Semi-global Exponential Output Feedback Control of Ships

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Abstract

This paper describes a semi-global exponentially stable output feedback control law for automatic heading control of ships described by a nonlinear model. Only compass (yaw angle) feedback is used. The yaw rate signal is reconstructed by a linear observer. A wave filter is included in the observer to reduce wear and tear on the rudder servo due to 1st-order wave-induced disturbances. Integral action is included in the control law to compensate for wave drift (2nd-order wave disturbances), low-frequency wind and current disturbances. The performance and robustness of the controller are demonstrated in a simulation study of two medium-sized ships.

1 Introduction

Conventional autopilots for ships require feedback from the yaw angle and yaw rate. Most ships, however, are only equipped with a gyro or a magnetic compass to measure the yaw angle. Hence, the yaw rate must be estimated by using a state estimator (observer). In addition to this, the yaw angle measurement is corrupted with wave-induced coloured noise. The wave disturbances are usually divided into 1st-order wave disturbances (oscillatoric behaviour) and 2nd-order wave disturbances (wave drift), see (Faltinsen 1990) for instance. Hence, the control system must generate the rudder signal by using filtered measurements. Moreover, the states are not directly available for feedback. This paper proposes an output feedback controller with wave filtering properties and integral action. Output feedback of ships with emphasis placed on nonlinear damping has also been discussed by (Paulsen 1996). An input error adaptive output feedback control law for ships has been proposed by (Lauvdal and Fossen 1995). The control law in this paper is based on an extension of the work of (Berghuis 1993).

2 Ship and Wave Disturbance Modelling

The wave filter problem is usually solved by modelling the compass measurement as the sum (Fossen 1994):

$$\psi = \psi_L + \psi_H \tag{1}$$

where ψ_L represents the the low-frequency ship dynamics and ψ_H is coloured measurement noise due to the 1st-order wave disturbances. Hence ψ_L must be reconstructed from ψ by the observer since this is the signal which should be used for feedback. The 1st-order wave disturbances can be described by a 2nd-order linear approximation to the Pierson-Moskowitz spectral density function, see (Fossen 1994) and references therein:

$$\dot{\xi}_H = \psi_H \tag{2}$$

$$\dot{\psi}_H = -2\zeta_n\, \omega_n\, \psi_H - \omega_n^2\, \xi_H + K_\omega\, \omega_H \tag{3}$$

where ζ_n is the relative damping ratio, ω_n is the dominating wave frequency, $K_w > 0$ is a constant and ω_H is Gaussian white noise. The yaw dynamics of the ship can be conveniently described by the nonlinear model of (Norrbin 1963), that is

$$\text{(Dynamics)} \qquad m\dot{r}_L + d(r_L) = \delta \tag{4}$$

$$\text{(Kinematics)} \qquad \dot{\psi}_L = r_L \tag{5}$$

where $m > 0$ is a positive constant and $d(r_L)$ is a nonlinear function describing the maneuvering characteristics of the ship, usually modelled as:

$$d(r_L) = d_3 r_L^3 + d_1 r_L \tag{6}$$

It will be assumed that $d(z)$ *is continuously differentiable in* z *such that a bound on* z *implies that* $\frac{\partial d(z)}{\partial z}$ *is bounded.* The resulting ship-wave model (2) – (5) is described by the four states ξ_H, ψ_H, r_L, and ψ_L, whereas the measurement equation is $\psi = \psi_L + \psi_H$. An output feedback control law using feedback from ψ will now be designed such that the signal ψ_H is suppressed in the feedback loop (wave filtering).

3 Main Results

For notational simplicity the following variables are introduced:

$$\begin{aligned} &\text{(Tracking error)} & \bar{\psi} &= \psi - \hat{\psi}_H - \psi_d & (7)\\ &\text{(Total estimation error)} & \tilde{\psi} &= \psi - \hat{\psi}_H - \hat{\psi}_L & (8)\\ &\text{(HF estimation error)} & \tilde{\psi}_H &= \psi_H - \hat{\psi}_H & (9) \end{aligned}$$

where the hat denotes the state estimates and ψ_d is the desired yaw angle. The performance of the observer and the control law is monitored by defining the sliding surfaces:

$$s_1 = \dot{\bar{\psi}} + 2\lambda_1\bar{\psi} + \lambda_1^2 \int_0^t \bar{\psi} d\tau, \qquad \lambda_1 > 0 \tag{10}$$

$$s_2 = \dot{\tilde{\psi}} + \lambda_2\tilde{\psi}, \qquad \lambda_2 > 0 \tag{11}$$

Based on these definitions we propose to adapt the results of (Berghuis 1993) to the ship-wave model discussed in Section 2. The main problem in doing this, is the wave model (2) – (3) which is driven by unknown white noise. We propose the following ship control law and observer:

$$\text{Controller}: \quad \left\{ \tau = m\ddot{\psi}_d + d(\dot{\psi}_d) - K_d(s_1 - s_2) \right. \tag{12}$$

$$\text{LF observer}: \quad \left\{ \begin{aligned} \dot{\hat{\psi}} &= \hat{z} + L_{d3}\bar{\psi} + L_{d1}\tilde{\psi} \\ \dot{\hat{z}} &= \ddot{\psi}_d + L_{p3}\bar{\psi} + L_{p1}\tilde{\psi} \end{aligned} \right. \tag{13}$$

$$\text{HF observer}: \quad \left\{ \begin{aligned} \dot{\hat{\xi}}_H &= \hat{\psi}_H + L_{p2}\tilde{\psi} \\ \dot{\hat{\psi}}_H &= -c_1\hat{\psi}_H - c_2\hat{\xi}_H + L_{d2}\tilde{\psi} \end{aligned} \right. \tag{14}$$

where $c_1 = 2\zeta_n\omega_n$ and $c_2 = \omega_n^2$ are given by the wave model, $K_d > 0$ is the feedback gain and $L_{d1}, L_{d2}, L_{d3}, L_{p1}, L_{p2}$, and L_{p3} are 6 observer gains which must be chosen such that notch filtering of the yaw angle measurements at $\omega = \omega_n$ is obtained at the same time as stability is guaranteed. Like (Berghuis 1993) we will tune the observer gains to the underlying controller to ensure stability of the entire system. The following algorithm is proposed for this purpose:

Observer Pole-Placement Algorithm

The observers (13) and (14) can be combined to give:

$$\frac{\hat{\psi}_L}{\psi}(s) = \frac{[(L_{d1} + L_{d3})s + (L_{p1} + L_{p3})][s^2 + 2\zeta_n\omega_n s + \omega_n^2]}{s^4 + \beta_1 s^3 + \beta_2 s^2 + \beta_3 s + \beta_4} \tag{15}$$

where

$$\beta_1 = 2\zeta_n\omega_n + L_{d2} + L_{d1} \tag{16}$$

$$\beta_2 = (1 - L_{p2})\omega_n^2 + L_{p1} + 2\zeta_n\omega_n L_{d1} - L_{d3}L_{d2} \tag{17}$$

$$\beta_3 = L_{d1}\omega_n^2 + 2\zeta_n\omega_n L_{p1} - L_{p3}L_{d2} + L_{d3}L_{p2}\omega_n^2 \tag{18}$$

$$\beta_4 = L_{p1} + L_{p2}L_{p3}\omega_n^2 \tag{19}$$

The transfer function (15) is shown in Figure 1.

