ROBUST ADAPTIVE SHIP AUTOPILOT WITH WAVE FILTER AND INTEGRAL ACTION

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SUMMARY

A stable minimum phase transfer function from rudder angle to yaw angle is used to design a globally convergent adaptive ship autopilot. 1st-order wave disturbances in yaw are filtered by applying a notch filter. Integral action is introduced by augmenting the plant. Global convergence is proven for the total system which include the observer, the parameter update law, the feedback controller, the notch filter and the integral part of the controller. The simulation results showed that the performance is excellent, even with no a priori knowledge of the ship parameters.

KEY WORDS Adaptive control Ship autopilot Marine systems

1. INTRODUCTION

The design of ship autopilots have been analyzed extensively in the literature, and the control laws applied ranges from conventional PID-type controllers to more advanced self-tuning and adaptive algorithms. Adaptive ship autopilots have been discussed by numerous authors, see e.g. Sælid and Jenssen[1] and Holzhüter and Strauch[2]. Model Reference Adaptive Control (MRAC) of ship was first discussed by Van Amerongen and co-authors[3–5], and this work was based on the assumption of full-state feedback. In this paper, the input error direct adaptive control scheme of Sastry and Bodson[6] is used to derive an adaptive output feedback autopilot. This approach to the adaptive ship autopilot problem was first presented by Lauvdal and Fossen[7].

In order to make an autopilot work properly, there are several things needed to be analyzed and addressed. First, varying ship dynamics due to e.g. changing load and weather conditions and cruise speed have to be addressed. In conventional schemes, speed-scaling is introduced to compensate for the speed dependent change in the dynamics, but variations due to changing load conditions are usually not addressed. This is the motivation for introducing an adaptive control law. Secondly, knowledge about the environmental forces acting on the ship is crucial in order to obtain robustness and satisfactory performance. The robustness issues are particularly important in an adaptive controller, since disturbances may cause parameter drift. Thirdly, robustness issues related to unmodeled dynamics and nonlinearities in the input need to be addressed.

The problem of high-frequency rudder motions due to 1st-order wave disturbances in the feedback loop is explicitly addressed. This is usually solved by using a Kalman-filter to estimate the low-frequency motion components of the ship. Wave filtering in terms of the Kalman filter algorithm has been discussed my numerous authors[8–12] the last decades.

However, if the wave filter, the control law and the parameter update law are designed independently, global stability/convergence and robustness cannot be guaranteed. In this paper, the ship model is augmented with a notch filter and the augmented plant is used for control design. Hence, stability properties of the ship *and* wave filter can be analyzed.

In open seas there are low-frequency forces caused by waves, current and wind acting on the ship. In conventional autopilots integral action is included in order to avoid a steady-state error in the yaw angle. However, adaptive schemes will, in general, not have integral action in the controller but instead rely on the integration in the parameter identifier to obtain a zero stationary error. This will work when the identifier is on-line, but in some cases it may be desirable to turn it off (e.g. in rough weather which may cause parameter drift) and then the fixed controller will be of PD-type and steady-state error cannot be avoided. An important contribution of this paper is an approach guaranteeing integral action even in the case of fixed control parameters.

The input error MRAC presented has several advantages. First, observers are used instead of full state feedback, and thus only compass measurement of the yaw angle is needed. Secondly, using the cascaded notch filtering approach it is, at least in theory, possible to filter the 1st-order wave disturbances sufficiently and prove global convergence at the same time. Thirdly, low-frequency disturbances due to currents and wind were compensated for by adding integral action in the controller. Finally, the input error scheme is robust to saturating inputs if this input can be measured and used in the identifier. Hence, the controlled ship showed to be robust and globally convergent also under rudder saturation.

Denoting the yaw angle and desired yaw angle by ψ_p and ψ_r, respectively, and the input by δ, the adaptive control scheme can be schematically illustrated as in Figure 1.

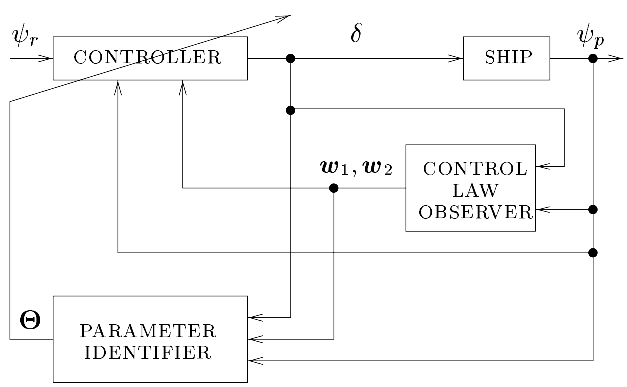

Figure 1: Block diagram showing MRAC principle.

2. PRELIMINARIES

Before deriving the adaptive ship autopilot, some preliminary assumptions are needed. Let $P(s)$ be the plant to be controlled and $M(s)$ be the reference model, i.e. the desired dynamics of the closed-loop system. Define:

$$P(s) \triangleq K_p \frac{n_p(s)}{d_p(s)}, \quad M(s) \triangleq K_m \frac{n_m(s)}{d_m(s)}, \tag{1}$$

where $P(s)$ and $M(s)$ are linear time-invariant (LTI) and single-input single-output (SISO) transfer functions, and both having monic numerator and denumerator. Let the relative degree of $P(s)$ and $M(s)$ be denoted γ_p and γ_m, respectively. The following assumptions will be in effect throughout the paper.

Assumptions Let $P(s)$ and $M(s)$ be given by (1). Then it is assumed that:

(A1) The plant $P(s)$ is minimum phase (i.e. $n_p(s)$ is Hurwitz) and the pair $n_p(s)$, $d_p(s)$ are coprime and have degrees m and n, respectively.

(A2) The reference model $M(s)$ is stable and minimum phase (i.e. $n_m(s)$ and $d_m(s)$ are Hurwitz) and the pair $n_m(s)$, $d_m(s)$ are coprime and have degrees m and n, respectively.

(A3) The reference signal ψ_r is piecewise continuous and bounded.

(A4) The gains K_p, K_m are positive and K_p is upper bounded by a known bound, $K_{\max}$, i.e. $K_p \leq K_{\max}$.

It is important to notice that the reference model assumption (A2) is easy to satisfy, i.e. it does not have the strictly positive real assumption usually needed. The reason for this is that input error is used instead of the usual output error. Hence, the approach taken allows for a large degree of freedom in choice of reference model and essential features like integral action and wave filter can be included.

