Modelling and Control of Underwater Vehicle–Manipulator Systems

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Abstract

In this paper a dynamic model of an underwater vehicle–manipulator system is derived. The iterative Newton–Euler algorithm has been extended to include the most dominating hydrodynamical forces. The model is written in a closed form. It is emphasized on representing the nonlinear model such that mechanical system properties like symmetry, skew-symmetry and positiveness can be used in the control design. Feedback linearization is used for trajectory control of the system.

Keywords: Underwater manipulator, vehicle, vehicle–manipulator system, dynamics, feedback linearization.

1 Introduction

Semi-autonomous robotic systems have been suggested for space and underwater operations. These operations include inspection, maintenance, repair and service work on space and underwater installations. Accurate control of both the vehicle and the manipulator is essential and hence a dynamic model of the total system is necessary. Several authors have studied spacecraft–manipulator systems. A review of this research is given in Dubowsky and Papadopoulos (1993) and references therein. The work in this paper is inspired by this research. Control of underwater vehicles has been studied by many authors; see for instance Fossen (1994) and Yuh (1990). Further information on the subject can also be found in Healey and Lienard (1993). Several methods have been used to derive the dynamics of an underwater vehicle–manipulator system. Ioi and Itoh (1990) have used the classical Newton–Euler formalism to compute a recursive formula for the dynamic model. A hybrid force controller of the manipulator was simulated. Mahesh, Yuh and Lakshmi (1991) have derived a discrete-time model of the underwater system and a discrete-time adaptive controller is used to control the system. A simulation study on the dynamic behavior of an underwater robot was also done by Janocha and Papadimitriou (1991). They presented a recursive algorithm for generating the dynamic model. Finally McMillan, Orin and McGhee (1994) have developed an efficient dynamic simulation algorithm which includes a mobile base and the hydrodynamic forces.

In this paper the closed form dynamic equations are suggested for the underwater system whereas feedback linearization is applied for trajectory control of the system. The paper is outlined as follows: in Section 2 a complete model of the system is presented whereas some of the system properties is described in Section 3. Feedback linearization is applied to the system in Section 4. Section 5 holds the conclusions.

2 System Dynamics

In this section the ROV, manipulator and ROV-manipulator equations of motion are discussed.

2.1 ROV Equations of Motion

The equations of motion of an underwater vehicle can be written in an abbreviated form as, Fossen (1994) :

$$\boldsymbol{M}_r\dot{\boldsymbol{\nu}} + \boldsymbol{C}_r(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{D}_r(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{g}_r(\boldsymbol{\eta}) = \boldsymbol{\tau}_r + \boldsymbol{\sigma}_c \tag{1}$$

The kinematic transformation between the earth-fixed and the body-fixed reference frame is given by $\dot{\boldsymbol{\eta}} = \boldsymbol{J}_R(\boldsymbol{\eta})\boldsymbol{\nu}$ where

$\boldsymbol{J}_R(\boldsymbol{\eta})$	defined in Fossen (1994)
$\boldsymbol{M}_r$	inertia matrix (included added inertia)
$\boldsymbol{C}_r(\boldsymbol{\nu})$	contains Coriolis and centripetal terms
$\boldsymbol{D}_r(\boldsymbol{\nu})$	hydrodynamic lift and damping matrix
$\boldsymbol{g}_r(\boldsymbol{\eta})$	vector of gravity and buoyancy forces
$\boldsymbol{\tau}_r$	propulsion forces
$\boldsymbol{\sigma}_c$	current induced disturbances (constant)
$\boldsymbol{\nu}$	linear and angular velocity vector with coordinates in the body-fixed frame
$\boldsymbol{\eta}$	position and orientation vector with coordinates in the earth-fixed frame

The orientation vector is represented in Euler angles. The inertia matrix is positive definite and the Coriolis and centripetal matrix is skew-symmetrical. In the earth-fixed reference frame the equations of motion are given by:

$$\boldsymbol{M}_{r_\eta}\dot{\boldsymbol{\eta}} + \boldsymbol{C}_{r_\eta}(\boldsymbol{\nu},\boldsymbol{\eta})\dot{\boldsymbol{\eta}} + \boldsymbol{D}_{r_\eta}(\boldsymbol{\nu},\boldsymbol{\eta})\dot{\boldsymbol{\eta}} + \boldsymbol{g}_{r_\eta}(\boldsymbol{\eta}) = \boldsymbol{\tau}_{r_\eta} + \boldsymbol{\sigma}_{c_\eta} \tag{2}$$

where the expressions for $\boldsymbol{M}_{r_\eta}$,$\boldsymbol{C}_{r_\eta}(\boldsymbol{\nu},\boldsymbol{\eta})$,$\boldsymbol{D}_{r_\eta}(\boldsymbol{\nu},\boldsymbol{\eta})$, $\boldsymbol{g}_{r_\eta}(\boldsymbol{\eta})$, $\boldsymbol{\tau}_{r_\eta}$ and $\boldsymbol{\sigma}_{c_\eta}$ are given in Fossen (1994).