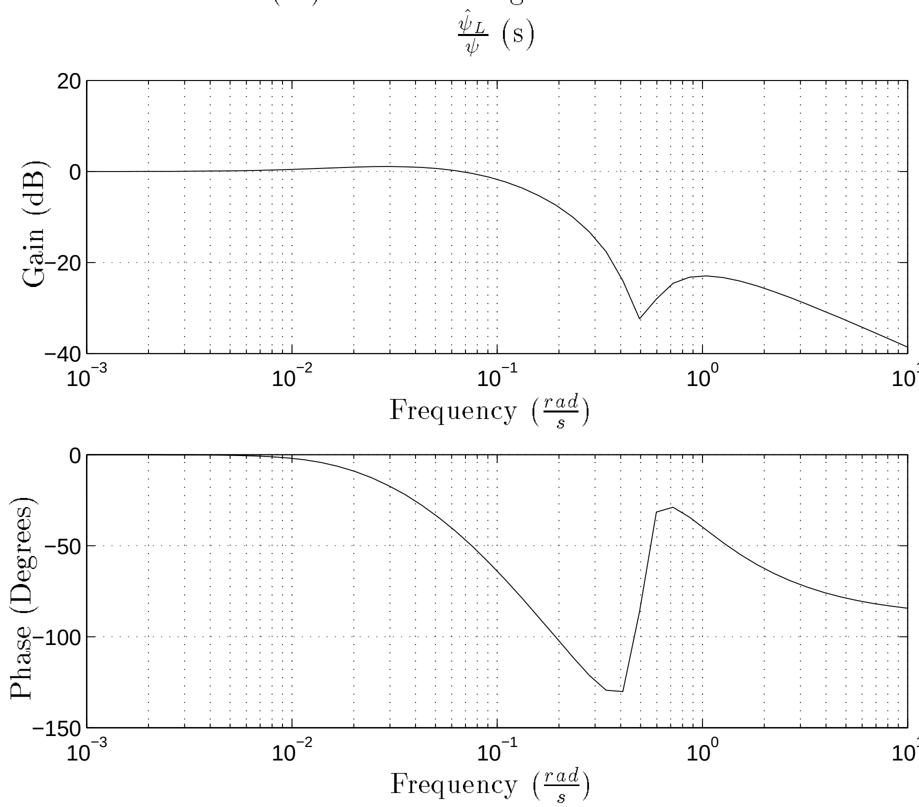

Figure 1: *Transfer function from measured yaw angle, ψ, to estimated low frequency yaw angle, $\hat{\psi}_L$. Parameters: $L_{d1} = 0.1183$, $L_{p1} = 0.0018$, $L_{d2} = 0.8827$, $L_{p2} = -0.3497$, $L_{p3} = 0$, $L_{d3} = 0.0050$.*

The 6 gains $L_{d1}, L_{d2}, L_{d3}, L_{p1}, L_{p2}$, and L_{p3} can be determined by proper placement of the 4 poles and the zero in 15. It is convenient to define:

$$\begin{array}{rclcrcl} L_{d1} & = & L_d + \lambda_2 & \quad & L_{p1} & = & \lambda_2 L_d \\ L_{d3} & = & \alpha \, L_d & \quad & L_d & = & \frac{2K_d}{m} \end{array} \tag{20}$$

where $\alpha > 0$ and $L_d > 0$ are two new design parameters. These equations link three of the parameters to the controller gain K_d via the design parameter L_d. Next, the following assumptions are made:

Assumption 1 *It is convenient to choose $L_{p3} = 0$. Notice that this will not reduce the flexibility in the design. Hence only 5 gains are required to place the 4 poles and the zero.*

Assumption 2 *For simplicity it is assumed that $\zeta_n = 0.1$, and $\omega_n = 0.5$, which gives $c_1 = 0.1$, and $c_2 = 0.5$. The stability proof in Theorem 1, however holds for $\zeta_n > 0$ and $\omega_n \in [\omega_{min}, \infty]$ with $\omega_{min} > \omega_b$ where ω_b is the bandwidth of the ship. The lower bound on ω_n is necessary since the additional phase lag from the notch filter will destabilize the control loop when the notch filter center frequency is close to the ship's bandwidth. In cases where the actual dominating wave frequency is lower than ω_{min}, acceptable performance can still be achieved with $\omega_n = \omega_{min}$. When wave frequencies are within the bandwidth of the ship, we do not require wave filtering. In practice, the dominating wave frequency can be estimated with a frequency tracker and fed to the observer (Holtzhüter and Strauch 1987).*

Assumption 3 *Two of the poles are chosen equal to ω_n, see (Fossen 1993). Hence it can be shown that (Appendix A):*

$$L_{d2} = \frac{A_2 L_d^2 + A_1 L_d + A_0}{B_2 L_d^2 + B_1 L_d + B_0}; \qquad L_{p2} = \frac{C_2 L_d^2 + C_1 L_d + C_0}{D_2 L_d^2 + D_1 L_d + D_0} \tag{21}$$

where $A_0, A_1, A_2, ...D_0, D_1, D_2$ depend on α and λ_2, but not on L_d. This means that if $K_d = mL_d/2$ is increased, and $\alpha \neq 0$, the gains L_{d2} and L_{p2} will be bounded by:

$$L_{d2} \leq \frac{A_2}{B_2} = -\frac{0.9}{\alpha}, \qquad L_{p2} \leq \frac{C_2}{D_2} = \frac{3.2\lambda_2}{\alpha} \tag{22}$$

Assumption 4 *The gain α is bounded as follows:*

$$0 < \alpha^2 < \frac{1}{2} \tag{23}$$

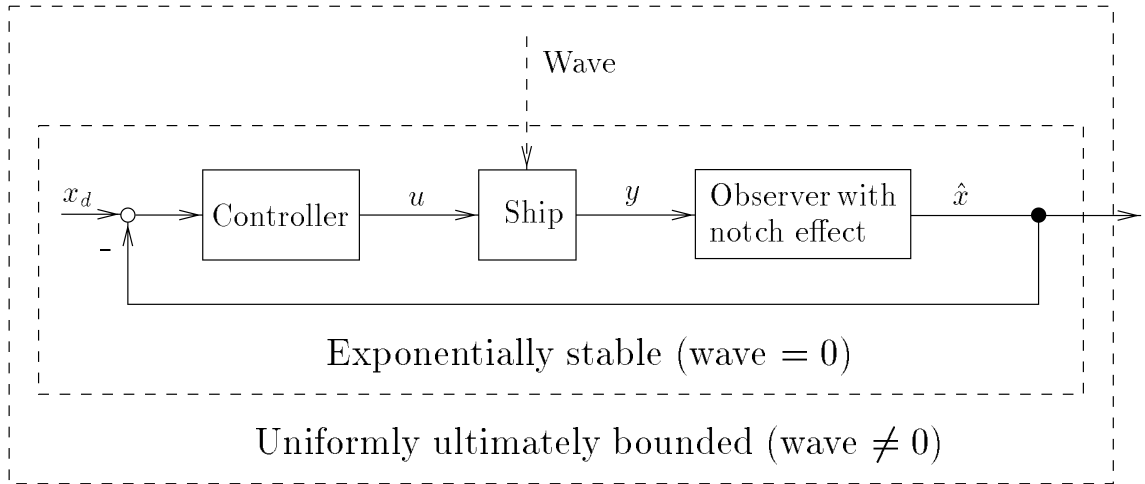

Figure 2: *Block diagram showing the stability properties of the system.*

The upper limit is found by requiring that the matrix $\boldsymbol{Q}$ in (38) is positive definite. From Appendix A, it can be seen that for $\alpha = 0$, limits for L_{d2} and L_{p2} do not exist. Hence α^2 must be greater than zero as assumed in (23).