3. AUGMENTED SHIP MODEL

The first step is to derive a plant model used for control design. According to (A1), the plant must be minimum phase. This constraint is usually not hard to satisfy since most ships are minimum phase from the rudder (δ) to the yaw angle (ψ_p). In this paper, the ship model is augmented with a notch filter, the actuator dynamics and an integral term to improve the robustness and performance of the adaptive autopilot.

3.1. Ship Dynamics

The ship model considered is the 2nd-order model of Son and Nomoto[13] given by

$$\begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\psi}_p \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \\ \psi_p \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ 0 \end{bmatrix} \delta, \tag{2}$$

where ψ_p is the yaw angle, r is the angular velocity in yaw, v is the sway velocity and δ is the rudder angle. Applying the Laplace transformation to this model yields the 3rd-order transfer function

$$\frac{\psi_p}{\delta}(s) = K \frac{(1 + T_1 s)}{s(1 + T_2 s)(1 + T_3 s)}, \tag{3}$$

where (A1) implies that $T_1 > 0$. For simplicity, (3) is written

$$\frac{\psi_p}{\delta}(s) = P_s(s) = K_s \frac{(s+\beta_0)}{s(s^2+\alpha_2 s+\alpha_1)}, \tag{4}$$

where $K_s = KT_1/T_2T_3$ and with obvious definitions of β_0, α_1 and α_2. Notice that the numerator and denumerator of $P_s(s)$ are monic and coprime since $T_1 \neq T_2$ and $T_1 \neq T_3$ and that $P_s(s)$ has relative degree $\gamma_p = 2$.

3.2. *Actuator Dynamics*

The ship rudder is not a high bandwidth actuation device since it includes hard nonlinearities like magnitude and rate saturations. Thus, the rudder dynamics should be included in control design. In this paper, the following model of the rudder dynamics is considered:

$$\delta(s) = \text{sat}\left(\frac{1}{1+T_a s}\delta_c(s)\right), \tag{5}$$

where δ_c is the commanded rudder angle and $\text{sat}(\cdot) = \text{sign}(\cdot)\min(\delta_{max}, |\cdot|)$. Since the gain of the linear rudder dynamics is of unity value, it follows that if $\delta_c \leq \delta_{max}$, $t \geq 0$ then $\delta \leq \delta_{max}$, $t \geq 0$. Thus, if it is assumed that $\delta_c \leq \delta_{max}$, the rudder dynamics can be assumed to be linear, i.e.

$$\frac{\delta}{\delta_c}(s) = P_a(s) = K_a \frac{1}{s+\alpha_a}, \tag{6}$$

where $K_a = \alpha_a = 1/T_a$. Hence, the autopilot should guarantee that $\delta_c \leq \delta_{max}$.

3.3. *Wave Filter*

An important part of a ship autopilot is the wave filter which is included to avoid high-frequency rudder motions due to 1st-order wave disturbances. In addition, in an adaptive autopilot it will reduce the problem of drift in the parameter identification algorithm if the filtered signal is used in the identifier. In conventional autopilots, filtering is usually obtained by applying a Kalman filter to estimate the low-frequency motion components. In the adaptive control law to be derived, this approach is not suitable since convergence cannot be proven, and we suggest an approach where the ship model is augmented with a wave filter. Consequently, we will use the augmented model for control design.

The wave filter is a notch filter, $P_n(s)$, given by

$$P_n(s) = \frac{(s^2+2\zeta\omega_n s+\omega_n^2)}{(s+\omega_n)^2}, \tag{7}$$

where ω_n is the wave encounter frequency and $0 < \zeta < 1$. Then the output of the system is redefined to be $\psi_p^L(s)$ such that

$$\psi_p^L(s) = P_n(s)\psi_p(s), \tag{8}$$

where $\psi_p^L(s)$ is the low frequency part of $\psi_p(s)$ which will be used for feedback. Hence, it is seen that high-frequency rudder motions can be avoided by proper tuning of the notch filter $P_n(s)$.

Remark 1 It should be noted that better filtering properties can be obtained by taking the wave filter to be k notch filters in series, all having different notch frequencies, i.e.

$$P_n(s) = P_{n_1}(s)P_{n_2}(s)\cdots P_{n_k}(s). \tag{9}$$

However, since the complexity of the controller is increasing with increasing plant order, it is assumed that the wave frequency is known or estimated on-line using a wave frequency tracker such that (7) will give sufficient filtering. $\triangle$

3.4. Integral Action

In ship autopilots it is necessary to introduce integral action to avoid steady-state errors due to slowly-varying disturbances like currents, 2nd-order wave drift and wind. In a conventional autopilot like e.g. a PID-controller

$$P_{PID}(s) = K_r \frac{T_d s + 1}{\kappa T_d s + 1} \frac{T_i s + 1}{T_i s}, \tag{10}$$

where $\kappa > 0$, this is taken care of by the integral term $(T_i s + 1)/T_i s$. The output control law of Sastry and Bodson[6] does not give integral action, and can be interpreted as a PD-controller. Thus, we redefine the input to be δ_c' such that

$$\frac{\delta_c}{\delta_c'}(s) = P_i(s) = \frac{s + \beta_i}{s}, \tag{11}$$

where $\beta_i = 1/T_i$ and δ_c' is the output of the PD-part of the controller. Hence, the output feedback control law to be derived will, in combination with (11), give integral action in the control loop.

Remark 2 Although an adaptive control law with integral action in the parameter identifier will give zero steady-state errors, this will not be the case if the identification algorithm is turned off. Moreover, (11) guarantees integral action even in the case of frozen control parameters. This is useful when the ship is in a cruise condition (constant yaw angle). $\triangle$

3.5. Augmented Ship Model

To summarize the discussion in this section, the plant considered is not only the ship dynamics, but it also contains a notch filter, a linear model of the actuator dynamics and integral action. Hence, the resulting plant used for control design is defined to be

$$\frac{\psi_p^L}{\delta_c'}(s) \triangleq P_n(s)P_s(s)P_a(s)P_i(s) = K_p \frac{n_p(s)}{d_p(s)}, \tag{12}$$

or equivalently,

$$\frac{\psi_p^L}{\delta_c'}(s) = P(s) = K_p \frac{s^4 + \alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0}{s^2(s^5 + \beta_6 s^4 + \beta_5 s^3 + \beta_4 s^2 + \beta_3 s + \beta_2)}, \tag{13}$$

where $K_p = K_s K_a$ and the definitions of α_k $(k = 0, \cdots, 3)$ and β_l $(l = 2, \cdots, 6)$ follows from (4), (6), (7), (11), and (12). Notice that $\gamma_p = 3$ and that (13) satisfies (A1) since it is assumed that $T_1 > 0$. The plant is shown in Figure 2.