2.2 Underwater Manipulator Equations of Motion

It is assumed in this paper that the robot is built up of cylindrical elements. The effect of the hydrodynamical forces on cylindrical elements is described by Faltinsen (1990), Olson

(1980) and Sarpkaya (1978). For a circular cylindrical element, with diameter D, radius r and length L, moving in deep water with density ρ, the following hydrodynamical forces are acting on the element:

- **Added mass and added moment of inertia**:

$$\boldsymbol{M}_A = \mathrm{diag}(M_{A_{11}}, M_{A_{22}}, M_{A_{33}}, M_{A_{44}}, M_{A_{55}}, M_{A_{66}})$$

  where

$$\begin{aligned} M_{A_{11}} &= 10\% \text{ of the vehicle mass} & M_{A_{44}} &= 0 \\ M_{A_{22}} &= \pi \rho r^2 L & M_{A_{55}} &= \tfrac{1}{12}\pi \rho r^2 L^3 \\ M_{A_{33}} &= \pi \rho r^2 L & M_{A_{66}} &= \tfrac{1}{12}\pi \rho r^2 L^3 \end{aligned}$$

  Notice that the off-diagonal elements have been neglected.

- **Frictional forces and moments**:

  1. Lift and drag forces: lift and drag are pressure and viscous shear forces and also forces due to vortex shedding. The lift forces are orthogonal to the flow velocity and the drag forces are parallel to the flow velocity. Drag and lift forces are thoroughly described in Stevens and Lewis (1992). For underwater systems only vortex shedding contributes to the lift force. The drag force can be written as a nonlinear expansion:

$$F_D = D_S v_r + D_Q |v_r| v_r + O(v_r^3) \tag{3}$$

     where D_S and D_Q are defined below. The vector v_r is the velocity of the rigid-body relative to the flow velocity. In this paper the 3rd-order and higher order terms are neglected. The first term in (3) is mainly due to the linear skin-friction:

$$F_S = D_S v_r$$

     where D_S is the linear skin-friction coefficient. The second term in (3) is due to the quadratic drag:

$$F_Q = D_Q |v_r| v_r$$

     Here $D_Q = \frac{1}{2}\rho C_D(\mathrm{Rn}, \alpha) A(\alpha)$ is the drag coefficient, where A is a projected frontal area of the body, α is the angle of attach, Rn is the Reynold number, C_D is the drag coefficient and ρ is the water density. The lift force can be expressed in a similar manner

$$F_L = \frac{1}{2}\rho C_L(\mathrm{Rn}, \alpha) A(\alpha) |v_r| v_r$$

     The lift coefficient $C_L(\mathrm{Rn}, \alpha)$ and the drag coefficient $C_D(\mathrm{Rn}, \alpha)$ depend on the angle of attach α, the Reynold number Rn and the Keulegan-Carpenter number KC. In the control model it is reasonable to assume Keulegan-Carpenter independence. The coefficients are usually empirical determined and some values are given in Table 1. The Reynold's number, lift and drag coefficients and the Strouhal number for a smooth cylinder are as follows, Walderhaug (1991):

Table 1: Lift and drag coefficients for a cylinder.

		C_D	C_L	S_n
$\mathrm{Rn} < 2 \cdot 10^5$	subcritical flow	1.0	$[3, 0.6]$	0.2
$2 \cdot 10^5 < \mathrm{Rn} < 5 \cdot 10^5$	critical flow	$[1.0, 0.4]$	0.6	0.2
$5 \cdot 10^5 < \mathrm{Rn} < 3 \cdot 10^6$	transcritical flow	0.4	0.6	0.28

2. Vortex induced forces: the vortex shedding induces oscillatory forces on the rigid-body in both the drag and the lift direction. In subcritical and trans-critical flow these vortices are known as the Von Karman vortex street. For a single vortex shedding frequency the force in the lift direction can be approximated as, Faltinsen (1990):

$$F_{L_v} = F_L \cos(2\pi f_v t + \gamma)$$

where $f_v = S_n |v_r| / D$ is the vortex shedding frequency; S_n is the Strouhal number and D is the cylinder diameter and γ is the phase angle of the traverse force. In the drag direction the total drag is, Faltinsen (1990) :

$$F_{D_v} = F_Q + A_D \cos(4\pi f_v t + \phi)$$

where A_D is the area of the cross-section and ϕ is the phase shift. The last term of F_{D_v} is small and can therefore be neglected, leaving only the mean drag value, which is equal to the first term in Morrison's equation, Faltinsen (1990). The lift and drag forces are conveniently defined along the relative velocity axis, see Figure 1. The formulas for the frictional forces can be extended to 3D, by writing:

$$\boldsymbol{F}_S = \boldsymbol{D}_S \boldsymbol{v}, \quad \boldsymbol{F}_L = |\boldsymbol{v}|^T \boldsymbol{D}_L \boldsymbol{v}, \quad \boldsymbol{F}_D = |\boldsymbol{v}|^T \boldsymbol{D}_D \boldsymbol{v}$$

where $\boldsymbol{v} = \boldsymbol{v}_c - \boldsymbol{U}_f$ is the velocity of the rigid-body relative to the flow. The vector $\boldsymbol{v}_c$ is the velocity of the rigid-body mass center and $\boldsymbol{U}_f$ is the flow velocity. The matrix $\boldsymbol{D}_S$ contains the linear skin-friction coefficients and $\boldsymbol{D}_L$ and $\boldsymbol{D}_D$ are diagonal matrices containing the lift and drag coefficients. The diagonal elements are in the form $\frac{1}{2}\rho C_D(\mathrm{Rn})\bar{A}(\alpha, \beta)$ where $\bar{A}(\alpha, \beta)$ is the projected frontal area of the body in body-fixed coordinates.

3. Current loads: these loads are denoted the Munk moment, and are given by Faltinsen (1990).

$$\boldsymbol{M}_{m_l} = \boldsymbol{S}(\boldsymbol{v})[\mathrm{diag}(M_{A_{11}}, M_{A_{22}}, M_{A_{33}})\boldsymbol{v}]$$

where $\boldsymbol{S}$ is a skew-symmetric matrix operator defined such that: $\boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{S}(\boldsymbol{a})\boldsymbol{b}$.

