NONLINEAR RUDDER-ROLL DAMPING OF NON-MINIMUM PHASE SHIPS USING SLIDING MODE CONTROL

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Abstract

A non-minimum phase nonlinear ship model from rudder angle to roll angle is used to design a stable controller for simultaneous roll damping and course keeping. The controller is stable in presence of 1st-order wave disturbances and modeling errors provided that the magnitude of the model errors and disturbances are known. Stability is proved using Lyapunov theory. The simulation results show excellent performance and robustness.

1 Introduction

Besides controlling the heading of a ship it is often desirable to reduce the wave-induced roll motion. The main reasons for introducing a roll damping system are to prevent cargo damage and to increase crew effectiveness and passenger comfort. Criteria for the maximum roll angle are given in [4], and they suggest that the root mean square of the roll angle should be less than 6 degrees for light manual work and 3 degrees for intellectual work.

There are several approaches that can be used to reduce the roll motion, e.g., bilge keels, anti-rolling tanks, fin stabilizers and rudder roll stabilizers (RRS). For a detailed discussion of the different roll stabilizers see [5] and the references therein. In this paper we consider the rudder roll damping approach which is attractive since existing equipment can be used and thus it is a relative inexpensive solution. Successful implementation of full-scale RRS system were first reported in [1] and later in e.g., [9] and [13]. For other results on RRS, see [2, 3, 10, 14, 15, 16].

The common assumption in the above papers is that the ship dynamics is linear and can be expressed by the linear state-space model

\[ \dot{x} = Ax + bu. \]  

Clearly, this assumption will not be satisfied since the ship dynamics is nonlinear, and in this paper a nonlinear controller based on a nonlinear affine model is proposed. The ship dynamics is described by the following nonlinear differential equation

\[ \dot{x} = f(x) + bu, \]  

where \( f(x) \) is a nonlinear function of the states. The nonlinear ship dynamics is not perfectly known due to e.g., changing load conditions, and thus for control design only the estimates \( \hat{f}(x) \) and \( \hat{b} \) can be used.

It is well-known that the dynamics in roll has non-minimum phase behavior, see e.g., [6]. This corresponds to zeros in the right half-plane for the linear model (1) and unstable zero dynamics for the nonlinear model (2). This results in an inverse response in the roll angle for a fast change in the rudder angle and gives limitations on the gain in the closed-loop system.

In this paper the problem of controlling a system with non-minimum phase behavior is solved by defining a sliding surface that gives the system stable zero dynamics if the sliding surface is used as output. Moreover, modeling errors is compensated for using a switching term in the controller provided an upper bound on the error can be found. The resulting nonlinear controller gives ultimately boundedness in the presence of wave disturbances (and stability in the absence of wave disturbances) under a set of weak assumptions.

2 Nonlinear Ship Model

Consider the following SISO nonlinear ship model:

\[ \begin{align*}
\dot{x} &= f(x) + b(\delta + w_H) \\
y &= c^T x,
\end{align*} \]  

where \( x \in \mathbb{R}^5 \) is the state vector defined as:

\[ x \triangleq [\phi \ p \ v \ r \ (\psi - \psi_d)]^T. \]  

Here \( \phi \) is the roll angle, \( p \) is the angular velocity in roll, \( v \) is the sway velocity, \( r \) is the angular velocity in yaw,
ψ and is the yaw angle, ψd is the (constant) desired yaw angle, δ ∈ R is the rudder angle, y ∈ R is the roll angle measurement (y = φ), wH ∈ R is used to describe 1st-order wave disturbances, b = [0 b2 b3 b4 0]T and f(x) depends on the ship.

The estimated model of the ship dynamics used for control design is usually linear and it is written

\[ \dot{x} = \hat{A}x + \hat{b}(\delta + w_H) \]  

(5)

To simplify the analysis, the following assumption is made:

**Assumption 1** Assume that \( \hat{b} \) is exactly known, that is \( b = \hat{b} \).

Then, expanding the nonlinear term in (3), that is \( f(x) = Ax + g(x) \), the ship dynamics can be rewritten as

\[ \dot{x} = (\hat{A} + \Delta A)x + g(x) + bu \]

(6)

where \( \Delta A = A - \hat{A} \) describes the modeling errors of the linear part of the dynamics and \( g(x) \) is an unknown nonlinear function.