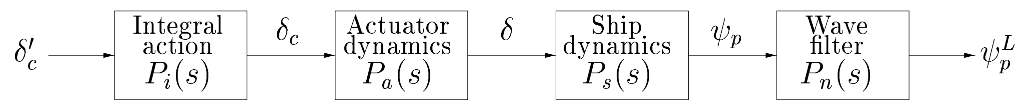

Figure 2: Block diagram showing the augmented plant.

4. OUTPUT FEEDBACK CONTROL LAW

The control design methodology of Sastry and Bodson[6,14,15], is applied to design a globally convergent output feedback autopilot. The control law is defined to be

$$\delta'_c(s) \triangleq c_0\,\psi_r(s) + \frac{c(s)}{\lambda(s)}\,\delta(s) + \frac{d'(s)}{\lambda(s)}\,\psi_p^L(s), \tag{14}$$

where $\psi_r(s)$ is the desired yaw angle. The term $\frac{d'(s)}{\lambda(s)}\psi_p(s)$ can be rewritten as

$$\frac{d'(s)}{\lambda(s)}\,\psi_p^L(s) \triangleq d_0\,\psi_p^L(s) + \frac{d(s)}{\lambda(s)}\,\psi_p(s). \tag{15}$$

Here $\lambda(s)$ must be a Hurwitz polynomial satisfying $\lambda(s) = \lambda_0(s)n_m(s)$, where $n_m(s)$ is the numerator of the reference model. Since (A2) requires that $n_m(s)$ must be Hurwitz, the only constraint in the design of $\lambda(s)$ is that $\lambda_0(s)$ must be Hurwitz. In addition the polynomials have to satisfy $\dim(\lambda(s)) = n-1$ and $\dim(c(s)) = \dim(d(s)) = n-2$.

Below it will be shown that the control law given by (14) and (15) is linear in its parameters and a *generalized observer* reconstructing the states will be given.

4.1. *Generalized Observer*

For the control law (14), a generalized observer (*state variable filters*) can be derived. Consider the polynomial

$$\lambda(s) = \lambda_0(s)n_m(s) = s^{n-1} + \sum_{i=0}^{n-2} \lambda_{i+1} s^i, \tag{16}$$

and define the generalized observers to be

$$\boldsymbol{w}_1(s) \triangleq \frac{1}{\lambda(s)} \begin{bmatrix} 1 \\ \vdots \\ s^{n-2} \end{bmatrix} \delta'_c(s), \tag{17}$$

$$\boldsymbol{w}_2(s) \triangleq \frac{1}{\lambda(s)} \begin{bmatrix} 1 \\ \vdots \\ s^{n-2} \end{bmatrix} \psi_p^L(s). \tag{18}$$

It should be noted that these observers reconstruct the states of the plant without using the plant parameters.

Next, using the polynomials

$$c(s) = \sum_{i=0}^{n-2} c_{i+1} s^i, \quad d(s) = \sum_{i=0}^{n-2} d_{i+1} s^i, \tag{19}$$

it is easily verified that by defining the vectors

$$\boldsymbol{c} \triangleq \begin{bmatrix} c_1, & \cdots, & c_{n-1} \end{bmatrix}^T, \quad \boldsymbol{d} \triangleq \begin{bmatrix} d_1, & \cdots, & d_{n-1} \end{bmatrix}^T, \tag{20}$$

the following identities are obtained

$$\frac{c(s)}{\lambda(s)} \delta_c'(s) = \boldsymbol{c}^T \boldsymbol{w}_1(s), \quad \frac{d(s)}{\lambda(s)} \psi_p^L(s) = \boldsymbol{d}^T \boldsymbol{w}_2(s). \tag{21}$$

Hence the control law (14) and (15) can be written

$$\delta_c'(t) = \boldsymbol{\Theta}^T \boldsymbol{w}(t), \tag{22}$$

where

$$\boldsymbol{\Theta} \triangleq \begin{bmatrix} c_0 \, , \boldsymbol{c}^T \, , d_0 \, , \boldsymbol{d}^T \end{bmatrix}^T \tag{23}$$

$$\boldsymbol{w}(t) \triangleq \begin{bmatrix} \psi_r(t) \, , \boldsymbol{w}_1^T(t) \, , \psi_p^L(t) \, , \boldsymbol{w}_2^T(t) \end{bmatrix}^T \tag{24}$$

4.2. Control Law Parameters

Based on the control law (14) and the generalized observer (17)–(18), the closed-loop dynamics can be shown to satisfy

$$\frac{\psi_p^L}{\psi_r}(s) = \frac{c_0 K_p \lambda(s) n_p(s)}{[\lambda(s) - c(s)] \, d_p(s) - K_p n_p(s) d'(s)}. \tag{25}$$

The goal of the control design is to make the closed-loop system (25) equal to the reference model (1). This is obtained if the model matching equality

$$K_m n_m(s) \big([\lambda(s) - c(s)] \, d_p(s) - K_p n_p(s) d'(s) \big) = c_0 K_p \lambda(s) n_p(s) d_m(s) \tag{26}$$

has a solution. A sufficient condition for a solution to exist is that $\gamma_s = \gamma_m$. However, in this case the solution is not unique, and in order to obtain a unique solution satisfying the model matching equality, it is required that $\deg(n_s(s)) = \deg(n_m(s))$ and $\deg(d_s(s)) = \deg(d_m(s))$. In either case, the solution of (26) is given by

$$\begin{aligned} c_0^* &= \tfrac{K_m}{K_p}, \\ c^*(s) &= \lambda(s) - q(s) n_p(s), \\ d'^*(s) &= \tfrac{1}{K_p} \big[q(s) d_p(s) - \lambda_0(s) d_m(s) \big]. \end{aligned} \tag{27}$$

In order to calculate the control gains (assuming known ship parameters) a reference model with dynamic properties equal to the desired closed-loop ship dynamics has to be chosen. A natural choice is a 3nd-order critical damped system

$$\frac{\psi_m}{r}(s) = K_m \frac{n_m(s)}{d_m(s)} = \frac{\omega_0^2}{s^3 + 3\omega_0 s^2 + 3\omega_0^2 s + \omega_0^3}, \tag{28}$$

where ω_0 is the natural frequency and with obvious choices of $n_m(s)$ and $d_m(s)$. Notice that the reference model has relative degree $\gamma_m = \gamma_p = 3$. Then, using $d'^*(s)$ and (15) it is easy to find d_0^* and $d^*(s)$. Consequently, application of the control law

$$\delta(t) = \mathbf{\Theta}^{*T} \boldsymbol{w}(t), \tag{29}$$

to the augmented plant (12) makes the closed-loop dynamics equal to the reference model dynamics (28).

