\psi}$$

The results are shown in Fig. 6 and Fig. 7, respectively. The simulations show that the rudder action for the output feedback controller with wave filtering is significantly reduced compared to the conventional PD-controller.

5.3. Comments on Parameter Tuning

SISO Systems:

Section 4 and the numerical example suggest that the parameters of the controller should be chosen in a systematic manner. It seems reasonable to choose the parameter m_1 smaller than the ship's moment of inertia m, (approximately $0.1\,m - 0.3\,m$). The values of m_1 and m_i in our case study are chosen by studying the frequency responses and simulation results. The ω_i's can be chosen equal to the dominant wave frequency ω_n. The values of ξ_i determine the damping amplitude around the dominant wave frequency. We see that K_{Pi} and K_{Di}, $i \in \{2..N\}$, should be chosen as functions of m_i, ξ_i and ω_i. K_{P1} and K_{D1} will have influence on the bandwidth of the controller. K_{P0} should be chosen larger than K_{P1} in order to make the observer faster than the controller.

MIMO Systems:

The constant regulator design matrices, $\boldsymbol{M}_i$, $\boldsymbol{K}_{Pi}$, and $\boldsymbol{K}_{Di}$ can be chosen diagonal. The tuning of the parameters will then follow approximately the same procedure as for the SISO case.

6. Concluding Remarks

A marine vehicle output feedback controller with wave filter has been derived. The controller could be thought of as a virtual system of passive elements, like dampers, springs, and masses. Asymptotic stability has been proven.

A frequency response analysis showed that the rudder action could be suppressed in the area around the dominant wave frequency by proper choices of the controller parameters. Numerical examples showed that the frequency range where wave filtering could be obtained was dependent on the number of virtual masses.

In the simulation study, we showed that the rudder action for our controller was significantly reduced compared to a PD-controller without wave filter.

Appendix A - Proof of Theorem 1

Consider the Lyapunov function candidate

$$\begin{aligned} V(\boldsymbol{x}) \;=\; & \frac{1}{2}[\dot{\boldsymbol{\eta}}^T\boldsymbol{M}_\eta\dot{\boldsymbol{\eta}} + \dot{\boldsymbol{\eta}}_1^T\boldsymbol{M}_1\dot{\boldsymbol{\eta}}_1 + \cdots \\ & + \dot{\boldsymbol{\eta}}_i^T\boldsymbol{M}_i\dot{\boldsymbol{\eta}}_i + \cdots + \dot{\boldsymbol{\eta}}_N^T\boldsymbol{M}_N\dot{\boldsymbol{\eta}}_N \\ & + (\boldsymbol{\eta} - \boldsymbol{\eta}_0)^T\boldsymbol{K}_{P0}(\boldsymbol{\eta} - \boldsymbol{\eta}_0) \\ & + (\boldsymbol{\eta}_0 - \boldsymbol{\eta}_1)^T\boldsymbol{K}_{P1}(\boldsymbol{\eta}_0 - \boldsymbol{\eta}_1) + \cdots \\ & + (\boldsymbol{\eta}_i - \boldsymbol{\eta}_{i+1})^T\boldsymbol{K}_{P(i+1)}(\boldsymbol{\eta}_i - \boldsymbol{\eta}_{i+1}) + \cdots \\ & + (\boldsymbol{\eta}_N - \boldsymbol{\eta}_{N+1})^T\boldsymbol{K}_{P(N+1)}(\boldsymbol{\eta}_N - \boldsymbol{\eta}_{N+1})] \end{aligned}$$

which can be thought of as a sum of the kinetic and potential energy of the system. Differentiating $V(\boldsymbol{x})$ with respect to time, substituting (3), (4), (5), and (6) into $\dot{V}$, and using Property 6 gives

$$\begin{aligned} \dot{V} \;=\; & -\dot{\boldsymbol{\eta}}^T\boldsymbol{D}_\eta(\boldsymbol{\nu},\boldsymbol{\eta})\dot{\boldsymbol{\eta}} - (\dot{\boldsymbol{\eta}}_1 - \dot{\boldsymbol{\eta}}_0)^T\boldsymbol{K}_{D1}(\dot{\boldsymbol{\eta}}_1 - \dot{\boldsymbol{\eta}}_0) \\ & - (\dot{\boldsymbol{\eta}}_1 - \dot{\boldsymbol{\eta}}_2)^T\boldsymbol{K}_{D2}(\dot{\boldsymbol{\eta}}_1 - \dot{\boldsymbol{\eta}}_2) - \cdots \\ & - (\dot{\boldsymbol{\eta}}_{i-1} - \dot{\boldsymbol{\eta}}_i)^T\boldsymbol{K}_{Di}(\dot{\boldsymbol{\eta}}_{i-1} - \dot{\boldsymbol{\eta}}_i) - \cdots \\ & - (\dot{\boldsymbol{\eta}}_{N-1} - \dot{\boldsymbol{\eta}}_N)^T\boldsymbol{K}_{Dn}(\dot{\boldsymbol{\eta}}_{N-1} - \dot{\boldsymbol{\eta}}_N) \\ & - \dot{\boldsymbol{\eta}}_N^T\boldsymbol{K}_{D(N+1)}\dot{\boldsymbol{\eta}}_N \;\le 0 \end{aligned}$$

Finally, by applying La Salle's invariant set theorem [10], we find that the equilibrium $\boldsymbol{x} = \boldsymbol{0}$ is asymptotically stable.

Note that the system will be asymptotically stable even when $\boldsymbol{D}_\eta(\boldsymbol{\nu},\boldsymbol{\eta})$ is equal to zero. This is because we can conclude that $\boldsymbol{\eta}$ is constant and thus that $\dot{\boldsymbol{\eta}}$ is zero from $\dot{V} = 0$.

□

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