

# A Passive Output Feedback Controller with Wave Filter for Marine Vehicles

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## Abstract

An output feedback controller with wave filter for regulation of nonlinear marine vehicles is derived. Only measurements of position and attitude are needed. Asymptotic stability for the position and attitude around the desired values, and the velocity about zero is proven by applying Lyapunov stability analysis. Even though the wave filter in this paper has a notch filter structure, it is incorporated in the controller such that the system from the vehicles velocity to the control input is passive. This is opposed to conventional notch filters which are usually designed separately from the controller itself, possibly complicating the stability analysis. Finally, a simulation study of a ship illustrates the design procedure of the controller.

**Keywords:** Marine vehicle control, passive controllers, wave filtering, output feedback controllers.

# 1 Introduction

In the design of marine vehicle controllers, one of the main problems is wave disturbances. To avoid unnecessary use of and wear on the actuators we do not want to compensate for first order (oscillating) wave induced disturbances. It is therefore desirable to reduce the effect of these disturbances on the control input. A solution to this problem is to use wave filtering.

Often, it is also desirable to avoid the need for velocity measurements because of expensive measurement equipment and possibly noisy and unreliable measurements.

Previous work on position/attitude control of marine vehicles includes papers on dynamic positioning and autopilot design, see [8]. Wave filtering can be performed by inserting a notch or dead-band filter in the control loop, see for instance [18]. In addition, to avoid the need for velocity measurements an observer or a filter has to be included in the control design. Since the wave filter, observer and controller usually are designed separately, it might be difficult to analyze the stability properties for the closed-loop system. Therefore, an optimal controller (LQG-controller) with an integrated wave filter and velocity observer was proposed in [3]. A Kalman filter provides a wave filtered estimate of the states which in turn is used for feedback control. The robustness of this design with respect to nonlinearities and parameter variations may, however, be difficult to analyze.

Another approach for the steering of linear ships is based on  $H_\infty$ -control, see for instance [19]. In this linear case, the specifications for the controller can be made to include wave filtering.

In [11], a passive controller with a wave filter for dynamic positioning for surface vessels is derived. Asymptotic stability for the total closed-loop system is proven even when the vessel is nonlinear. However, the controller requires measurements of the ship's velocities. In our paper, the approach by [11] is modified to avoid the need for measurements of the velocities, and asymptotic stability for the position and attitude about a desired reference is proven. Our output feedback controller including a wave filter is also shown to be passive from the vehicle's velocity to the control input as long as the controller parameters are chosen positive. Knowledge about the frequency characteristics of the wave disturbances is exploited in the wave filter design. If the passive and bounded controller is applied to marine vehicles that are output strictly passive as defined by [10], the system will be passive and  $L_2$ -stable, and thus robust with regard to some disturbances and model parameter uncertainties, see for instance [21]. This paper also shows how wave filtering in a wider frequency range than the one in [11] can be obtained.

The proposed controller can be applied to several marine vehicle control problems, including dynamic positioning and autopilot design. In the next section we present two models on which the controller design can be based, a six degrees of freedom model for a marine vehicle and a SISO maneuvering model in yaw. However, the following control design and stability analysis will be based on the multivariable model only, since the control design and analysis for the monovariable model follows trivially. However, in the frequency analysis of the wave filtering properties of the controller, a SISO model will be used for convenience. Finally, the simulation study demonstrates the performance of the method when applied to a nonlinear SISO maneuvering model.

## 2 Dynamical Models of Marine Vehicles

For marine vehicle control purposes, it is assumed that the dynamics of a marine vehicle affected by first order wave disturbances can be divided into a low frequency dynamics part describing the vehicle's motion without the presence of wave disturbances, and a high frequency dynamics part

describing the vehicle's response to wave disturbances. The measured position and attitude is then given as the sum of the low and high frequency components of the position/attitude.

Other disturbances, such as wind and currents, are not considered in this paper.

In this section, two low frequency models which are suitable for the proposed controller are presented.

## 2.1 Nonlinear model of marine vehicle in six DOF

The following model in six degrees of freedom for the low frequency is considered, [8],

$$\mathbf{M}\dot{\boldsymbol{\nu}} + \mathbf{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{D}(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{g}(\boldsymbol{\eta}) = \boldsymbol{\tau} \quad (1)$$

$$\dot{\boldsymbol{\eta}} = \mathbf{J}(\boldsymbol{\eta})\boldsymbol{\nu} \quad (2)$$