### 3 Transformation to Normal Form

In order to analyze the nonlinear ship model the equations of motion are transformed to normal form, see [8]. For a detailed description on ship dynamics in normal form, see [6]. Let \( r \) be the relative degree of the system (the number of time differentiations of the output \( y \) before the input \( u \) explicitly appears). For linear systems the relative degree corresponds to the number of poles minus the number of zeros. Let the Lie derivative (the derivative of \( \lambda \) along \( f \)) be given by

\[ L_f \lambda(x) = \frac{\partial \lambda}{\partial x} f(x). \]

(7)

Then, the notation \( L_f^k \lambda \) can be used to denote the recursion

\[ L_f^k \lambda(x) = \frac{\partial L_f^{k-1} \lambda}{\partial x} f(x). \]

(8)

The system (3) is transformed to normal form by applying the transformation \( z = \phi(x) = [\phi_1(x), \cdots, \phi_r(x), \phi_{r+1}(x), \cdots, \phi_k(x)]^T \) where the first two transformations are defined as:

\[ z_1 = \phi_1(x) = c^T x, \quad z_2 = \phi_2(x) = L_f c^T x \]

(9)

The inverse transformation will be denoted \( x = \phi^{-1}(z) \).

Applying this transformation to (3) yields the normal equations:

\[ \begin{align*}
    z_1 &= z_2 \\
    z_2 &= L_f^2 h(\phi^{-1}(z)) + L_g L_f h(\phi^{-1}(z)) u \\
    z_i &= L_f \phi_i(\phi^{-1}(z)) + L_g \phi_i(\phi^{-1}(z)) u \\
    y &= z_1
\end{align*} \]

where \( i = 3, 4, 5 \). This system can be further simplified by choosing the last last three transformations such that

\[ L_{y_i} \phi_i(\phi^{-1}(z)) = 0, \quad i = 3, 4, 5 \]

(11)

This implies that the input \( u \) is eliminated from the last three differential equations. In order to simplify the notation, the system (10) is written as two subsystems with two and three state variables by defining \( \xi = [z_1, z_2]^T \) and \( \eta = [z_3, z_4, z_5]^T \). If

\[ \begin{bmatrix} \xi \\ \eta \end{bmatrix} = \begin{bmatrix} \phi \\ \cdots \\ (\psi - \psi_d) \\ b_2 p - b_2 v \\ b_{2p} - b_{2r} \end{bmatrix}, \]

(12)

it is easily verified that \( \phi_i, \ i = 3, 4, 5, \) satisfy (11). Hence, (10) can be expressed as:

\[ \begin{align*}
    \dot{\xi}_1 &= \xi_2 \\
    \dot{\xi}_2 &= a^*(\xi, \eta) + b^*(\xi, \eta) \delta \\
    \dot{\eta} &= q^*(\xi, \eta) \\
    y &= \xi_1
\end{align*} \]

(13)

where \( a^*(\xi, \eta) = L_f^2 h(\phi^{-1}(z)) \) and \( b^*(\xi, \eta) = L_g L_f h(\phi^{-1}(z)) \). Expanding (13) and using the fact that \( L_g L_f h(\phi^{-1}(z)) = b_2 \), the following equations are obtained:

\[ \begin{align*}
    \dot{\xi}_1 &= \xi_2 \\
    \dot{\xi}_2 &= (\beta + \Delta \beta)^T \xi + (\delta + \Delta \delta)^T \eta + a(\xi, \eta) + b_2 u \\
    \dot{\eta} &= (P + \Delta P) \xi + (Q + \Delta Q) \eta + q(\xi, \eta) \\
    y &= \xi_1
\end{align*} \]

(14)

where the linear and nonlinear terms corresponds to the linear and nonlinear terms in (6).