5. DIRECT ADAPTIVE CONTROL: AN INPUT ERROR APPROACH

In the calculation of the control gains it was assumed that the ship parameters in (3) and (5) are exactly known. Indeed, this is not the case since e.g. changing load and weather conditions will change the dynamics of the ship. Clearly, this motivates an adaptive approach which can be expected to give better performance under different work and load conditions.

5.1. Identifier Structure

To derive an error equation, notice that the reference model given by (1) can be written

$$\psi_r(s) = M^{-1}(s)\psi_m(s). \tag{30}$$

Based on this, the plant input can be defined as

$$\psi_{p,r}(s) \stackrel{\Delta}{=} M^{-1}(s)\psi_p^L(s), \tag{31}$$

and the *input error* $e_i(s)$ can be defined as

$$e_i(s) \stackrel{\Delta}{=} \psi_{p,r}(s) - \psi_r(s) = M^{-1}(s)(\psi_p^L(s) - \psi_m(s)). \tag{32}$$

Next, consider the matching equality (26) and divide both sides by $K_m n_m(s)\lambda(s)d_p(s)$. Then it follows that

$$1 - \frac{c(s)}{\lambda(s)} - \frac{d'(s)}{\lambda(s)}P(s) = c_0 M^{-1}(s)P(s) \tag{33}$$

This form of the matching equality implies that it is possible to derive an expression for the input error

$$e_i(s) = \frac{1}{c_0}\mathbf{\Phi}^T(s)\boldsymbol{z}(s), \tag{34}$$

where $\boldsymbol{z}(s) = [\,\psi_{p,r}(s),\ \overline{\boldsymbol{w}}^T(s)\,]^T$, $\mathbf{\Phi}(s) = \mathbf{\Theta}(s) - \mathbf{\Theta}^*$ and

$$\overline{\boldsymbol{w}}(s) = \left[\ \boldsymbol{w}_1^T(s)\ ,\ \psi_p^L(s),\ \ \boldsymbol{w}_2^T(s)\ \right]^T. \tag{35}$$

In practice, the error equation (34) is not particularly useful since the signal $\psi_{p,r}(s)$ is generated by a non-proper transfer function $M^{-1}(s)$ and a modification of the input error is necessary. Let $L^{-1}(s)$ be any stable and minimum phase transfer function with relative degree $\gamma_l = n - m = 3$ and multiply (33) with $L^{-1}(s)\delta_c'(s)$. Then

$$L^{-1}(s)\frac{d'(s)}{\lambda(s)}P(s)\delta_c'(s) + c_0\big(M(s)L(s)\big)^{-1}P(s)\delta_c'(s) = L^{-1}(s)\delta_c'(s) - \frac{c(s)}{\lambda(s)}\delta_c'(s), \tag{36}$$

which can easily be rewritten

$$\left(L^{-1}(s)\frac{d'(s)}{\lambda(s)} + c_0\big(M(s)L(s)\big)^{-1}\right)\psi_p^L(s) = \left(L^{-1}(s) - \frac{c(s)}{\lambda(s)}\right)\delta_c'(s). \tag{37}$$

Using (37) and defining

$$\begin{aligned} \boldsymbol{v}(s) &\triangleq L^{-1}(s)\boldsymbol{z}(s) \\ &= \begin{bmatrix} (M(s)L(s))^{-1}\psi_p^L(s) \\ L^{-1}(s)\overline{\boldsymbol{w}}(s) \end{bmatrix}^T, \end{aligned} \tag{38}$$

leads to (for details, see Sastry and Bodson[6], pp. 111–114) the definition of the *modified input error*

$$e_2(s) \triangleq \boldsymbol{\Phi}^T(s)\boldsymbol{v}(s) = \boldsymbol{\Theta}^T(s)\boldsymbol{v}(s) - L^{-1}(s)\delta_c'(s), \tag{39}$$

where $e_2(t) \in \mathbb{R}$. Notice that the signal $(M(s)L(s))^{-1}\psi_p(s)$ is available since $M(s)L(s)$ has relative degree $\gamma_{ml} = 0$.

Remark 3 It is important to notice that an identification algorithm using the modified input error is robust to input saturation provided that the saturated signal is used in the identifier. This is in contrast to the output error approach which will give erroneous parameter estimates when the input saturates. △

5.2. *Identification Algorithm*

Based on the *modified input error* (39), an identification algorithm can be used to update the control parameters in (22). Since the controller is linear in its parameters, other choices than the gradient algorithm can be used. In this paper we use the *normalized least square algorithm* given by

$$\dot{\boldsymbol{\Theta}}(t) = -g\frac{e_2(t)\boldsymbol{P}(t)\boldsymbol{v}(t)}{1+\gamma\boldsymbol{v}^T(t)\boldsymbol{P}(t)\boldsymbol{v}(t)}, \tag{40}$$

$$\dot{\boldsymbol{P}}(t) = -g\frac{\boldsymbol{P}(t)\boldsymbol{v}(t)\boldsymbol{v}^T(t)\boldsymbol{P}(t)}{1+\gamma\boldsymbol{v}^T(t)\boldsymbol{P}(t)\boldsymbol{v}(t)}, \tag{41}$$

where $g,\gamma > 0$, $\boldsymbol{P}(t) \in \mathbb{R}^{2n\times 2n}$ and $\boldsymbol{P}(0) > \boldsymbol{0}$.

6. STABILITY PROPERTIES

The properties of the adaptive scheme can be summarized in the following lemma.