Assumptions (1) – (4) imply that the two poles and the zero can be specified by three gains (K_d, α and λ_2). The other two poles are placed such that wave filtering is accomplished at $\omega = \omega_n$. Hence α and λ_2 can be chosen within the limits of (4) and (11), whereas K_d must be chosen according to Theorem 1 below. In the proof of Theorem 1, it will be assumed that $K_w = 0$, which means that there is no wave-induced disturbances. This will not represent any loss of generality, because Theorem 1 only states that the nonlinear ship model, with the output feedback control system including wave filtering properties and integral action, is exponentially stable. Hence, it is not necessary to include environmental disturbances in the proof, although the purpose of the wave-filter and integral action is to deal with these disturbances. The problem with environmental disturbances belongs naturally in a robustness analysis, and, as indicated in Figure 2, it can be shown that the system is uniformly ultimately bounded if the environmental disturbances (and model uncertainties) are assumed bounded. This problem will not be addressed here.

Theorem 1 (Semi-global Exponential Output Feedback Control of

Ships)
The system (4) and (5) with control law (12), observers (13) and (14) and state vector:

$$\mathbf{x} = \left[\dot{\bar{\psi}},\ 2\lambda_1 \bar{\psi},\ \lambda_1^2 \int_0^t \bar{\psi},\ \dot{\tilde{\psi}},\ \lambda_2 \tilde{\psi},\ \dot{\tilde{\psi}}_H,\ \lambda_2 \tilde{\psi}_H \right]^T \tag{24}$$

is semi-globally exponentially stable about $\mathbf{x} = \mathbf{0}$ *under Assumptions 1–4 and if* $K_d > 0$ *is chosen sufficiently large.*

Proof of Theorem 1:
The closed-loop for the HF dynamics (HF model and HF observer) given by (2), (3), and (14) can be written:

$$\ddot{\tilde{\psi}}_H = -c_1 \dot{\tilde{\psi}}_H - c_2 \tilde{\psi}_H + L_{p2} \tilde{\psi} - L_{d2} \dot{\tilde{\psi}} \tag{25}$$

The closed loop for the ship model and the controller is obtained by substituting (12) into (4) and (5) which yields

$$m \ddot{\psi}_L - m \ddot{\psi}_d + d(\dot{\psi}_L) - d(\dot{\psi}_d) + K_d(s_1 - s_2) = 0 \tag{26}$$

Application of the *Mean Value Theorem* yields

$$d(\dot{\psi}_L) - d(\dot{\psi}_d) = \gamma(\dot{\psi}_L - \dot{\psi}_d); \qquad \gamma = \left. \frac{\partial d(z)}{\partial z} \right|_{z=\mu} \tag{27}$$

Here μ is a point on the line segment between $\dot{\psi}_L$ and $\dot{\psi}_d$. Notice that $\gamma = \gamma(\boldsymbol{x})$ is a function of the state vector. By using $\dot{\psi}_L - \dot{\psi}_d = \dot{\bar{\psi}} - \dot{\tilde{\psi}}_H$, and (27) we obtain

$$m \ddot{\bar{\psi}} - m \ddot{\tilde{\psi}}_H + \gamma \dot{\bar{\psi}} - \gamma \dot{\tilde{\psi}}_H + K_d(s_1 - s_2) = 0 \tag{28}$$

substituting (25) into the expression for $\ddot{\tilde{\psi}}_H$ gives:

$$m \ddot{\bar{\psi}} = -\gamma \dot{\bar{\psi}} + (\gamma - m c_1) \dot{\tilde{\psi}}_H - m c_2 \tilde{\psi}_H + m L_{p2} \tilde{\psi} - m L_{d2} \dot{\tilde{\psi}} - K_d(s_1 - s_2) \tag{29}$$

The closed-loop for the LF part of the observer is obtained from (13), that is:

$$\ddot{\hat{\psi}}_L = \ddot{\psi}_d + L_{p1} \tilde{\psi} + L_{d1} \dot{\tilde{\psi}} + L_{d3} \dot{\bar{\psi}} \tag{30}$$

Using (20) gives:

$$\ddot{\hat{\psi}}_L = \ddot{\psi} - \ddot{\hat{\psi}}_H - \ddot{\bar{\psi}} + \lambda_2 L_d \tilde{\psi} + (L_d + \lambda_2) \dot{\tilde{\psi}} + \alpha L_d \dot{\bar{\psi}} \tag{31}$$

Finally, from (8) and (11) we get:

$$\dot{s_2} = \ddot{\tilde{\psi}} - L_d s_2 - \alpha L_d \dot{\tilde{\psi}} \tag{32}$$

The following Lyapunov function candidate is proposed:

$$\begin{aligned} V(s_1, \bar{\psi}, \int_0^t \bar{\psi} d\tau, s_2, \tilde{\psi}, \tilde{\psi}_H) &= \frac{1}{2}\ ms_1^2 + \frac{1}{2}\ \left[\bar{\psi},\ \int_0^t \bar{\psi} d\tau\right]\ \mathbf{K_1}\ \left[\bar{\psi},\ \int_0^t \bar{\psi} d\tau\right]^T \\ &+ \frac{1}{2}\ ms_2^2 + \frac{1}{2}\ K_2 \tilde{\psi}^2 + \frac{1}{2}\ \left[\dot{\tilde{\psi}}_H,\ \tilde{\psi}_H\right]\ \mathbf{K_3}\ \left[\dot{\tilde{\psi}}_H,\ \tilde{\psi}_H\right]^T \end{aligned} \tag{33}$$

where

$$\mathbf{K_1} = \begin{bmatrix} 4K_d\lambda_1 - 5m{\lambda_1}^2 & 2K_d{\lambda_1}^2 - 2m{\lambda_1}^3 \\ 2K_d{\lambda_1}^2 - 2m{\lambda_1}^3 & 4K_d{\lambda_1}^3 - m{\lambda_1}^4 \end{bmatrix} \tag{34}$$

$$K_2 = 2\lambda_2 K_d + mL_{d2}\lambda_2 - mL_{p2}c_2 \tag{35}$$

$$\mathbf{K_3} = \begin{bmatrix} (\frac{1}{c_1} + \frac{\lambda_2}{c_1}) & \lambda_2 \\ \lambda_2 & ((\frac{c_2}{\lambda_2^2 c_1} + \frac{c_2}{\lambda_2 c_1}) + \lambda_2^{-1} c_1)\lambda_2^2 \end{bmatrix} \tag{36}$$

In Appendix B, it is shown that:

$$\dot{V}(\mathbf{x}) = -\mathbf{x^T Q x} < \mathbf{0} \quad \forall\ \boldsymbol{x} \neq \mathbf{0} \tag{37}$$