4. Diffractions forces: the diffraction forces can be neglected by assuming that the current velocity is constant.

The total friction forces and moments acting at the center of mass are:

$$\boldsymbol{p} = \boldsymbol{F}_L + \boldsymbol{F}_D + \boldsymbol{F}_S, \quad \boldsymbol{n} = \boldsymbol{M}_{m_l}$$

- **Buoyancy**: $B = \rho g \nabla$ where ∇ is the deplacement volume and g is the acceleration of gravity. For neutrally buoyant bodies $B = mg$. The buoyant force attacks in the body's center of buoyance.

Including these forces in the force and moment equations for a manipulator link k gives the following equations written in reference frame k:

$$\begin{aligned} \boldsymbol{F}_k &= \boldsymbol{M}_k({}^k\boldsymbol{a}_k + {}^k\boldsymbol{\alpha}_k \times {}^k\boldsymbol{d}_{k/k_c} + {}^k\boldsymbol{\omega}_k \times ({}^k\boldsymbol{\omega}_k \times {}^k\boldsymbol{d}_{k/k_c})) & (4) \\ \boldsymbol{T}_k &= \boldsymbol{I}_k\,{}^k\boldsymbol{\alpha}_k +{}^k \boldsymbol{\omega}_k \times (\boldsymbol{I}_k\,{}^k\boldsymbol{\omega}_k) & (5) \end{aligned}$$

where

$\boldsymbol{F}_k$	total forces acting at the center of mass of link k
$\boldsymbol{f}_k$	forces exerted by link $k-1$ on link k
$\boldsymbol{R}_k^{k+1}$	the rotation matrix between frame k+1 and frame k
$\boldsymbol{M}_k$	the mass and added mass of link k, located at the center of mass
$\boldsymbol{I}_k$	moment of inertia and added moment of inertia of link k, about the center of mass
$\boldsymbol{T}_k$	total moments acting about the center of mass of link k

See Appendix 1 for further details. The force $\boldsymbol{f}_k$ and moment interaction $\boldsymbol{t}_k$ between link k and link $k-1$ written in frame k are:

$$\begin{aligned} \boldsymbol{f}_k &= \boldsymbol{R}_k^{k+1\ k+1}\boldsymbol{f}_{k+1} + \boldsymbol{F}_k - m_k\boldsymbol{g}_k + \boldsymbol{b}_k + \boldsymbol{p}_k & (6) \\ \boldsymbol{t}_k &= \boldsymbol{R}_k^{k+1\ k+1}\boldsymbol{t}_{k+1} + \boldsymbol{d}_{k/k+1} \times (\boldsymbol{R}_k^{k+1\ k+1}\boldsymbol{f}_{k+1}) + \boldsymbol{d}_{k/k_c} \times \boldsymbol{F}_k + \boldsymbol{T}_k \\ &\quad +\boldsymbol{d}_{k/k_c} \times (-m_k\boldsymbol{g}_k + \boldsymbol{p}_k) + \boldsymbol{d}_{k/k_b} \times \boldsymbol{b}_k + \boldsymbol{n}_k & (7) \end{aligned}$$

where

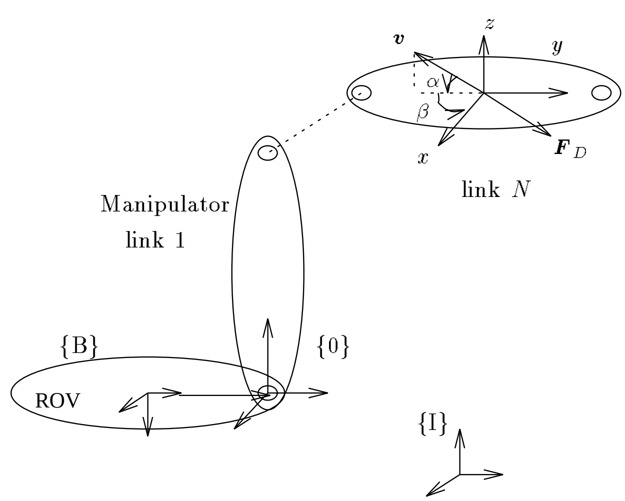

Figure 1: ROV–manipulator system showing the ROV rigid-body and the manipulator links. The vector $\boldsymbol{v}$ is the relative velocity, $\boldsymbol{F}_D$ is the drag force, α and β are the angle of attach and sideslip. Frame {I} and {B} the earth-fixed and body-fixed reference frames, and frame {0} is the reference frame located at the manipulator base. The number of manipulator links is N.

$\boldsymbol{p}_k$	linear and quadratic hydrodynamic friction forces
$\boldsymbol{b}_k$	the buoyant force
$\boldsymbol{d}_{k/k_b}$	vector from the center of buoyance of link k to frame k
${}^k\boldsymbol{d}_{k/k+1}$	the vector from joint k to joint k+1
$\boldsymbol{n}_k$	hydrodynamic friction moments

The joint torques are:

$$\boldsymbol{\tau}_k = \boldsymbol{z}^T\,{}^k\boldsymbol{t}_k \tag{8}$$

where $\boldsymbol{z}^T$ is the unit vector along the z-axis. A recursive Newton-Euler algorithm for an underwater manipulator is achieved when substituting (4–7) into the iterative Newton-Euler dynamics algorithm for manipulators, Spong and Vidyasagar (1989).