From (14) the zero-dynamics of the system state-space model is extracted. This corresponds to a zero output \( y = 0 \), and is obtained by choosing the control law as

\[ u = -\frac{1}{b_2} (a(0, \eta) + (\delta + \Delta \delta)^T \eta). \]

(15)

Hence, the \( \eta \)-dynamics (zero-dynamics) takes the form:

\[ \dot{\eta} = (Q + \Delta Q) \eta + q(0, \eta). \]

(16)

If the zero-dynamics is unstable the system (3) is said to be non-minimum phase. From (14) it is easily verified that the zero-dynamics is independent of the input, and cannot be altered by feedback. Thus if the zero-dynamics is unstable, perfect tracking is impossible and the goal becomes asymptotic tracking.

It is well known that all ships show a non-minimum phase behavior in roll whereas yaw is minimum phase for most ships, see [6]. This implies that the zero-dynamics given by the output \( y = \phi \) is unstable. Hence, output feedback linearization cannot be used in roll since the inverse dynamics will be unstable.
4 Design of Autopilot

In this section we derive a nonlinear control law for simultaneous roll-damping and course-keeping.

4.1 Sliding Surface in Roll

Define a sliding surface, see [7]:

\[
\sigma = \xi_2 + \lambda \xi_1 \\
\dot{\sigma} = \dot{\xi}_2 + \lambda \dot{\xi}_1
\]

where \(\xi_2, \xi_1\) and \(y\) are defined in (14) and \(\lambda > 0\). Taking the sliding surface as the output, the ship dynamics can be rewritten as:

\[
\dot{x} = (A + \Delta A)x + g(x) + bu \\
\sigma = x_1 + \lambda x_2
\]

The control objective is to regulate \(\sigma\) to zero such that \(\phi = p = 0\). It is easily verified that the modified ship model (19) has relative degree 1. Thus when defining a new vector

\[
\dot{\eta} = [\xi_1, \eta]^T 
\]

it is seen that (14) with

\[
P_\xi = [p_\xi_1, p_\xi_2] \begin{bmatrix} \xi_2 \\ \xi_1 \end{bmatrix}, \ r^T \xi = [r_\xi_1, r_\xi_2] \begin{bmatrix} \xi_2 \\ \xi_1 \end{bmatrix}
\]

can be written in normal form:

\[
\begin{align*}
\dot{\sigma} & = \dot{y} + \lambda \dot{y}_1 \\
\dot{\xi}_1 & = \lambda \dot{\xi}_2 + (\xi_1 + \Delta \xi_1)^T \xi_2 + \dot{\xi}_2 + \lambda (r + \Delta r)^T \xi + a(z, \xi, \eta) + b_2 u \\
\dot{\eta} & = \dot{\xi}_1 + \lambda \dot{\xi}_2 + \dot{\xi}_2 + \lambda (r + \Delta r)^T \xi + a(z, \xi, \eta) + b_2 u
\end{align*}
\]

where

\[
\begin{align}
\dot{\xi}_1 & = (\lambda \dot{r} + \Delta r) + \lambda (\dot{r}_1, \dot{r}_2) \\
\dot{\xi}_2 & = (\dot{r}_1 + \Delta r_1 - \lambda (\dot{r}_2 + \Delta r_2) - \lambda \xi_2)^T, (\xi + \Delta \xi)^T \\
\dot{p} & = [1, (\rho_\xi_2 + \Delta p_\xi_2)^T]^T \\
\dot{q} & = \left[\begin{array}{c}
-\lambda \\
(\rho_\xi_2 + \Delta p_\xi_2 - \lambda \rho_\xi_2 + \Delta p_\xi_2) \\
\dot{Q} + \Delta Q
\end{array}\right]
\end{align}
\]

It is easily verified that the linear part of the zero-dynamics, \(\dot{\eta} = \dot{Q} \eta\) still is unstable. This problem can, however, be solved by modifying the sliding surface.