where

$$\mathbf{Q} = \begin{bmatrix} Kd - \gamma - 2\lambda_1 m & \frac{1}{2}\gamma & \frac{1}{2}\gamma & \frac{1}{2}(\gamma + mL_{d2}) + \alpha K_d & \frac{1}{2}(\gamma - mc_2L_{p2}\lambda_2^{-1}) + \alpha K_d & -\frac{1}{2}(\gamma - mc_1) & \frac{1}{2}mc_2\lambda_2^{-1} \\ \frac{1}{2}\gamma & Kd & 0 & +\frac{1}{2}mL_{d2} & -\frac{1}{2}mc_2L_{p2} & -\frac{1}{2}(\gamma - mc_1) & \frac{1}{2}mc_2\lambda_2^{-1} \\ \frac{1}{2}\gamma & 0 & K_d & +\frac{1}{2}mL_{d2} & -\frac{1}{2}mc_2L_{p2}\lambda_2^{-1} & -\frac{1}{2}(\gamma - mc_1) & \frac{1}{2}mc_2\lambda_2^{-1} \\ \frac{1}{2}(\gamma + mL_{d2}) + \alpha K_d & +\frac{1}{2}mL_{d2} & \frac{1}{2}mL_{d2} & K_d + mL_{d2} & 0 & \frac{1}{2}((1+\lambda_2)\frac{L_{d2}}{c_1} - (\gamma - mc_1)) & \frac{1}{2}(L_{d2} + mc_2L_{p2}\lambda_2^{-1}) \\ \frac{1}{2}(\gamma - mc_2L_{p2}\lambda_2^{-1}) + \alpha K_d & -\frac{1}{2}mc_2L_{p2} & -\frac{1}{2}mc_2L_{p2}\lambda_2^{-1} & 0 & K_d - mc_2L_{p2}\lambda_2^{-1} & \frac{1}{2}((1+\lambda_2)\frac{L_{p2}}{c_1}\lambda_2 - 1 - (\gamma - mc_1)) & \frac{1}{2}(mc_2 - L_{p2})\lambda_2^{-1} \\ -\frac{1}{2}(\gamma - mc_1) & -\frac{1}{2}(\gamma - mc_1) & -\frac{1}{2}(\gamma - mc_1) & \frac{1}{2}((1+\lambda_2)\frac{L_{d2}}{c_1} - (\gamma - mc_1)) & \frac{1}{2}((1+\lambda_2)\frac{L_{p2}}{c_1}\lambda_2 - 1 - (\gamma - mc_1)) & 1 & 0 \\ \frac{1}{2}mc_2\lambda_2^{-1} & \frac{1}{2}mc_2\lambda_2^{-1} & \frac{1}{2}mc_2\lambda_2^{-1} & \frac{1}{2}(L_{d2} + mc_2L_{p2}\lambda_2^{-1}) & \frac{1}{2}(mc_2 - L_{p2})\lambda_2^{-1} & 0 & c_2\lambda_2^{-1} \end{bmatrix} \tag{38}$$

Since γ is a function of $\boldsymbol{x}$ we must assume that γ is bounded by γ_{max}. From (27) this means that $\dot{\psi}_L - \dot{\psi}_d$ must be bounded, which is a reasonable assumption. The rest of the off-diagonal elements are also bounded. If K_d is chosen sufficiently large and α is chosen according to Assumption 4, $\boldsymbol{Q}$ will be positive definite. Consequently, $\dot{V}$ satisfies:

$$\dot{V}(\mathbf{x}) \leq -\kappa \| \mathbf{x} \|^2 \tag{39}$$

where $\kappa = \underset{\gamma \leq \gamma_{max}}{\lambda_{min}(\boldsymbol{Q})} > 0$. Further it can be shown that

$$\frac{1}{2} P_m \| \mathbf{x} \|^2 \leq V(\mathbf{x}) \leq \frac{1}{2} P_M \| \mathbf{x} \|^2 \tag{40}$$

where $P_m = \frac{1}{3} R_m$ and $P_M = 3 R_M$. R_m and R_M are the minimum and maximum eigenvalues of $R = \mathrm{diag}(R_1, R_2)$, where

$$\boldsymbol{R_1} = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & \lambda_1^{-1} K_d - \frac{5}{4} m & \lambda_1^{-1} K_d - m & 0 \\ 0 & \lambda_1^{-1} K_d - m & 4\lambda_1^{-1} K_d - m & 0 \\ 0 & 0 & 0 & m \end{bmatrix} \tag{41}$$

and

$$\boldsymbol{R_2} = \begin{bmatrix} 2\lambda_2^{-1} K_d + m L_{d2} \lambda_2^{-1} - m l_{p2} \lambda_2^{-2} c_2 & 0 & 0 \\ 0 & (\frac{1}{c_1} + \frac{\lambda_2}{c_1}) & \lambda_2 \\ 0 & \lambda_2 & ((\frac{c_2}{\lambda_2^2 c_1} + \frac{c_2}{\lambda_2 c_1}) + \lambda_2^{-1} c_1) \lambda_2^2 \end{bmatrix} \tag{42}$$

Hence it can be concluded that the system (2) – (5) and (12) – (14) is semi-globally exponentially stable about **x=0**.

4 Simulation Study

The controller and observer are tuned to obtain maximum tracking capabilities at the same time as the wave-induced disturbances are suppressed at higher frequencies. The chosen parameters do not match all those used in the stability proof, because the stability results are rather conservative.

4.1 Case Study 1: (Perfect Model Structure)

The model (4) – (6) is used. Model data is taken from a medium sized cargo ship, see Table 1. For the controller it is assumed 20 % error in m, and 40 % error in d_1 and d_3. The ship parameters and control parameters are shown in Table 1, whereas the control gains are given in Table 2. Figures (3) and (4) show the results when the controller is tuned with low and high gains.

Table 1: Parameters used in the ship and control model.

$Parameters$	$Model$	$Controller$
m	82.86	99.43
d_1	2.86	1.72
d_3	5.72	3.44

Table 2: *Controller and observer gains for the cargo ship.*

$Control\ Gains$	$Low-Gain$	$High-Gain$
k_d	1	7
L_{d1}	0.3793	0.8232
L_{p1}	0.0113	0.0233
L_{d2}	1.0307	0.9868
L_{p2}	-1.5832	-3.1333
L_{d3}	0.005	0.005
λ_1	0.1	0.1
λ_2	0.1	0.1

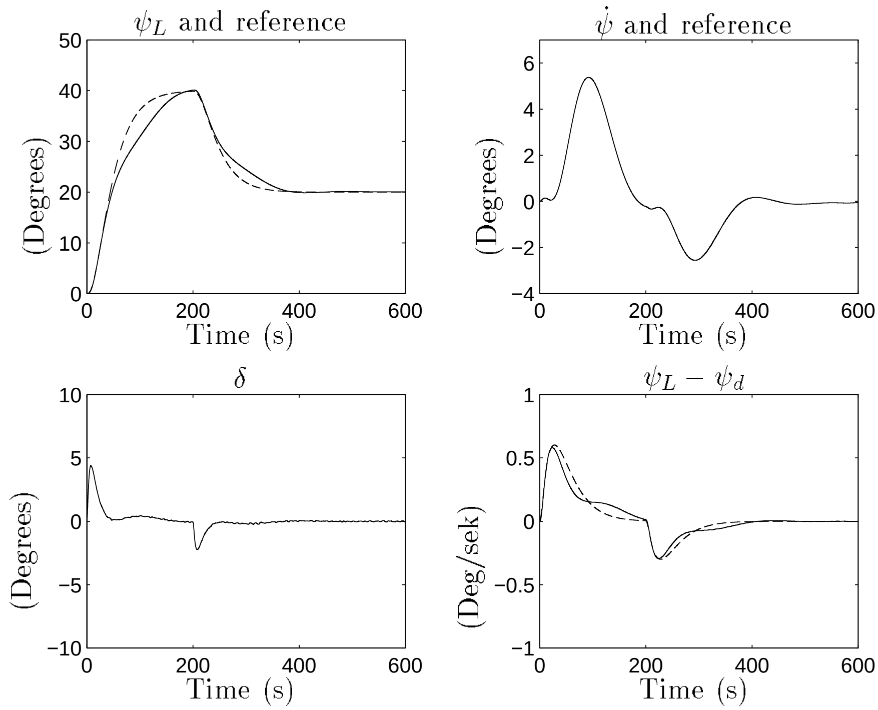

Figure 3: *Simulation results for the cargo ship with the low-gain controller. The wave-induced motion is approximately* $\pm$ *3 degrees in yaw.*