Evaluating (22–27) and (4–8) symbolically yields the equations of motion for the underwater manipulator. The result can be written

$$\boldsymbol{M}_m(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}_m(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{D}_m(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}_m(\boldsymbol{q}) = \boldsymbol{\tau}_m \tag{9}$$

where

$\boldsymbol{M}_m(\boldsymbol{q})$	inertia matrix (including added inertia)
$\boldsymbol{C}_m(\boldsymbol{q},\dot{\boldsymbol{q}})$	contains Coriolis and centripetal terms
$\boldsymbol{D}_m(\boldsymbol{q},\dot{\boldsymbol{q}})$	hydrodynamic lift and damping matrix
$\boldsymbol{g}_m(\boldsymbol{q})$	vector of gravity and buoyancy forces
$\boldsymbol{\tau}_m$	external input forces
$\boldsymbol{q}$	generalized link coordinates

2.3 ROV–Manipulator Equations of Motion

The manipulator has an initial velocity equal to the velocity of the ROV. Evaluating the iterative Newton-Euler algorithm symbolically, with initial velocity $\boldsymbol{\nu} = [{}^0\boldsymbol{v}_0^T, {}^0\boldsymbol{\omega}_0^T]^T$ in (22–27), the equations of motion for the manipulator part of the system can be written in the form

$$\begin{aligned}\boldsymbol{M}_m(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}_m(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{D}_m(\boldsymbol{q})\dot{\boldsymbol{q}} + \boldsymbol{g}_m(\boldsymbol{q}) + \boldsymbol{D}_5(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{\nu})\dot{\boldsymbol{q}} +& \\ \boldsymbol{M}_c^T(\boldsymbol{q})\dot{\boldsymbol{\nu}} + \boldsymbol{C}_3(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{D}_4(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{\nu})\boldsymbol{\nu} &= \boldsymbol{\tau}_m\end{aligned} \tag{10}$$

where

$\boldsymbol{M}_c^T(\boldsymbol{q})\dot{\boldsymbol{\nu}}$	are the reaction forces and moments between the ROV and the manipulator
$\boldsymbol{C}_i(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{\nu})\boldsymbol{\nu}$	are the Coriolis and centripetal forces due to the interaction between the ROV and the manipulator (i=1..3)
$\boldsymbol{D}_i(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{\nu})$	is the quadratic drag due to the manipulator links and ROV (i=1..5)
$\boldsymbol{D}_m(\boldsymbol{q})$	is the linear skin-friction affecting the manipulator

The weight and movement of the manipulator links will affect the ROV dynamics. These forces and moments disturbances $\boldsymbol{\sigma} = [\boldsymbol{\sigma}_f^T, \boldsymbol{\sigma}_m^T]^T$ can be calculated from (6–7).

$$\boldsymbol{\sigma}_f = \boldsymbol{R}_0^B \boldsymbol{f}_1 \tag{11}$$

$$\boldsymbol{\sigma}_m = \boldsymbol{R}_0^B \boldsymbol{t}_1 - \boldsymbol{d}_{rov/man} \times (\boldsymbol{R}_0^B \boldsymbol{f}_1) \tag{12}$$

where $\boldsymbol{d}_{rov/man}$ is the vector between the body-fixed frame of the ROV and the base frame of the manipulator and $\boldsymbol{f}_1$ and $\boldsymbol{t}_1$ are the forces and moments acting on the base of the manipulator. The matrix $\boldsymbol{R}_0^B$ is the rotation matrix between the body-fixed ROV reference frame and the manipulator-base frame. Locating the body-fixed reference frame of the ROV in the manipulator base gives

$$\boldsymbol{\sigma} = [(\boldsymbol{R}_0^1\boldsymbol{f}_1)^T, (\boldsymbol{R}_0^1\boldsymbol{t}_1)^T]^T$$

Evalutating the manipulator dynamics and the interaction forces symbolically yields the following equations of motion for the ROV system:

$$\begin{aligned}\boldsymbol{M}_r\dot{\boldsymbol{\nu}} + \boldsymbol{C}_r(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{D}_r(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{g}_r(\boldsymbol{\eta}) + \boldsymbol{H}(\boldsymbol{q})\dot{\boldsymbol{\nu}} + \boldsymbol{C}_1(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{\nu})\boldsymbol{\nu}+ \\ \boldsymbol{D}_1(\boldsymbol{q})\boldsymbol{\nu} + \boldsymbol{D}_2(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{M}_c(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}_2(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{D}_3(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{\nu})\dot{\boldsymbol{q}} + \boldsymbol{g}_E(\boldsymbol{q}) &= \boldsymbol{\tau}_r \end{aligned} \quad (13)$$

where

$\boldsymbol{H}(\boldsymbol{q})\dot{\boldsymbol{\nu}}$	is the added inertia due to the manipulator
$\boldsymbol{D}_1(\boldsymbol{q})\boldsymbol{\nu}$	is the linear skin friction due to the manipulator
$\boldsymbol{C}_2(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$	are the Coriolis and centripetal forces due to the manipulator
$\boldsymbol{g}_E(\boldsymbol{q})$	is the gravity force and moment vector due to the manipulator