4.2 Combined Sliding Surface for Roll and Yaw

Consider the modified sliding surface, see [7]:

\[
\dot{\sigma} = \sigma - k^T \eta
\]

where \(k = [k_1, k_2, k_3, k_4]^T\). Hence the modified \(\sigma\)- and \(\eta\)-dynamics are:

\[
\begin{align}
\dot{\sigma} &= (\dot{r} - k^T \dot{p}) \sigma + \dot{a}(\sigma + k^T \dot{Q}) \eta \\
&+ (\dot{r}k^T + \dot{\xi}^T - k^T (\dot{p}k^T + \dot{Q})) \eta \\
&- k^T \xi (\sigma + k^T \dot{Q}) \eta + b_2 u \\
\dot{\eta} &= \dot{\xi} \sigma + \dot{p} k^T + \dot{Q} \eta + \dot{a}(\sigma + k^T \dot{Q}) \eta
\end{align}
\]

From (26) and (27) the zero dynamics in \((\sigma, \eta)\)-coordinates are obtained:

\[
\dot{\eta} = (\dot{p} k^T + \dot{Q}) \eta + \dot{a}(k^T \dot{Q}) \eta
\]

Since \((A, b)\) in (6) is a controllable pair, it can be shown that \((\dot{p}, \dot{Q})\) is a controllable pair. Hence, there exists a \(k\) such that \(\dot{p} k^T + \dot{Q}\) is Hurwitz. Moreover, the Lyapunov equation

\[
(\dot{p} k^T + \dot{Q})^T P_0 + P_0 (\dot{p} k^T + \dot{Q}) = -Q_0
\]

has a unique positive definite solution \(P_0\) if \(Q_0\) is positive definite. Then, consider the Lyapunov function

\[
V = \eta^T P_0 \eta
\]

with time derivative

\[
\dot{V} = -\eta^T Q_0 \eta + 2 \eta^T P_0 \dot{a}(k^T \dot{Q}) \eta
\]

The following assumption is made.

Assumption 2 Assume that \(\eta^T P_0 \dot{a}(k^T \dot{Q}) \eta \leq 0\).

Remark 1 Assumption 2 implies that only the linear part of the zero-dynamics in roll can become unstable. This behavior is also observed in yaw where linear damping can have both signs (cause stable and unstable ships) whereas the nonlinear damping always is dissipative.

Using Assumption 2 we obtain

\[
\dot{V} \leq -\eta^T Q_0 \eta
\]

and regulation of \(\sigma\) to zero implies that \(\eta \to 0\) and finally (25) implies that \(\sigma \to 0\), i.e.,

\[
\sigma \to 0 \implies \phi, p, v, r, (\psi - \psi_d) \to 0
\]

4.3 Nonlinear Control Law

Rewrite (24) such that

\[
\begin{align}
\dot{r} &= \dot{r} + \Delta r, \quad s = \dot{s} + \Delta s, \quad p = \dot{p} + \Delta p, \quad Q = \dot{Q} + \Delta Q
\end{align}
\]

where \(\dot{r}, \dot{s}, \dot{p}\) and \(\dot{Q}\) are known. Then to obtain the desired \(\sigma\)-dynamics (\(\sigma \to 0\) as \(t \to \infty\) in the case of no disturbances), we propose the following controller

\[
\delta = \frac{1}{b_2} \left( k_\sigma \dot{\sigma} + k_\eta \dot{\eta} - \mu \tanh(\mu \dot{\sigma}) \right).
\]
Figure 1: Block diagram showing the roll damping system and course-keeping controller.

Here $k_\delta$ is the feedback gain defined as

$$k_\delta = k^T \hat{p} - \hat{\gamma} - \gamma$$

(36)

where $\gamma \geq 0$, $k_\eta$ is a feed-forward gain vector,

$$k_\eta^T = k^T (\hat{p}k^T - \hat{Q}) - \hat{\eta}k^T - \hat{s}^T$$

(37)

whereas $\mu > 0$ is the switching gain and $\varphi > 0$ is a boundary layer parameter. The definition of the switching gain is given by the stability proof in Section 5 while the boundary layer parameter is a design parameter. The ship with the control law is shown in Figure 1.

5 Main Result

The main results of the paper is summarized in the following theorem.