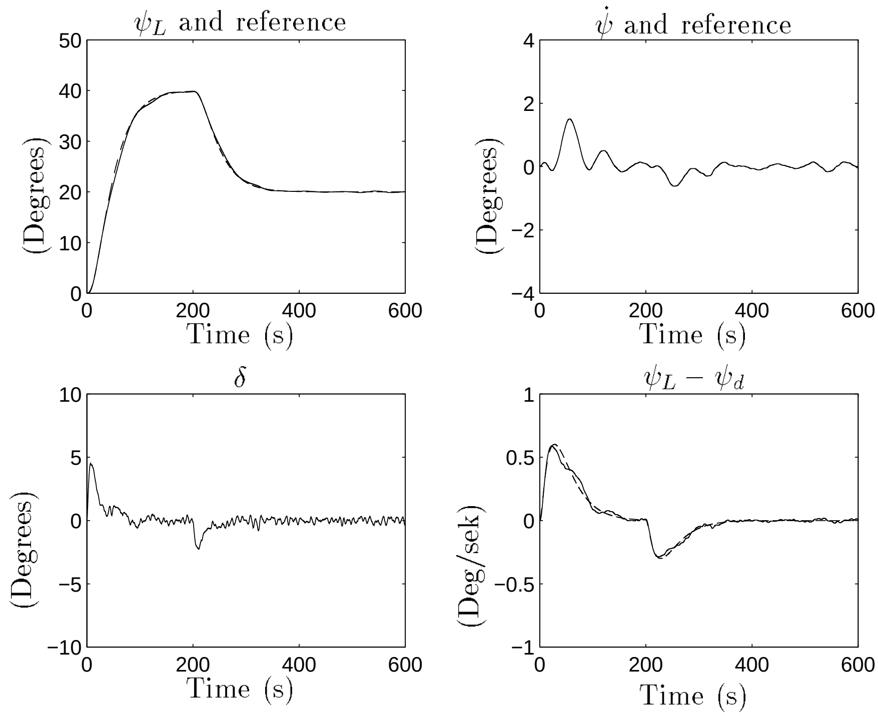

Figure 4: *Simulation results for the cargo ship with the high gain controller. The wave-induced motion is approximately ± 3 degrees in yaw.*

4.2 Case Study 2: (Unknown Model Structure)

In this case, a nonlinear 3-DOF model (surge, sway, and yaw) of the Mariner Class Vessel (Chislett and Strøm-Tejsen 1965) is simulated. The model is given in Appendix C. Notice that this model has a rudder offset. In this case, we have also included a slowly varying disturbance to simulate a current. The control law is based on the following approximation of the Chislett and Strøm-Tejsen model to the model (4) – (6):

$$m = 540, \quad d_1 = 4, \quad d_3 = 80 \tag{43}$$

The controller and observer gains are given in Table 3.

Table 3: *Controller and observer gains for the Mariner Class Vessel*

L_{d1}	=	0.2784	k_d	=	30
L_{p1}	=	0.0115	L_{d3}	=	0.005
L_{d2}	=	1.3216	λ_1	=	0.05
L_{p2}	=	-1.8470	λ_2	=	0.01

The simulation results are shown in Figures (5) – (7). The wave-induced motion is approximately ± 3 degrees. Notice that excellent results are obtained also for the case where an approximated model structure is used. The rudder offset in figure 7 is due to the current and the rudder offset in the model.

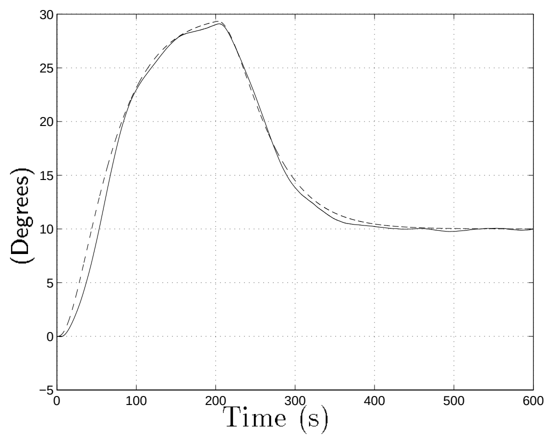

Figure 5: *Low-frequency yaw angle (filtered yaw measurement), and desired yaw angle as a function of time for the Mariner Class Vessel.*

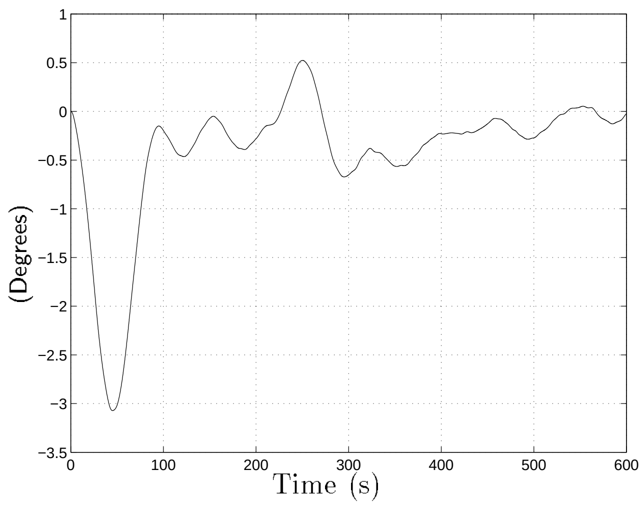

Figure 6: *Tracking error in yaw as a function of time.*

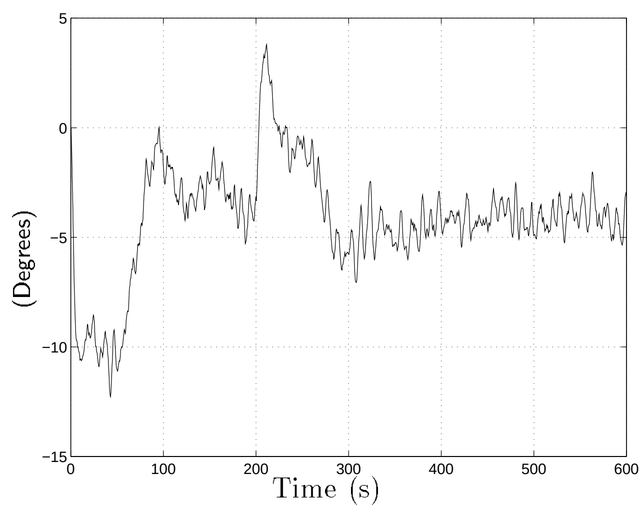

Figure 7: *Rudder angle as a function of time for the Mariner Class Vessel.*

5 Conclusion

In this paper we have described an output feedback control law for automatic heading control of ships described by a nonlinear model. Only feedback from a compass (yaw angle) was used, and the yaw rate signal was reconstructed by a linear observer. A wave filter was included in the observer to reduce wear and tear on the rudder servo due to 1st-order wave-induced disturbances. Integral action was also included in the control law to compensate for wave drift (2nd-order wave disturbances), low-frequency wind and current disturbances. The total system was proved to be semi-globally exponentially stable. Finally, the performance and robustness of the control law were demonstrated in a simulation study of two medium-sized ships.