The equations of motion for the total system in a body-fixed reference frame, located at the manipulator base, can be expressed in the form

$$\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{\zeta}} + \boldsymbol{C}(\boldsymbol{q},\boldsymbol{\zeta})\boldsymbol{\zeta} + \boldsymbol{D}(\boldsymbol{q},\boldsymbol{\zeta})\boldsymbol{\zeta} + \boldsymbol{g}(\boldsymbol{q},\boldsymbol{\eta}) = \boldsymbol{\tau} \quad (14)$$

where $\boldsymbol{\zeta} = [\boldsymbol{\nu}^T, \dot{\boldsymbol{q}}^T]^T$ and

$$\boldsymbol{M}(\boldsymbol{\zeta}) = \begin{bmatrix} \boldsymbol{M}_r + \boldsymbol{H}(q) & \boldsymbol{M}_c(\boldsymbol{q}) \\ \boldsymbol{M}_c^T(\boldsymbol{q}) & \boldsymbol{M}_m(\boldsymbol{q}) \end{bmatrix}$$

$$\boldsymbol{C}(\boldsymbol{q},\boldsymbol{\zeta}) = \begin{bmatrix} \boldsymbol{C}_r(\boldsymbol{\nu}) + \boldsymbol{C}_1(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{\nu}) & \boldsymbol{C}_2(\boldsymbol{q},\dot{\boldsymbol{q}}) \\ \boldsymbol{C}_3(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{\nu}) & \boldsymbol{C}_m(\boldsymbol{q},\dot{\boldsymbol{q}}) \end{bmatrix}$$

$$\boldsymbol{D}(\boldsymbol{q},\boldsymbol{\zeta}) = \begin{bmatrix} \boldsymbol{D}_r(\boldsymbol{\nu}) + \boldsymbol{D}_1(\boldsymbol{q}) + \boldsymbol{D}_2(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{\nu}) & \boldsymbol{D}_3(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{\nu}) \\ \boldsymbol{D}_4(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{\nu}) & \boldsymbol{D}_m(\boldsymbol{q}) + \boldsymbol{D}_5(\boldsymbol{q},\dot{\boldsymbol{q}},\boldsymbol{\nu}) \end{bmatrix}$$

$$\boldsymbol{g}(\boldsymbol{q},\boldsymbol{\eta}) = \begin{bmatrix} \boldsymbol{g}_r(\boldsymbol{\eta}) + \boldsymbol{g}_E(\boldsymbol{q}) \\ \boldsymbol{g}_m(\boldsymbol{q}) \end{bmatrix}, \qquad \boldsymbol{\tau} = [\boldsymbol{\tau}_r^T, \boldsymbol{\tau}_m^T]^T$$

The equations of motion for the total system can be written in the earth-fixed reference system by applying the following kinematic transformations

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}_R(\boldsymbol{\eta})\boldsymbol{\nu} \quad (15)$$

$$\dot{\boldsymbol{x}}_e = \boldsymbol{J}_1(\boldsymbol{\eta})\boldsymbol{\nu} + \boldsymbol{J}_2(\boldsymbol{q})\dot{\boldsymbol{q}} \quad (16)$$

where $\boldsymbol{x}_e$ is the position of the manipulator end-effector in the inertial frame and $\dot{\boldsymbol{x}}_e = [{}^I\boldsymbol{v}_N^T, {}^I\boldsymbol{\omega}_N^T]^T$. The matrices $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$ can be determind from the following equations:

$$ {}^I\boldsymbol{\omega}_N = \boldsymbol{R}_I^0({}^0\boldsymbol{w}_0 + \sum_{j=1}^{N} \boldsymbol{R}_0^j \boldsymbol{z}\dot{\boldsymbol{q}}_j) \tag{17} $$

$$ {}^I\boldsymbol{v}_N = \boldsymbol{R}_I^0 \left[{}^0\boldsymbol{v}_0 + \sum_{j=1}^{N} \boldsymbol{R}_0^j ({}^j\boldsymbol{\omega}_j \times {}^j\boldsymbol{d}_j) \right] \tag{18} $$

Manipulating these equations gives:

$$ \boldsymbol{J}_1 = \boldsymbol{R}_I^0 \begin{bmatrix} \boldsymbol{I}_{3\times3} & -\boldsymbol{S}(\sum_{j=1}^{N} \boldsymbol{R}_0^j \boldsymbol{d}_j) \\ \boldsymbol{0}_{3\times3} & \boldsymbol{I}_{3\times3} \end{bmatrix}, \quad \boldsymbol{J}_2 = \boldsymbol{R}_I^0 \begin{bmatrix} \boldsymbol{\alpha}_1 & \dots & \boldsymbol{\alpha}_N \\ \boldsymbol{\beta}_1 & \dots & \boldsymbol{\beta}_N \end{bmatrix} $$

where $\boldsymbol{\alpha}_k = -\sum_{j=k}^{N} \boldsymbol{S}(\boldsymbol{R}_0^j \boldsymbol{d}_j)\boldsymbol{\beta}_k$ and $\boldsymbol{\beta}_k = \boldsymbol{R}_0^k \boldsymbol{z}$ whereas $\boldsymbol{S}$ is the skew-symmetric matrix operator.