Lemma 1 Assume that $K_{\max}$ is known and serves as an upper bound on K_p, i.e. $K_p \le K_{\max}$. Then, the augmented plant (13) with a reference model satisfying (A2), the output control law (22), identifier structure (39) and identification algorithm (40) and (41) is globally convergent. Moreover, $e_2(t) \to 0$, $\psi_p(t) \to \psi_m(t)$ as $t \to \infty$ and $\boldsymbol{\Phi}(t)$ is bounded.

Proof: The system satisfies the conditions of the input error direct adaptive control algorithm of Sastry and Bodson[6] pp. 115–117 and a proof is found on pp. 142–149. △

7. SIMULATION STUDY OF SHIP AUTOPILOT

In this section an extensive simulation study is done for the container ship presented in Fossen[16], page 174. The ship length is 161 meters and the parameters in (3) and (5) are given by

$$\begin{aligned} T_1 &= 18.5, & T_2 &= 7.8, & T_a &= 3, \\ T_3 &= 118.0, & K &= 0.185. \end{aligned} \tag{42}$$

A Bode plot of the ship transfer function neglecting the actuator dynamics is shown in Figure 3 (solid line). If not explicitly stated otherwise, the initial parameter vectors are

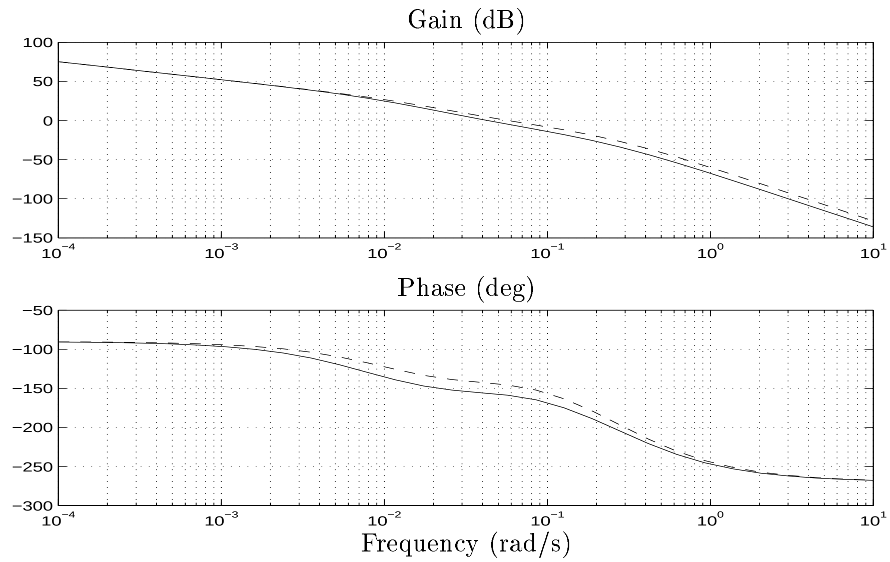

Figure 3: Bode plot showing the nominal ship dynamics (solid line). The perturbed ship dynamics of (50) is also shown (dashed line).

$$\mathbf{\Theta}(0) = [0.1, \boldsymbol{c}^T(0), 0, \boldsymbol{d}^T(0)]^T, \tag{43}$$

$$\boldsymbol{c}(0) = \boldsymbol{d}(0) = \mathbf{0}. \tag{44}$$

The adaption parameters are chosen as:

$$\boldsymbol{P}(0) = 10\boldsymbol{I}, \quad g = \gamma = 1.0, \tag{45}$$

where $\boldsymbol{I}$ is the identity matrix of appropriate dimension. For simplicity, the transfer function $L^{-1}(s)$ is chosen to be equal the reference model, i.e.

$$L^{-1}(s) = M(s). \tag{46}$$

If not stated otherwise, the reference trajectory used in the simulations are a sequence of step inputs of amplitude 10, i.e. $\psi_r = \pm 10$ where the sign is changed at time $t = 300, 600, ...$ (s).

7.1. Parameter Variations

In this section parameter uncertainties in the ship dynamics are considered. The plant used in the simulations describes the ship and the actuator dynamics, i.e.

$$\frac{\psi_p}{\delta_c}(s) = P_s(s)P_a(s). \tag{47}$$

Assuming the parameters given in (42) are known, an output feedback controller in the form (14) can be derived. Since $n = 4$ and $\gamma_p = 3$ let the reference model $M(s)$ be given by (28) with $\omega_0 = 0.05$ and $\lambda(s)$

$$\lambda(s) = \lambda_0(s) = (s + 0.1)^3, \tag{48}$$

Then, the control vector $\mathbf{\Theta} \in \mathbb{R}^8$ can be found using (15) and (27) or be identified through a simulation. Consider

$$\mathbf{\Theta}^* = \frac{1}{100}\left[\begin{array}{cccccccc} 13.0 & 0.03 & 1.31 & 16.2 & 1.41 & -0.01 & -0.84 & 7.37 \end{array}\right]^T, \tag{49}$$

which was found through identification. A simulation with the frozen control parameters given by (49) is shown in Figure 4 (solid line). It is seen that in this case the closed-loop system performs like the reference model. Next, assume that the ship is described by the following parameters

$$T_1 \;=\; 22.2, \quad T_2 \;=\; 6.24, \quad T_3 \;=\; 82.6\ . \tag{50}$$

Figure 3 show a Bode plot of the perturbed model (dashed line) without the actuator dynamics. The time response of the system given by (50) with the frozen controller (49) is shown in Figure 4 (dashed line). Clearly, the closed-loop performance is not satisfactory since oscillations occurs in the yaw and the rudder angles. This motivates the use of an adaptive control scheme. In Figure 5 the parameter identifier is turned on at $t = 300$ (s), and it is

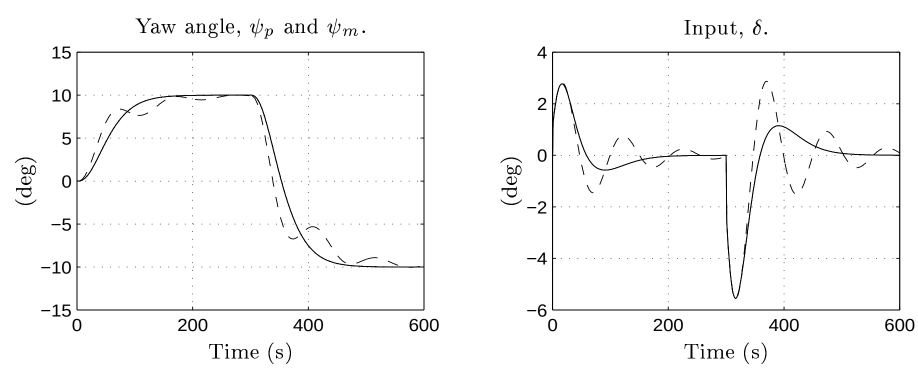

Figure 4: Response of fixed output feedback controller for the nominal ship dynamics (solid line) and the perturbed ship dynamics (dashed line). The yaw angle of the reference model ψ_m is indicated by the dotted line.

seen that after a short period of time the closed-loop dynamics matches the reference model. Also notice that no large transients appear in the yaw angle.