A Derivation of the Expressions for L_{p2} and L_{d2}

The poles in (15) are given by:

$$\pi(s) = \prod_{i=1}^{4}(s - p_i) \tag{44}$$

This gives the following equations:

$$\begin{aligned}
p_1p_2p_3p_4 &= (L_{p1} + L_{p3}L_{p2})\omega_n^2 && (45)\\
-p_1p_2p_3 - p_1p_2p_4 - p_1p_3p_4 - p_2p_3p_4 &= L_{d1}\omega_n^2 + 2\zeta_n\omega L_{p1} - L_{p3}L_{d2} + L_{d3}L_{p2}\omega_n^2 && (46)\\
p_1p_2 + p_1p_3 + p_1p_4 + p_2p_3 + p_2p_4 + p_3p_4 &= (1 - L_{p2})\omega_n^2 + L_{p1} + 2\zeta_n\omega L_{d1} - L_{d3}L_{d2} && (47)\\
-(p_1 + p_2 + p_3 + p_4) &= 2\zeta_n\omega_n + L_{d2} + L_{d1} && (48)
\end{aligned}$$

By solving (45) - (48) for L_{d2} and L_{p2}, and using (20), and $p_3 = p_4 = \omega_n = 0.5$, and $\zeta_n = 0.1$, we obtain:

$$\begin{aligned}
L_{d2} &= \frac{-0.9\alpha L_d^2 + 0.9(\alpha - \alpha\lambda_2 - \lambda_2)L_d + 0.225}{\alpha^2 L_d^2 + \alpha L_d + 0.25} && (49)\\
&= \frac{A_2L_d^2 + A_1L_d + A_0}{B_2L_d^2 + B_1L_d + B_0} && (50)\\
L_{p2} &= \frac{0.9\lambda_2B_2L_d^2 + (0.9\lambda_2B_1 - 0.225B_2 - 0.25 + A_1)L_d}{0.25\alpha B_2L_d^2 + 0.25\alpha B_1L_d + 0.25\alpha B_0} && (51)\\
&+ \frac{(0.9\lambda_2B_0 - 0.225B_1 + 0.25A_1)}{0.25\alpha B_2L_d^2 + 0.25\alpha B_1L_d + 0.25\alpha B_0} && (52)\\
&= \frac{C_2L_d^2 + C_1L_d + C_0}{D_2L_d^2 + D_1L_d + D_0} && (53)
\end{aligned}$$

where $A_0 = 0.225, A_1 = 0.9(\alpha - \alpha\lambda_2 - \lambda_2), A_2 = -0.9\alpha, B_0 = 0.25, B_1 = \alpha, B_2 = \alpha^2, C_0 = 0.9\lambda_2B_0 - 0.225B_1 + 0.25A_1, C_1 = 0.9\lambda_2B_1 - 0.225B_2 - 0.25 + A_1, C_2 = 0.9\lambda_2B_2, D_0 = 0.25\alpha B_0, D_1 = 0.25\alpha B_1, and D_2 = 0.25\alpha B_2.$

B Proof of Theorem 1

The Lyapunov function given in (33) can be divided into three parts: The ship-controller system, the low-frequency observer, and the high-frequency observer. To simplify the analysis, each system will be analysed separately.

1. Ship-controller System

Consider the Lyapunov function candidate:

$$V_1(s_1, \bar{\psi}, \int_0^t \bar{\psi}\ d\tau) = \frac{1}{2}\ m s_1^2 + \frac{1}{2}\ \left[\bar{\psi},\quad \int_0^t \bar{\psi}\ d\tau\right]\ \mathbf{K_1}\ \left[2\bar{\psi},\quad \int_0^t \bar{\psi}\ d\tau\right]^T \quad (54)$$

Differentiating (54) with respect to time gives:

$$\begin{aligned}\dot{V}_1 = \quad & m s_1 \dot{s_1} + (4\lambda_1 K_d - 5m\lambda_1^2)\bar{\psi}\dot{\bar{\psi}} + (2\lambda_1^2 K_d - 2m\lambda_1^3)(\dot{\bar{\psi}} \int_0^t \bar{\psi}\ d\tau + \bar{\psi}^2) \\ & + (4\lambda_1^3 K_d - m\lambda_1^4)\bar{\psi} \int_0^t \bar{\psi}\ d\tau \end{aligned} \quad (55)$$

The first term in (55) is given from (10), that is:

$$m s_1 \dot{s_1} = m s_1 (\ddot{\bar{\psi}} + 2\lambda_1 \dot{\bar{\psi}} + \lambda_1^2 \bar{\psi}) \quad (56)$$

Substituting (29) for $m\ddot{\bar{\psi}}$ gives:

$$\begin{aligned} m s_1 \dot{s_1} \quad = \quad & -\gamma \dot{\bar{\psi}} s_1 + (\gamma - m c_1)\dot{\tilde{\psi}}_H s_1 - m c_2 \tilde{\psi}_H s_1 + m c_2 L_{p2} \tilde{\psi} s_1 \\ & -m L_{d2} \dot{\tilde{\psi}} s_1 - K_d s_1^2 + K_d s_1 s_2 + m 2\lambda_1 \dot{\bar{\psi}} s_1 + m\lambda_1^2 \bar{\psi} s_1 \end{aligned} \quad (57)$$

Taking the term $-K_d s_1^2$ from (57) and three more terms in (55) gives:

$$\begin{aligned} & -K_d s_1^2 + 4\lambda_1 K_d \bar{\psi}\dot{\bar{\psi}} + 4\lambda_1^3 K_d \bar{\psi} \int_0^t \bar{\psi}\ d\tau + 2\lambda_1^2 K_d \dot{\bar{\psi}} \int_0^t \bar{\psi}\ d\tau \\ & = -K_d \dot{\bar{\psi}}^2 - K_d (2\lambda_1 \bar{\psi})^2 - K_d (\lambda_1^2 \int_0^t \bar{\psi}\ d\tau)^2 \end{aligned} \quad (58)$$

Similarly $m 2\lambda_1 \dot{\bar{\psi}} s_1$ in (57) is combined with two of the terms in (55), according to :

$$2m\lambda_1 \dot{\bar{\psi}} s_1 - 4m\lambda_1^2 \dot{\bar{\psi}}\bar{\psi} - 2m\lambda_1^3 \dot{\bar{\psi}} \int_0^t \bar{\psi}\ d\tau = 2\lambda_1 m \dot{\bar{\psi}}^2 \quad (59)$$

Finally the term $m\lambda_1^2 \bar{\psi} s_1$ in (57) is combined with the last three terms in (55) such that:

$$m\lambda_1^2 \bar{\psi} s_1 - m\lambda_1^2 \bar{\psi}\dot{\bar{\psi}} - m\lambda_1^4 \bar{\psi} \int_0^t \bar{\psi}\ d\tau - 2m\lambda_1^3 \bar{\psi}^2 = 0 \quad (60)$$

The remaining terms in (57) and the right side of (58) and (59) are collected to give:

$$
\begin{aligned}
\dot{V}_1(s_1, \bar{\psi}, \int_0^t \bar{\psi}\, d\tau) \quad = \quad & -(K_d + \gamma - 2\lambda_1 m)\dot{\bar{\psi}}^2 - K_d(2\lambda_1\bar{\psi})^2 - K_d(\lambda_1^2 \int_0^t \bar{\psi}\, d\tau)^2 \\
& -\gamma\dot{\bar{\psi}}(2\lambda_1\bar{\psi}) - \gamma\dot{\bar{\psi}}(\lambda_1^2 \int_0^t \bar{\psi}\, d\tau) - mL_{d2}\dot{\bar{\psi}}\dot{\tilde{\psi}} \\
& +mc_2L_{p2}\lambda_2^{-1}\dot{\bar{\psi}}(\lambda_2\tilde{\psi}) + (\gamma - mc_1)\dot{\bar{\psi}}\dot{\tilde{\psi}}_H - mc_2\lambda_2^{-1}\dot{\bar{\psi}}(\lambda_2\tilde{\psi}_H) \\
& -mL_{d2}(2\lambda_1\bar{\psi})\dot{\tilde{\psi}} + mc_2L_{p2}(2\lambda_1\bar{\psi})(\lambda_2\tilde{\psi}) + (\gamma - mc_1)(2\lambda_1\bar{\psi})\dot{\tilde{\psi}}_H \\
& -mc_2\lambda_2^{-1}(2\lambda_1\bar{\psi})(\lambda_2\tilde{\psi}_H) - mL_{d2}(\lambda_1^2 \int_0^t \bar{\psi}\, d\tau)\dot{\tilde{\psi}} \\
& +mc_2L_{p2}\lambda_2^{-1}(\lambda_1^2 \int_0^t \bar{\psi}\, d\tau)(\lambda_2\tilde{\psi}) + (\gamma - mc_1)(\lambda_1^2 \int_0^t \bar{\psi}\, d\tau)\dot{\tilde{\psi}}_H \\
& -mc_2\lambda_2^{-1}(\lambda_1^2 \int_0^t \bar{\psi}\, d\tau)(\lambda_2\tilde{\psi}_H) + K_d s_1 s_2 \qquad (61)
\end{aligned}
$$