The transformation matrix and its inverse are given by:

$$ \boldsymbol{J}(\boldsymbol{q},\boldsymbol{\eta}) = \begin{bmatrix} \boldsymbol{J}_R(\boldsymbol{\eta}) & \boldsymbol{0} \\ \boldsymbol{J}_1(\boldsymbol{\eta}) & \boldsymbol{J}_2(\boldsymbol{q}) \end{bmatrix} \Rightarrow \boldsymbol{J}^{-1}(\boldsymbol{q},\boldsymbol{\eta}) = \begin{bmatrix} \boldsymbol{J}_R^{-1} & \boldsymbol{0} \\ -\boldsymbol{J}_2^{-1}\boldsymbol{J}_1\boldsymbol{J}_R^{-1} & \boldsymbol{J}_2^{-1} \end{bmatrix} $$

$$ \begin{aligned} \dot{\boldsymbol{\eta}} &= \boldsymbol{J}_R(\boldsymbol{\eta})\boldsymbol{\nu} & \Longleftrightarrow \quad \boldsymbol{\nu} &= \boldsymbol{J}_R^{-1}(\boldsymbol{\eta})\dot{\boldsymbol{\eta}} \\ \ddot{\boldsymbol{\eta}} &= \boldsymbol{J}_R(\boldsymbol{\eta})\dot{\boldsymbol{\nu}} + \dot{\boldsymbol{J}}_R(\boldsymbol{\eta})\boldsymbol{\nu} & \Longleftrightarrow \quad \dot{\boldsymbol{\nu}} &= \boldsymbol{J}_R^{-1}(\boldsymbol{\eta})(\ddot{\boldsymbol{\eta}} - \dot{\boldsymbol{J}}_R(\boldsymbol{\eta})\boldsymbol{J}_R^{-1}(\boldsymbol{\eta})\dot{\boldsymbol{\eta}}) \\ \dot{\boldsymbol{x}}_e &= \boldsymbol{J}_1(\boldsymbol{\eta})\boldsymbol{\nu} + \boldsymbol{J}_2(\boldsymbol{q})\dot{\boldsymbol{q}} & \Longleftrightarrow \quad \dot{\boldsymbol{q}} &= \boldsymbol{J}_2^{-1}(\dot{\boldsymbol{x}}_e - \boldsymbol{J}_1\boldsymbol{J}_R^{-1}\dot{\boldsymbol{\eta}}) \\ \ddot{\boldsymbol{x}}_e &= \boldsymbol{J}_1\dot{\boldsymbol{\nu}} + \boldsymbol{J}_2\ddot{\boldsymbol{q}} + \dot{\boldsymbol{J}}_1\boldsymbol{\nu} + \dot{\boldsymbol{J}}_2\dot{\boldsymbol{q}} & \Longleftrightarrow \quad \ddot{\boldsymbol{q}} &= \boldsymbol{J}_2^{-1}(\ddot{\boldsymbol{x}}_e - \boldsymbol{J}_1\boldsymbol{J}_R^{-1}\ddot{\boldsymbol{\eta}} - \\ & & & \quad \dot{\boldsymbol{J}}_2\boldsymbol{J}_2^{-1}\dot{\boldsymbol{x}}_e + \boldsymbol{J}_1\boldsymbol{J}_R^{-1}\dot{\boldsymbol{J}}_R\boldsymbol{J}_R^{-1}\dot{\boldsymbol{\eta}} - \\ & & & \quad \dot{\boldsymbol{J}}_1\boldsymbol{J}_R^{-1}\dot{\boldsymbol{\eta}} + \dot{\boldsymbol{J}}_2\boldsymbol{J}_2^{-1}\boldsymbol{J}_1\boldsymbol{J}_R^{-1}\dot{\boldsymbol{\eta}}) \end{aligned} $$

Consequently the equations of motion for the total system in the earth-fixed reference frame can therefore be written in the form

$$ \boldsymbol{M}_e(\boldsymbol{q},\boldsymbol{\eta},\boldsymbol{\zeta})\ddot{\boldsymbol{\xi}} + \boldsymbol{C}_e(\boldsymbol{q},\boldsymbol{\eta},\boldsymbol{\zeta})\dot{\boldsymbol{\xi}} + \boldsymbol{D}_e(\boldsymbol{q},\boldsymbol{\eta},\boldsymbol{\zeta})\dot{\boldsymbol{\xi}} + \boldsymbol{g}_e(\boldsymbol{q},\boldsymbol{\eta}) = \boldsymbol{\tau}_e \tag{19} $$

where $\boldsymbol{\xi} = [\boldsymbol{\eta}^T, \boldsymbol{x}_e^T]^T$ and :

$$ \begin{aligned} \boldsymbol{M}_e(\boldsymbol{q},\boldsymbol{\eta},\boldsymbol{\zeta}) &= \boldsymbol{J}^{-T}(\boldsymbol{q},\boldsymbol{\eta})\boldsymbol{M}(\boldsymbol{q})\boldsymbol{J}^{-1}(\boldsymbol{q},\boldsymbol{\eta}) \\ \boldsymbol{C}_e(\boldsymbol{q},\boldsymbol{\eta},\boldsymbol{\zeta}) &= \boldsymbol{J}^{-T}(\boldsymbol{q},\boldsymbol{\eta})[\boldsymbol{C}(\boldsymbol{q},\boldsymbol{\zeta}) - \boldsymbol{M}(\boldsymbol{q})\boldsymbol{J}^{-1}(\boldsymbol{q},\boldsymbol{\eta})\dot{\boldsymbol{J}}(\boldsymbol{q},\boldsymbol{\eta})]\boldsymbol{J}^{-1}(\boldsymbol{q},\boldsymbol{\eta}) \\ \boldsymbol{D}_e(\boldsymbol{q},\boldsymbol{\eta},\boldsymbol{\zeta}) &= \boldsymbol{J}^{-T}(\boldsymbol{q},\boldsymbol{\eta})\boldsymbol{D}(\boldsymbol{q},\boldsymbol{\zeta})\boldsymbol{J}^{-1}(\boldsymbol{q},\boldsymbol{\eta}) \\ \boldsymbol{g}_e(\boldsymbol{q},\boldsymbol{\eta}) &= \boldsymbol{J}^{-T}(\boldsymbol{q},\boldsymbol{\eta})\boldsymbol{g}(\boldsymbol{q},\boldsymbol{\eta}) \\ \boldsymbol{\tau}_e &= \boldsymbol{J}^{-T}(\boldsymbol{q},\boldsymbol{\eta})\boldsymbol{\tau} \end{aligned} $$