**Theorem 1** Consider the ship model (3) with controller (35)–(37) and let Assumptions 1 and 2 be satisfied. Then, there exists a $\mu$ in (35) such that for $w_H = 0$ the closed-loop system is globally asymptotically stable and for $w_H \neq 0$ and $w_H$ bounded, the closed-loop is globally ultimately bounded.

**Proof:** Since the zero-dynamics (28) is stable and independent of the control variable, $\dot{\delta}$, then according to [8] $\dot{\eta} \to 0$ if $\delta = 0$. Hence, stability of (26) and (27), and thus the ship model (3) with the controller (35)–(37) can be analyzed using the following Lyapunov-function candidate

$$V(\dot{\delta}(x)) = \frac{1}{2} \dot{\sigma}^2(x) \geq 0.$$  

(38)

Writing $\dot{\sigma}(x) = \dot{\delta}$ the time derivative of the Lyapunov-function candidate becomes:

$$\dot{V}(\dot{\delta}) = \dot{\sigma} \dot{\delta}$$  

(39)

Substitution of the $\dot{\delta}$-dynamics (26) into $\dot{V}$ implies:

$$\dot{V}(\dot{\delta}) = \dot{\sigma}(\dot{\delta} - k^T \hat{p}) \dot{\delta} + \dot{\delta} (\dot{\delta} + k^T \hat{\eta}, \dot{\eta})$$

$$+ \dot{\delta} \left(k^T \hat{p}k^T + \hat{\eta} - k^T \hat{Q} + \dot{Q} \right) \dot{\eta}$$

$$- \dot{\sigma} \left(k^T \hat{p} \dot{\delta} + k^T \dot{\delta} \hat{\eta}, \dot{\eta} \right) + \dot{\delta} \dot{\eta}$$

(40)

Substituting the controller$^1$ (35) into this expression yields:

$$\dot{V}(\dot{\delta}) = \dot{\sigma}(\dot{\delta} - k^T \hat{p} + k_\delta \dot{\delta} - \dot{\delta} \mu \cdot \text{sign}(\dot{\delta})$$

$$+ \dot{\delta} \left(k^T \hat{p}k^T + \hat{\eta} - k^T (\hat{p}k^T + \hat{Q}) + k_\eta^T \right) \dot{\eta}$$

$$- \dot{\sigma} k^T \dot{\delta} (\dot{\delta} + k^T \hat{\eta} + \hat{\eta})$$

(41)

Next, the choice of control parameters given by (36) and (37) implies that

$$\dot{V}(\dot{\delta}) = -\gamma \dot{\delta}^2 - \dot{\delta} \mu \cdot \text{sign}(\dot{\delta}) + \dot{\delta} \left(k^T \dot{\delta} + \dot{\delta} \hat{\eta} - k^T (\hat{p}k^T + \hat{Q}) \right) \dot{\eta}$$

$$+ \dot{\delta} \left(k^T \hat{p}k^T + \dot{\eta} \right) - k^T \dot{\delta} \dot{\eta} + \dot{\delta} \dot{\eta}$$

(42)

$$= -\gamma \dot{\delta}^2 - \dot{\delta} \mu \text{sign}(\dot{\delta}) + \dot{\delta} \dot{\eta}$$

(43)

with obvious definition of $\Delta h(\dot{\delta}, \dot{\eta})$. Finally, by choosing the switching gain $\mu$ according to

$$\mu > |\Delta h(\dot{\delta})|$$

(44)

the time derivative of the Lyapunov-function becomes:

$$\dot{V}(\dot{\delta}) \leq -\gamma \dot{\delta}^2 + |\dot{\delta} \dot{\eta}|$$

$$\leq -\gamma |\dot{\delta}| \left(|\dot{\delta}| - \frac{|\dot{\delta}|}{\gamma} |\dot{\delta}| \right)$$

(45)

Hence $\dot{\delta}$ will be bounded and approach a small neighborhood of the origin which proves boundedness. Moreover, when $w_H = 0$, $\dot{V}(\dot{\delta})$ satisfy $\dot{V}(\dot{\delta}) \geq 0$ and $\dot{V}(\dot{\delta}) \leq 0$ and application of LaSalle's invariant set theorem, see e.g. [11], implies that

$$\dot{\delta} \to 0 \text{ as } t \to \infty$$

(46)

which proves global asymptotic stability. □

The performance of the nonlinear controller is illustrated in a simulation study in the following section.