2. Low Frequency Observer

The Lyapunov function for the low frequency observer is chosen as:

$$
V_2(s_2, \tilde{\psi}) \quad = \quad \frac{1}{2}\, m s_2^2 + \frac{1}{2}\, K_2\tilde{\psi}^2 \qquad (62)
$$

Differentiation of (62) with respect to time gives:

$$
\dot{V}_2 = m s_2 \dot{s_2} + (2\lambda_2 K_d - mc_2L_{p2} + mL_{d2}\lambda_2)\tilde{\psi}\dot{\tilde{\psi}} \qquad (63)
$$

Substituting (32) for $m\dot{s_2}$ gives:

$$
\begin{aligned}
m s_2 \dot{s_2} \quad = \quad & -\gamma\dot{\bar{\psi}}s_2 + (\gamma - mc_1)\dot{\tilde{\psi}}_H s_2 - mc_2\tilde{\psi}_H s_2 + mc_2L_{p2}\tilde{\psi}s_2 \\
& -mL_{d2}\dot{\tilde{\psi}}s_2 + K_d s_2^2 - K_d s_1 s_2 - mL_d s_2^2 - \alpha m L_d\dot{\bar{\psi}}s_2 \\
& +(2\lambda_2 K_d - mc_2L_{p2} + mL_{d2}\lambda_2)\tilde{\psi}\dot{\tilde{\psi}} \qquad (64)
\end{aligned}
$$

To get the expression for $\dot{V}_2$ in an appropriate form, some of the terms in (64) can be combined with (11) according to:

$$
\begin{aligned}
- \quad & (mL_d - K_d)s_2^2 - mL_{d2}\dot{\tilde{\psi}}s_2 + mc_2L_{p2}\tilde{\psi}s_2 + (2\lambda_2 K_d - mc_2L_{p2} + mL_{d2}\lambda_2)\tilde{\psi}\dot{\tilde{\psi}} \\
& = -(K_d + mL_{d2})\dot{\tilde{\psi}}^2 - (K_d - mc_2L_{p2}\lambda_2^{-1})(\lambda_2\tilde{\psi})^2 - (mL_d - 2K_d)s_2^2 \qquad (65)
\end{aligned}
$$

Adding the right side of (65) with the remaining terms from (64), and using (20), gives the final expression for $\dot{V}_2$:

$$\begin{aligned}\dot{V}_2(s_2, \bar{\psi}, \tilde{\psi}) &= -(K_d + mL_{d2})\dot{\tilde{\psi}}^2 - (K_d - mc_2 L_{p2}\lambda_2^{-1})(\lambda_2\tilde{\psi})^2 - \gamma\dot{\bar{\psi}}\dot{\tilde{\psi}} - \gamma\dot{\bar{\psi}}(\lambda_2\tilde{\psi}) \\ &+(\gamma - mc_1)\dot{\tilde{\psi}}_H\dot{\tilde{\psi}} + (\gamma - mc_1)\dot{\tilde{\psi}}_H(\lambda_2\tilde{\psi}) - mc_2\lambda_2^{-1}\dot{\tilde{\psi}}(\lambda_2\tilde{\psi}_H) \\ &-mc_2\lambda_2^{-1}(\lambda_2\tilde{\psi})(\lambda_2\tilde{\psi}_H) - 2\alpha K_d\dot{\bar{\psi}}\dot{\tilde{\psi}} - 2\alpha K_d\dot{\bar{\psi}}(\lambda_2\tilde{\psi}) - K_d s_1 s_2 \quad (66)\end{aligned}$$

Notice that the last term of (66) is cancelled by the last term in (61).

3. High Frequency Observer

The Lyapunov function for the high frequency observer is chosen as:

$$V_3(\dot{\tilde{\psi}}_H, \tilde{\psi}_H) = \frac{1}{2}\,[\dot{\tilde{\psi}}_H \;\; \tilde{\psi}_H]\,\mathbf{K_3}\,[\dot{\tilde{\psi}}_H \;\; \tilde{\psi}_H]^T \quad (67)$$

Differentiation of (67) leads to:

$$\dot{V}_3 = \frac{1}{c_1}(1+\lambda_2)\dot{\tilde{\psi}}_H\ddot{\tilde{\psi}}_H + \lambda_2\dot{\tilde{\psi}}_H^2 + \lambda_2\tilde{\psi}_H\ddot{\tilde{\psi}}_H + (\frac{c_2}{c_1}(1+\lambda_2) + \lambda_2 c_1)\tilde{\psi}_H\dot{\tilde{\psi}}_H \quad (68)$$

Substituting (25) into (68) for $\ddot{\tilde{\psi}}_H$ gives:

$$\begin{aligned}\dot{V}_3 &= (1+\lambda_2)\dot{\tilde{\psi}}_H^2 - (1+\lambda_2)\frac{c_2}{c_1}\dot{\tilde{\psi}}_H\tilde{\psi}_H + (1+\lambda_2)\frac{L_{p2}}{c_1}\dot{\tilde{\psi}}_H\tilde{\psi} \\ &-(1+\lambda_2)\frac{L_{d2}}{c_1}\dot{\tilde{\psi}}_H\dot{\tilde{\psi}} - c_1\lambda_2\dot{\tilde{\psi}}_H\tilde{\psi}_H - c_2\lambda_2\tilde{\psi}_H^2 + c_2 L_{p2}\lambda_2\tilde{\psi}_H\tilde{\psi} \\ &+L_{d2}\lambda_2\tilde{\psi}_H\dot{\tilde{\psi}} + \lambda_2\dot{\tilde{\psi}}_H^2 + \left(\frac{c_2}{c_1}(1+\lambda_2) + \lambda_2 c_1\right)\dot{\tilde{\psi}}_H\tilde{\psi}_H \quad (69)\end{aligned}$$

Notice that the cross terms involving $\dot{\tilde{\psi}}_H$ and $\tilde{\psi}_H$ cancel out. The final expression for $\dot{V}_3$ is:

$$\begin{aligned}\dot{V}_3(\dot{\tilde{\psi}}_H, \tilde{\psi}_H) &= -\dot{\tilde{\psi}}_H^2 - c_2\lambda_2^{-1}(\lambda_2\tilde{\psi}_H)^2 - (1+\lambda_2)\frac{L_{d2}}{c_1}\dot{\tilde{\psi}}\dot{\tilde{\psi}}_H \\ &-L_{d2}\dot{\tilde{\psi}}(\lambda_2\tilde{\psi}_H) + \frac{L_{p2}}{c_1}(1+\lambda_2)\lambda_2^{-1}(\lambda_2\tilde{\psi})\dot{\tilde{\psi}}_H \\ &+c_2 L_{p2}\lambda_2^{-1}(\lambda_2\tilde{\psi})(\lambda_2\tilde{\psi}_H) \quad (70)\end{aligned}$$

Adding the three Lyapunov function derivatives $\dot{V}_1$, $\dot{V}_2$, and $\dot{V}_3$ yields: $\dot{V} = -\boldsymbol{x}^T\boldsymbol{Q}\boldsymbol{x}$ where $\mathbf{Q}$ is given in (38).