2.4 Body-fixed Properties of the Equations of Motion

The inertia matrix for the total system is positive definite $\boldsymbol{M}(\boldsymbol{q},\boldsymbol{\nu}) = \boldsymbol{M}^T(\boldsymbol{q},\boldsymbol{\nu}) > \boldsymbol{0}$ due to positive kinetic energy, whereas symmetry is guaranteed by applying Newtons' third law (action-reaction principle).

The damping matrix is strictly positive $\boldsymbol{D}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\nu}) > \mathbf{0}$, due to the dissipative nature of the ROV–manipulator system.

For the total system it is also true that $\boldsymbol{\zeta}^T(\dot{\boldsymbol{M}}(\boldsymbol{q}, \boldsymbol{\nu}) - 2\boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\nu}))\boldsymbol{\zeta} = \mathbf{0}$.

3 Feedback Linearization

This section discusses the control system design in terms of feedback linearization. Feedback linearization transforms the nonlinear system dynamics into a linear system. The control laws are chosen so that zero tracking errors are achieved. The tracking errors are defined as $\tilde{\boldsymbol{\eta}} = \boldsymbol{\eta} - \boldsymbol{\eta}_d$ and $\tilde{\boldsymbol{q}} = \boldsymbol{q} - \boldsymbol{q}_d$ where d denotes the desired states. The equations of motion can also be expressed in the form

$$\boldsymbol{M}_{11}(\boldsymbol{q})\dot{\boldsymbol{\nu}} + \boldsymbol{M}_c(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{n}_1(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\nu}) = \boldsymbol{\tau}_r \tag{20}$$

$$\boldsymbol{M}_{22}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{M}_c^T(\boldsymbol{q})\dot{\boldsymbol{\nu}} + \boldsymbol{n}_2(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\nu}) = \boldsymbol{\tau}_m \tag{21}$$

where $\boldsymbol{M}_{11} = \boldsymbol{M}_r + \boldsymbol{H}$, $\boldsymbol{M}_{22} = \boldsymbol{M}_m$ and $\boldsymbol{n}_1(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\nu})$ and $\boldsymbol{n}_2(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\nu})$ are the vectors of the Coriolis and centripetal and friction forces. The following control input will cancel out the nonlinearities in the system

$$\boldsymbol{\tau}_r = \boldsymbol{M}_{11}(\boldsymbol{q})\boldsymbol{a}_\nu + \boldsymbol{M}_c(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{n}_1(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\nu})$$

where $\boldsymbol{a}_\nu = \boldsymbol{J}_R^{-1}(\boldsymbol{a}_\eta - \dot{\boldsymbol{J}}_R \boldsymbol{J}_R^{-1}\boldsymbol{\eta})$. Hence the PID control law

$$\boldsymbol{a}_\eta = \ddot{\boldsymbol{\eta}}_d - \boldsymbol{K}_{p_r}\tilde{\boldsymbol{\eta}} - \boldsymbol{K}_{d_r}\dot{\tilde{\boldsymbol{\eta}}} - \boldsymbol{K}_{i_r}\int_0^t \tilde{\boldsymbol{\eta}}\, dt$$

leads to the exponential stable dynamics

$$\ddot{\tilde{\boldsymbol{\eta}}} + \boldsymbol{K}_{d_r}\dot{\tilde{\boldsymbol{\eta}}} + \boldsymbol{K}_{p_r}\tilde{\boldsymbol{\eta}} + \boldsymbol{K}_{i_r}\int_0^t \tilde{\boldsymbol{\eta}}\, dt = \mathbf{0}$$

which implies that $\tilde{\boldsymbol{\eta}} \to \mathbf{0}$. For the manipulator a similar control law is chosen

$$\boldsymbol{\tau}_m = \boldsymbol{M}_{22}(\boldsymbol{q})\boldsymbol{v} + \boldsymbol{M}_c^T(\boldsymbol{q})\dot{\boldsymbol{\nu}} + \boldsymbol{n}_2(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\nu})$$

where

$$\boldsymbol{v} = \ddot{\boldsymbol{q}}_d - \boldsymbol{K}_{pm}\tilde{\boldsymbol{q}} - \boldsymbol{K}_{dm}\dot{\tilde{\boldsymbol{q}}} - \boldsymbol{K}_{im}\int_0^t \tilde{\boldsymbol{q}} dt$$

Hence the manipulator tracking error satisfies

$$\ddot{\tilde{\boldsymbol{q}}} + \boldsymbol{K}_{dm}\dot{\tilde{\boldsymbol{q}}} + \boldsymbol{K}_{pm}\tilde{\boldsymbol{q}} + \boldsymbol{K}_{im}\int_0^t \tilde{\boldsymbol{q}} dt = \mathbf{0}$$

which implies that $\tilde{\boldsymbol{q}} \to \mathbf{0}$.

4 Conclusions

In this paper the dynamic equations of an underwater vehicle–manipulator system are written in closed form. For simulation studies a recursive formulation of the inverse dynamics is more efficient. However a closed form formulation makes it possible to study the structure and properties of the system and it often simplifies the control design.