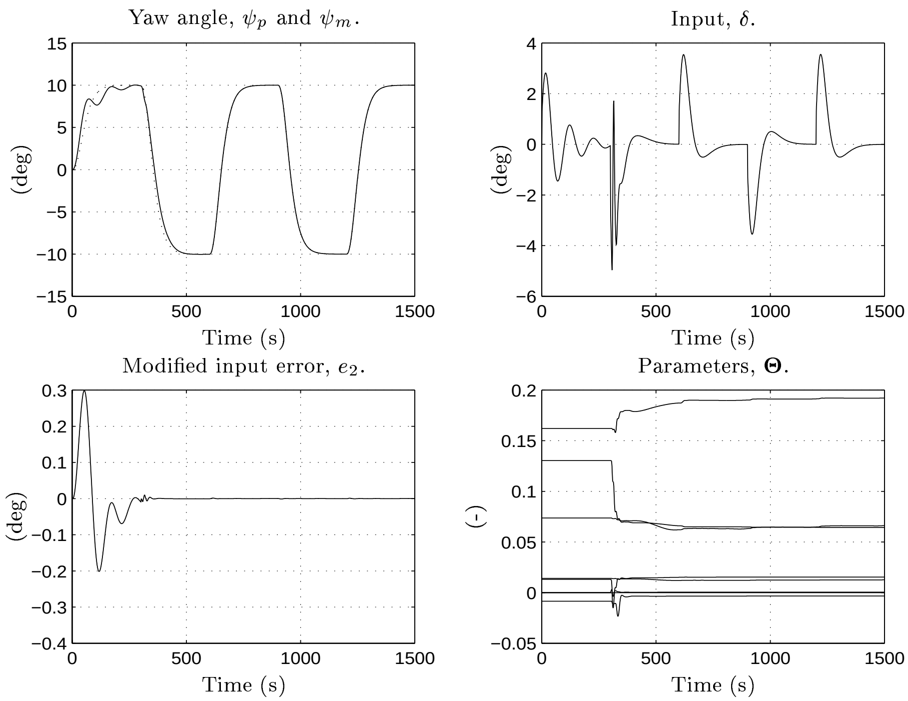

Figure 5: Response of the perturbed ship dynamics when the adaptive controller is turned on at $t = 300$ (s). The yaw angle of the reference model ψ_m is indicated by the dotted line.

7.2. Robustness Properties

To illustrate the robustness to saturating inputs, consider the ship dynamics described by (42). Let the reference trajectory be more aggressive (± 30 (deg)) and let $\boldsymbol{P}(0) = \boldsymbol{I}$. Then Figure 6 shows the time-response of the ship with the adaptive autopilot when $\delta_{max} = 30$ (deg). Notice that rudder saturation do not influence the convergence properties.

7.3. Low-Frequency Disturbances

In open seas there are low-frequency environmental forces acting on the ship. These disturbances are mainly caused by waves, ocean currents and wind. The effect of these slowly-varying forces will be a stationary error in the yaw angle if the controller is implemented without integral action. In the simulations of this section, the low-frequency disturbances are lumped together and modeled as an offset term δ_{dist} to the rudder angle, δ. Hence, if w_L describes Gaussian white noise, then

$$\psi_p(s) \stackrel{\Delta}{=} P_s(s)(P_a(s)\delta_c(s) + \delta_{\text{dist}}(s)), \tag{51}$$

where δ_{dist} are integrated white noise and given by

$$\delta_{\text{dist}}(s) = \frac{1}{s} w_L(s). \tag{52}$$

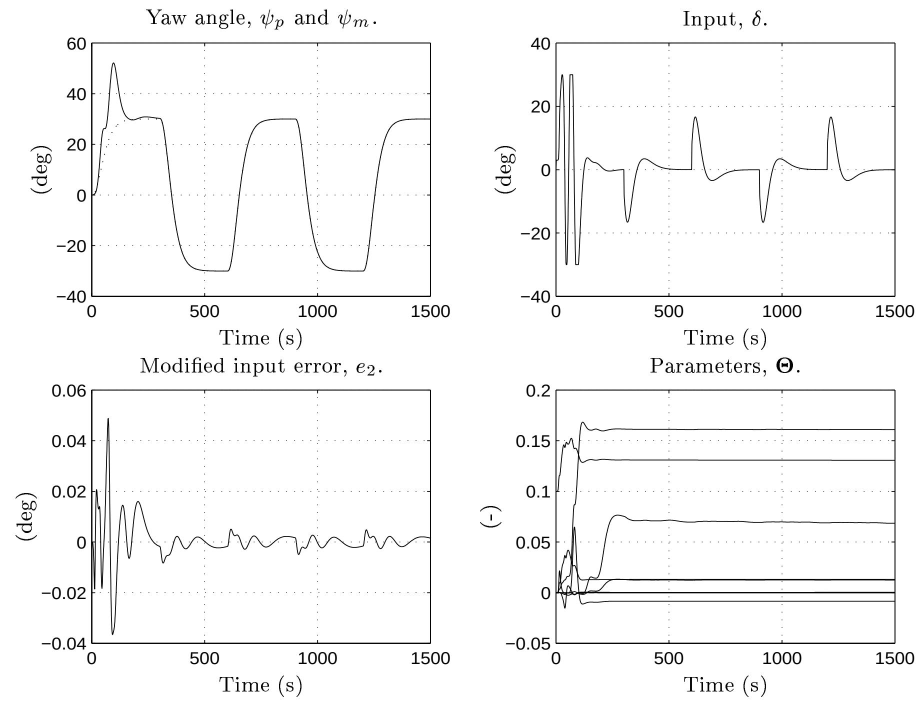

Figure 6: Response of the ship dynamics with the adaptive controller and saturating input.