$^1$Notice that $\tanh(\delta \dot{\delta})$ is replaced by $\text{sign}(\dot{\delta})$ in the stability proof in order to guarantee that $\dot{V} \leq 0$. The continuous function $\tanh(\delta \dot{\delta})$ is, however, used for implementation to reduce chattering and does not cause stability problems.
6 Simulation Study

The controller is applied to the nonlinear ship model given in [2]. The main ship data are given in Appendix A. The rudder saturation δ_{max} is set to 30 (deg) while the rudder rate δ̇_{max} is limited to 15 (deg/s). A measure of effectiveness of the rudder-roll damping system is taken from [12] and reads as follows:

\[
\text{Roll reduction} = 100 \cdot \frac{\text{AP} - \text{RRCS}}{\text{AP}} \% \quad (47)
\]

where RRCS and AP are the standard deviations with and without roll damping, respectively. The vector \( k \) in the transformation:

\[
\tilde{\sigma} = \sigma - k^T \tilde{\eta} \quad (48)
\]

was computed using the \texttt{matlab} pole-placement function \texttt{acker} and the eigenvalues of \( \dot{p}k^T + Q \) were placed at \(-0.5\), i.e. \( \lambda_i = -0.5 \), \( i = 1, 2, 3, 4 \). The feedback and feedforward gains, \( k_\sigma \) and \( k_\eta \) in (35) were chosen according to (36) and (37). Finally, \( \gamma = 1 \) whereas the switching gain \( \mu \) and the boundary layer parameter \( \varphi \) were chosen as 0.2 and 1, respectively.

In the first simulation study, the cruise speed was set to 10 m/s and the states of interest are shown in Figure 2. By comparing the roll angle \( \phi \) and the angular velocity in roll \( \dot{\phi} \) with a simulation without rudder-roll damping there were obtained a damping in roll and roll rate of 71% and 73%, respectively. These are remarkable good results.

Figure 3 shows the same controller (no speed-scaling) when the cruise speed is reduced to 8 (m/s). Naturally the effectiveness is reduced and the damping where measured to be 50 % in roll and 49 % in roll rate. This can be improved by scaling the controller with respect to speed. Notice that the autopilot in yaw has excellent performance even without speed scaling.

7 Conclusions

The combined rudder-roll damping and course controller derived in this paper proved to be easy to tune and the stability in the presence of 1st-order wave disturbances has been verified by simulations. The robustness to modeling errors has been demonstrated by applying the same controller at two different cruise speeds.

8 Acknowledgment

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A Ship Model Data

A detailed description of the nonlinear ship model used in the simulations is found in [2]. Some of the main ship data are given in Table 1.

<table>
<thead>
<tr>
<th>Main particulars for ship</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Length, ( L_{wp} )</td>
<td>48 m</td>
</tr>
<tr>
<td>Beam, ( B )</td>
<td>8.6 m</td>
</tr>
<tr>
<td>Draft, ( D )</td>
<td>2.2 m</td>
</tr>
<tr>
<td>Displacement, ( V )</td>
<td>350 m³</td>
</tr>
<tr>
<td>Inertia in yaw, ( I_y )</td>
<td>( 33.7 \times 10^6 ) kgm²</td>
</tr>
<tr>
<td>Inertia in roll, ( I_x )</td>
<td>( 3.4 \times 10^6 ) kgm²</td>
</tr>
</tbody>
</table>

Table 1: Data for the nonlinear ship model.
Figure 3: The figure shows the roll angle $\phi$, yaw angle $\psi$ and rudder angle $\delta$ in a simulation over 1000 (s) when the cruise speed $U$ is 8 (m/s). The controller is not active the first 300 (s) and is turned on after 300 (s). The reference angle in yaw changes from 0 (deg) to 10 (deg) after 600 (s). The wave encounter frequency is 0.8 (rad/s).

References