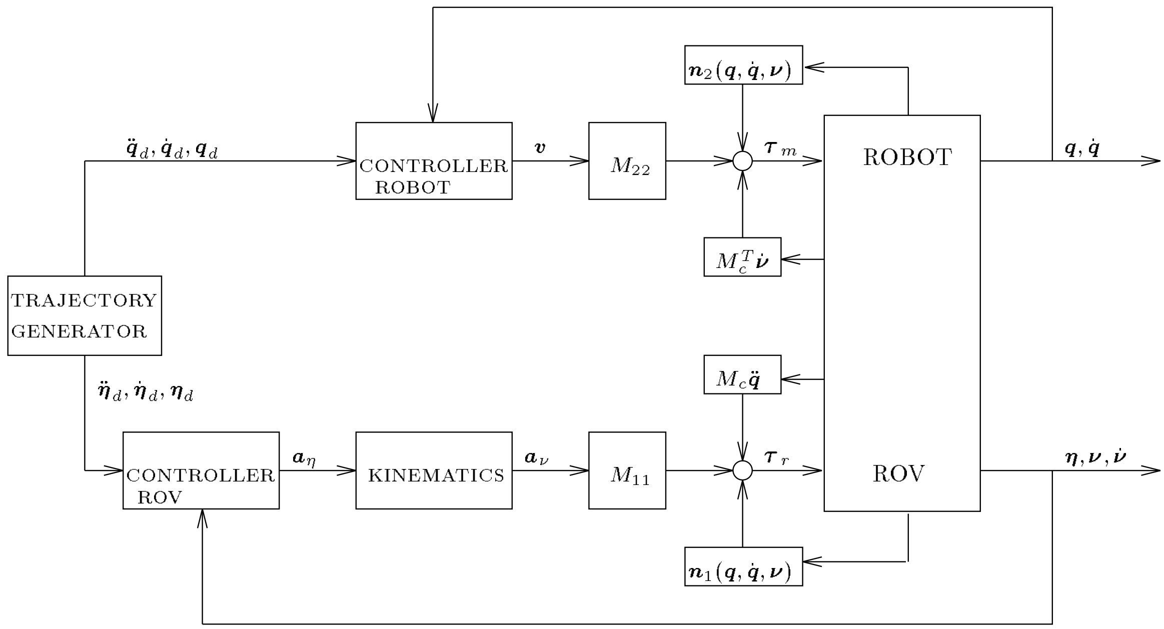

Figure 2: A closed loop control system employing feedback linearization

A Iterative Computation of Angular and Linear velocities

The iterative algorithm for revolute joints is written, Spong and Vidyasagar (1989):
Outward iterations k: 0..5 :

$$
\begin{aligned}
{}^{k+1}\boldsymbol{\omega}_{k+1} &= \boldsymbol{R}^k_{k+1}({}^{k}\boldsymbol{\omega}_k + \boldsymbol{z}\dot{q}_{k+1}) && (22)\\
{}^{k+1}\boldsymbol{\alpha}_{k+1} &= \boldsymbol{R}^k_{k+1}({}^{k}\boldsymbol{\alpha}_k + {}^{k}\boldsymbol{\omega}_k \times \boldsymbol{z}\dot{q}_{k+1} + \boldsymbol{z}\ddot{q}_{k+1}) && (23)\\
{}^{k+1}\boldsymbol{v}_{k+1} &= \boldsymbol{R}^k_{k+1}\,{}^{k}\boldsymbol{v}_k + {}^{k+1}\boldsymbol{\omega}_{k+1} \times {}^{k+1}\boldsymbol{d}_{k+1/k} && (24)\\
{}^{k+1}\boldsymbol{v}_{k+1_c} &= \boldsymbol{R}^k_{k+1}\,{}^{k}\boldsymbol{v}_k + {}^{k+1}\boldsymbol{\omega}_{k+1} \times {}^{k+1}\boldsymbol{d}_{k/k_c} && (25)\\
{}^{k+1}\boldsymbol{a}_{k+1} &= \boldsymbol{R}^k_{k+1}\,{}^{k}\boldsymbol{a}_k + {}^{k+1}\boldsymbol{\alpha}_{k+1} \times {}^{k+1}\boldsymbol{d}_{k+1/k} + {}^{k+1}\boldsymbol{\omega}_{k+1} \times ({}^{k+1}\boldsymbol{\omega}_{k+1} \times {}^{k+1}\boldsymbol{d}_{k+1/k}) && (26)\\
& && (27)
\end{aligned}
$$

where

$\boldsymbol{z}$	unit vector along the z-axis
q	generalized joint position
${}^{k}\boldsymbol{\omega}_k$	angular velocity of link k
${}^{k}\boldsymbol{\alpha}_k$	angular acceleration of link k
${}^{k}\boldsymbol{v}_k$	linear velocity of link k
${}^{k}\boldsymbol{v}_{k_c}$	linear velocity of the center of mass of link k
${}^{k}\boldsymbol{a}_k$	linear acceleration of link k
m_k	mass of link k
$\boldsymbol{g}_k$	the acceleration due to gravity of link k
${}^{k}\boldsymbol{d}_{k+1/k}$	the vector from joint k+1 to joint k
${}^{k}\boldsymbol{d}_{k/k_c}$	the vector from reference frame k (joint k+1) to the center of mass of link k

The coordinate frames are located at each joint according to the Denavit–Hartenberg convention, Spong and Vidyasagar (1989)

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