The adaptive controller is able to produce a zero stationary error in yaw if the parameter identifier contains integral action, and this is illustrated in Figure 7 (solid line) when $\psi_r = 0$. The major drawback with this approach is that the parameter estimates can be drifting if the reference signal is kept constant. This suggests that the parameter identification algorithm should be turned off in a cruise condition. However, if the parameter identifier is off, the fixed controller will be of PD-type, resulting in a stationary error in yaw, see Figure 7 (dashed line). Since it might be of interest to turn off the parameter identifier, the fixed controller should also give zero stationary error. Hence, consider the augmented plant

$$\psi_p(s) = P_s(s)\big(P_a(s)P_i(s)\delta_c'(s) + \delta_{\text{dist}}(s)\big), \tag{53}$$

where $P_i(s)$ is given by (11) with $T_i = 10/\omega_0$. Notice that $n = 5$ and the dimension of the controller needs to be extended. Thus, take

$$\lambda(s) = \lambda_0(s) = (s + 0.1)^4, \tag{54}$$

and with this plant, a simulation study gave the following steady-state control parameters

$$\mathbf{\Theta}^* = \frac{1}{100}\left[9.03\ -0.08\ -2.15\ -13.5\ -52.5\ -6.91\ -0.00\ -0.06\ -2.96\ -17.6\right]^T, \tag{55}$$

where $\mathbf{\Theta}^* \in \mathbb{R}^{10}$. The time response for this (fixed) controller is shown in Figure 8. It is seen that integral action is obtained with the parameter identifier turned off.

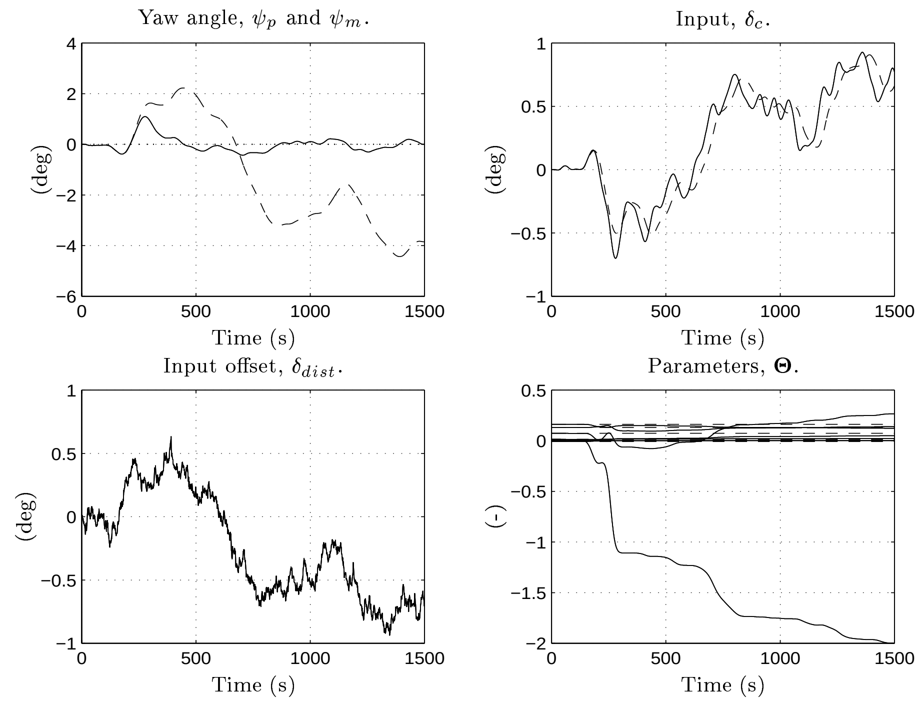

Figure 7: Response of the ship with low-frequency disturbances and with the identification turned on (dashed) and off (solid).

7.4. High-Frequency Disturbances

In Section 7.3, the low-frequency environmental forces in open seas were discussed. In addition to this, there are also high-frequency disturbances, mainly cased by waves, acting on the ship. The effect of these disturbances is an undesirable high-frequency rudder motion if the yaw angle measurements is used for feedback without any filtering. To model the wave disturbances, a 2nd-order linear approximation $P_w(s)$ to the Pierson-Moskowitz spectrum is used (see Fossen[16] for details). Then, if w_H is Gaussian white noise, the wave model is given by

$$\psi_p^H(s) = \frac{2\zeta_w \omega_e \sigma_w}{s^2 + 2\zeta_w \omega_e s + \omega_e^2} w_H(s), \tag{56}$$

where σ_w describe the wave intensity, ζ_w is a damping coefficient and ω_e is the encounter frequency. Since the yaw angle measurements will contain noise, an additional white noise term w_M is added. Hence, the dynamics of the ship with disturbances are given by

$$\psi_p(s) = P_s(s)\big(P_a(s)P_i(s)\delta_c'(s) + \delta_{\text{dist}}(s)\big) + P_w(s)w_H(s) + w_M(s). \tag{57}$$

The time response of this plant with the control law of Section 7.3 is shown in Figure 9 when the initial parameters are given by (55) and the wave model is described by

$$\omega_e = 0.7, \;\; \zeta_w = 0.1, \;\; \sigma_w = \sqrt{10}. \tag{58}$$

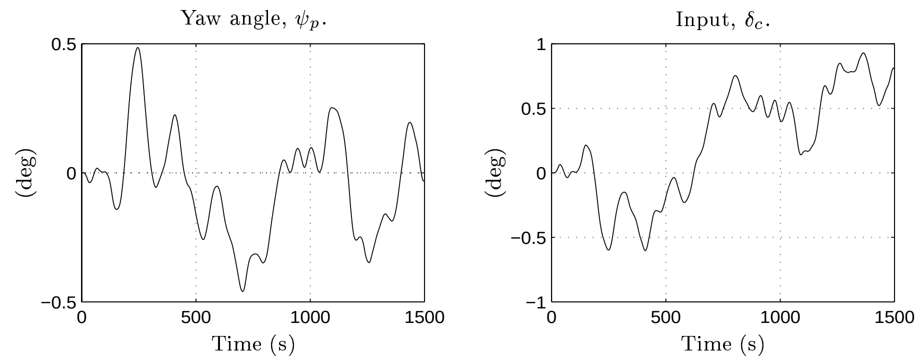

Figure 8: Response of fixed output feedback controller for the ship dynamics augmented with an integral term.