C Nonlinear Ship Model (Mariner Class Vessel)

The hydro- and aerodynamics laboratory in Lyngby, Denmark, has performed both planar motion mechanism (PMM) tests and full-scale steering and maneuvering predictions for the *Mariner Class Vessel*. The main data and dimensions of the Mariner Class Vessel are (Chislett and Strøm-Tejsen 1965):

Length overall (L_{oa})	171.80	(m)
Length between perpendiculars (L_{pp})	160.93	(m)
Maximum beam (B)	23.17	(m)
Design draft (T)	8.23	(m)
Design displacement (∇)	18541	(m^3)
Design speed (u_0)	15	(knots)

For this vessel the dynamic equations of motion in surge, sway and yaw are:

$$\begin{bmatrix} m' - X'_{\dot{u}} & 0 & 0 \\ 0 & m' - Y'_{\dot{v}} & m'x'_G - Y'_{\dot{r}} \\ 0 & m'x'_G - N'_{\dot{v}} & I'_z - N'_{\dot{r}} \end{bmatrix} \begin{bmatrix} \Delta\dot{u}' \\ \Delta\dot{v}' \\ \Delta\dot{r}' \end{bmatrix} = \begin{bmatrix} \Delta X' \\ \Delta Y' \\ \Delta N' \end{bmatrix} \tag{71}$$

where the nonlinear forces and moment $\Delta X'$, $\Delta Y'$ and $\Delta N'$ are defined as (Prime-System I with L_{pp} and U as normalization variables, see (Fossen 1994)):

$$\begin{aligned}
\Delta X' &= X'_u\Delta u' + X'_{uu}\Delta u'^2 + X'_{uuu}\Delta u'^3 + X'_{vv}\Delta v'^2 + X'_{rr}\Delta r'^2 + X'_{rv}\Delta r'\Delta v' \\
&+ X'_{\delta\delta}\Delta\delta'^2 + X'_{u\delta\delta}\Delta u'\Delta\delta'^2 + X'_{v\delta}\Delta v'\Delta\delta' + X'_{uv\delta}\Delta u'\Delta v'\Delta\delta' \\
\Delta Y' &= Y'_v\Delta v' + Y'_r\Delta r' + Y'_{vvv}\Delta v'^3 + Y'_{vvr}\Delta v'^2\Delta r' + Y'_{vu}\Delta v'\Delta u' + Y'_{ru}\Delta r'\Delta u' \\
&+ Y'_\delta\Delta\delta' + Y'_{\delta\delta\delta}\Delta\delta'^3 + Y'_{u\delta}\Delta u'\Delta\delta' + Y'_{uu\delta}\Delta u'^2\Delta\delta' + Y'_{v\delta\delta}\Delta v'\Delta\delta'^2 \\
&+ Y'_{vv\delta}\Delta v'^2\Delta\delta' + (Y0' + Y0'_u\Delta u' + Y0'_{uu}\Delta u'^2) \\
\Delta N' &= N'_v\Delta v' + N'_r\Delta r' + N'_{vvv}\Delta v'^3 + N'_{vvr}\Delta v'^2\Delta r' + N'_{vu}\Delta v'\Delta u' + N'_{ru}\Delta r'\Delta u' \\
&+ N'_\delta\Delta\delta' + N'_{\delta\delta\delta}\Delta\delta'^3 + N'_{u\delta}\Delta u'\Delta\delta' + N'_{uu\delta}\Delta u'^2\Delta\delta' + N'_{v\delta\delta}\Delta v'\Delta\delta'^2 \\
&+ N'_{vv\delta}\Delta v'^2\Delta\delta' + (N0' + N0'_u\Delta u' + N0'_{uu}\Delta u'^2)
\end{aligned}$$

The non-dimensional coefficients in the model are:

$$m' = 798 \cdot 10^{-5}; \qquad I'_z = 39.2 \cdot 10^{-5}; \qquad x'_G = -0.023$$

Table 4: Non-dimensional hydrodynamic coefficients for the Mariner Class Vessel (Chislett and Strøm-Tejsen 1965).

X-equation	Y-equation	N-equation
$X'_{\dot{u}} = -840 \cdot 10^{-5}$	$Y'_{\dot{v}} = -1546 \cdot 10^{-5}$	$N'_{\dot{v}} = 23 \cdot 10^{-5}$
	$Y'_{\dot{r}} = 9 \cdot 10^{-5}$	$N'_{\dot{r}} = -83 \cdot 10^{-5}$
$X'_{u} = -184 \cdot 10^{-5}$	$Y'_{v} = -1160 \cdot 10^{-5}$	$N'_{v} = -264 \cdot 10^{-5}$
$X'_{uu} = -110 \cdot 10^{-5}$	$Y'_{r} = -499 \cdot 10^{-5}$	$N'_{r} = -166 \cdot 10^{-5}$
$X'_{uuu} = -215 \cdot 10^{-5}$	$Y'_{vvv} = -8078 \cdot 10^{-5}$	$N'_{vvv} = 1636 \cdot 10^{-5}$
$X'_{vv} = -899 \cdot 10^{-5}$	$Y'_{vvr} = 15356 \cdot 10^{-5}$	$N'_{vvr} = -5483 \cdot 10^{-5}$
$X'_{rr} = 18 \cdot 10^{-5}$	$Y'_{vu} = -1160 \cdot 10^{-5}$	$N'_{vu} = -264 \cdot 10^{-5}$
$X'_{\delta\delta} = -95 \cdot 10^{-5}$	$Y'_{ru} = -499 \cdot 10^{-5}$	$N'_{ru} = -166 \cdot 10^{-5}$
$X'_{u\delta\delta} = -190 \cdot 10^{-5}$	$Y'_{\delta} = 278 \cdot 10^{-5}$	$N'_{\delta} = -139 \cdot 10^{-5}$
$X'_{rv} = 798 \cdot 10^{-5}$	$Y'_{\delta\delta\delta} = -90 \cdot 10^{-5}$	$N'_{\delta\delta\delta} = 45 \cdot 10^{-5}$
$X'_{v\delta} = 93 \cdot 10^{-5}$	$Y'_{u\delta} = 556 \cdot 10^{-5}$	$N'_{u\delta} = -278 \cdot 10^{-5}$
$X'_{uv\delta} = 93 \cdot 10^{-5}$	$Y'_{uu\delta} = 278 \cdot 10^{-5}$	$N'_{uu\delta} = -139 \cdot 10^{-5}$
	$Y'_{v\delta\delta} = -4 \cdot 10^{-5}$	$N'_{v\delta\delta} = 13 \cdot 10^{-5}$
	$Y'_{vv\delta} = 1190 \cdot 10^{-5}$	$N'_{vv\delta} = -489 \cdot 10^{-5}$
	$Y0' = -4 \cdot 10^{-5}$	$N0' = 3 \cdot 10^{-5}$
	$Y0'_{u} = -8 \cdot 10^{-5}$	$N0'_{u} = 6 \cdot 10^{-5}$
	$Y0'_{uu} = -4 \cdot 10^{-5}$	$N0'_{uu} = 3 \cdot 10^{-5}$

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