Clearly, the response is unsatisfactory due to parameter drift caused by the disturbances.

Next, include the notch filter such that the augmented model is given by

$$\psi_p^L(s) = P_n(s)\left[P_s(s)\big(P_a(s)P_i(s)\delta_c'(s) + \delta_{\text{dist}}(s)\big) + P_w(s)\eta_H(s) + \eta_M(s)\right]. \tag{59}$$

Figure 10 shows the time response of the ship when wave filtering is included by using the notch filter (7) with parameters

$$\zeta = 0.01, \;\; \omega_n = 0.7. \tag{60}$$

In addition, since $n = 7$ the dimension of the controller needs to be extended such that $\boldsymbol{\Theta} \in \mathbb{R}^{14}$ and

$$\lambda(s) = \lambda_0(s) = (s + 0.1)^6. \tag{61}$$

Notice the smooth behavior of the rudder for relatively large wave disturbances. Also notice that the parameters do not drift in the presence of 1st-order wave disturbances.

8. CONCLUSIONS

In this paper a globally convergent adaptive autopilot has been presented. Only output feedback in terms of compass measurements was used. A cascaded notch filter has been included in the design to obtain proper wave disturbance filtering whereas integral action is obtained by augmenting the plant. The adaptive autopilot applied to the ship with a cascaded notch filter proved to be easy to tune even with no a priori information about the ship. Worst case initial values on $\boldsymbol{\Theta}(t)$ may imply large tracking errors, but one step in the reference signal is usually sufficient to improve the performance significantly. In addition, the 1st-order wave disturbances did not introduce parameter drift.

9. ACKNOWLEDGMENT

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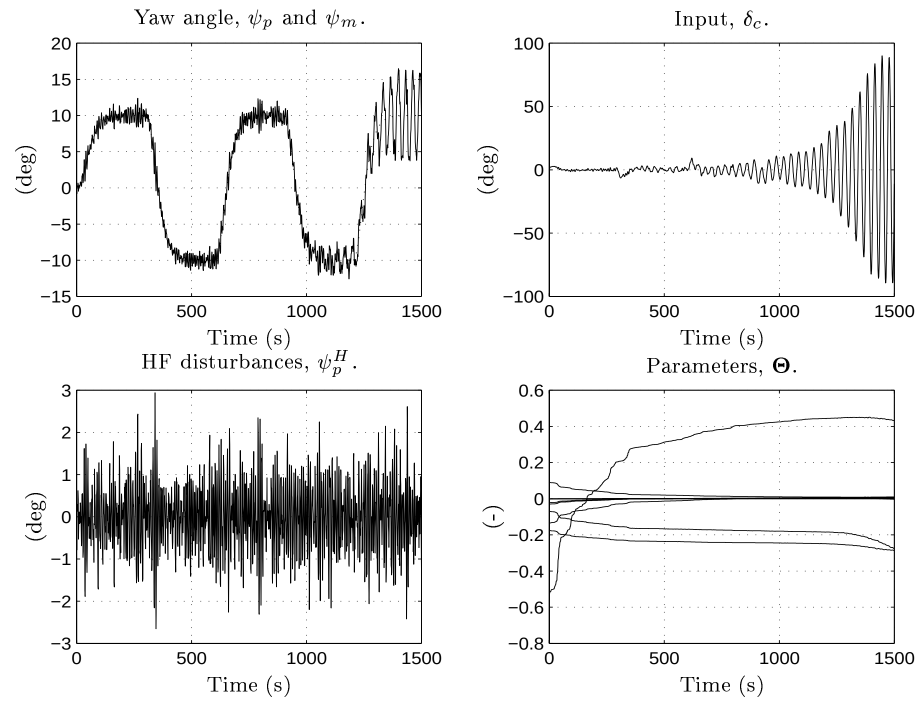

Figure 9: Response of the ship without wave filter and with high-frequency disturbances.

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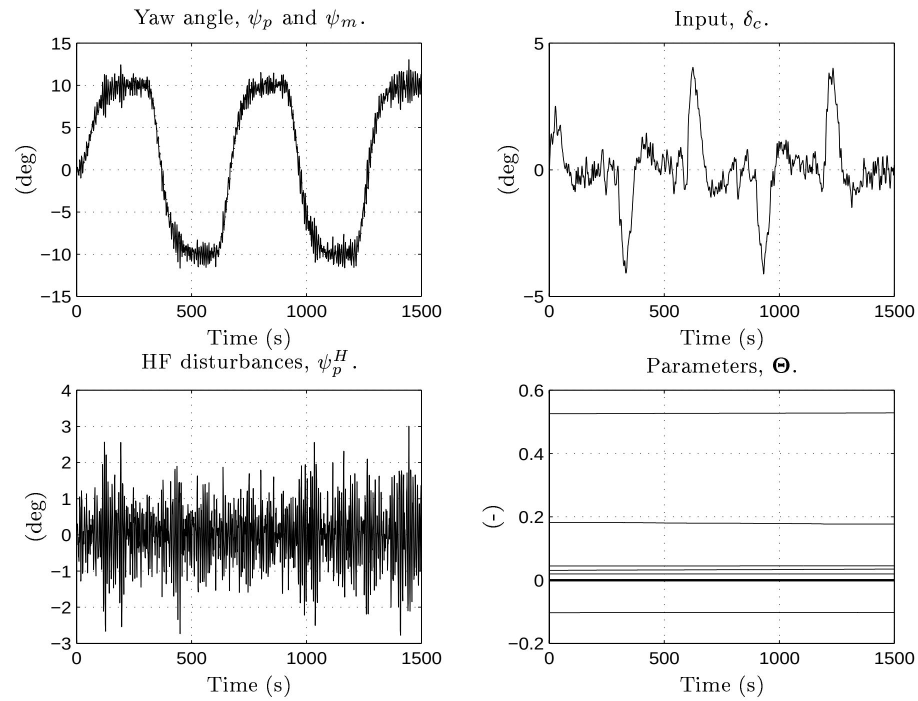

Figure 10: Response of the ship augmented with integral action and notch filter and with high-frequency disturbances. The output of the reference model ψ_m is indicated by the dotted line.

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