Eq. (1) is the equation of motion, and (2) is the kinematic differential equation. The vector  $\boldsymbol{\nu} = (u, v, w, p, q, r)^T$  is a composite vector of the velocity  $(u, v, w)^T$  and the angular velocity  $(p, q, r)^T$ , both in the body-fixed reference frame. The vector  $\boldsymbol{\eta} = (x, y, z, \phi, \theta, \psi)^T$  is a vector of vehicle position  $(x, y, z)^T$  and Euler angles  $(\phi, \theta, \psi)^T$ . The components of  $\boldsymbol{\nu}$  and  $\boldsymbol{\eta}$  correspond to the motion variables in surge, sway, heave, roll, pitch, and yaw, respectively. The vector  $\boldsymbol{\tau} \in R^6$  is the control vector, and  $\mathbf{g}(\boldsymbol{\eta}) \in R^6$  is a vector of restoring forces and moments. The block diagonal transformation matrix  $\mathbf{J}(\boldsymbol{\eta})$  relates the body-fixed frame to the inertial reference frame (usually the earth). The matrix  $\mathbf{J}(\boldsymbol{\eta})$  only depends on the Euler angles  $(\phi, \theta, \psi)$  and is not defined for a pitch angle at  $\theta = \frac{\pi}{2} \pm 2n\pi$ ,  $n \in N$ . In the following we therefore assume that the pitch angle does not approach the singularity points. The inertia matrix  $\mathbf{M}$  includes hydrodynamic added mass. The matrix  $\mathbf{C}(\boldsymbol{\nu})$  consists of Coriolis and centrifugal terms, and  $\mathbf{D}(\boldsymbol{\nu})$  is a matrix of hydrodynamic damping terms. The matrices  $\mathbf{M}$ ,  $\mathbf{C}$ , and  $\mathbf{D}$  have the following properties (assuming wave frequency independence), [8]:

- 1)  $\mathbf{M} = \mathbf{M}^T > \mathbf{0}$      $\dot{\mathbf{M}} = \mathbf{0}$
- 2)  $\mathbf{C}(\boldsymbol{\nu}) = -\mathbf{C}^T(\boldsymbol{\nu}) \quad \forall \boldsymbol{\nu} \in R^6$
- 3)  $\mathbf{D}(\boldsymbol{\nu}) > \mathbf{0} \quad \forall \boldsymbol{\nu} \in R^6, \boldsymbol{\nu} \neq \mathbf{0}$

The dynamical model (1) can alternatively be expressed in the earth-fixed reference frame as

$$\mathbf{M}_\eta(\boldsymbol{\eta})\ddot{\boldsymbol{\eta}} + \mathbf{C}_\eta(\boldsymbol{\nu}, \boldsymbol{\eta})\dot{\boldsymbol{\eta}} + \mathbf{D}_\eta(\boldsymbol{\nu}, \boldsymbol{\eta})\dot{\boldsymbol{\eta}} + \mathbf{g}_\eta(\boldsymbol{\eta}) = \mathbf{J}^{-T}(\boldsymbol{\eta})\boldsymbol{\tau} \quad (3)$$

where

$$\begin{aligned} \mathbf{M}_\eta(\boldsymbol{\eta}) &= \mathbf{J}^{-T}(\boldsymbol{\eta})\mathbf{M}\mathbf{J}^{-1}(\boldsymbol{\eta}) \\ \mathbf{C}_\eta(\boldsymbol{\nu}, \boldsymbol{\eta}) &= \mathbf{J}^{-T}(\boldsymbol{\eta})[\mathbf{C}(\boldsymbol{\nu}) - \mathbf{M}\mathbf{J}^{-1}(\boldsymbol{\eta})\dot{\mathbf{J}}(\boldsymbol{\eta})]\mathbf{J}^{-1}(\boldsymbol{\eta}) \\ \mathbf{D}_\eta(\boldsymbol{\nu}, \boldsymbol{\eta}) &= \mathbf{J}^{-T}(\boldsymbol{\eta})\mathbf{D}(\boldsymbol{\nu})\mathbf{J}^{-1}(\boldsymbol{\eta}) \\ \mathbf{g}_\eta(\boldsymbol{\eta}) &= \mathbf{J}^{-T}(\boldsymbol{\eta})\mathbf{g}(\boldsymbol{\eta}) \end{aligned}$$

The system (3) satisfies the following properties under the assumption that the pitch angle  $\theta \neq \frac{\pi}{2} \pm 2n\pi$ ,  $n \in N$ :

- 4)  $\mathbf{M}_\eta(\boldsymbol{\eta}) = \mathbf{M}_\eta^T(\boldsymbol{\eta}) > \mathbf{0} \quad \forall \boldsymbol{\eta} \in R^6$
- 5)  $\mathbf{D}_\eta(\boldsymbol{\nu}, \boldsymbol{\eta}) > \mathbf{0} \quad \forall \boldsymbol{\eta}, \boldsymbol{\nu} \in R^6, \boldsymbol{\nu} \neq \mathbf{0}$
- 6)  $(\dot{\mathbf{M}}_\eta - 2\mathbf{C}_\eta)$  is skew-symmetric, [7], i.e.  $\mathbf{x}^T[\dot{\mathbf{M}}_\eta(\boldsymbol{\eta}) - 2\mathbf{C}_\eta(\boldsymbol{\nu}, \boldsymbol{\eta})]\mathbf{x} = 0 \quad \forall \boldsymbol{\eta}, \boldsymbol{\nu}, \mathbf{x} \in R^6$

## 2.2 Nonlinear SISO maneuvering characteristics

The maneuvering characteristics in yaw for a course-stable ship can be given by, [14],

$$m\ddot{\psi} + d(\dot{\psi}) = \delta \quad (4)$$

where  $m$  is a positive inertia constant, and the function  $d(\dot{\psi}) = d_1\dot{\psi} + d_3\dot{\psi}^3$  where  $d_1$ , and  $d_3$  are positive damping constants. The variable  $\psi$  is the yaw angle, and  $\delta$  is the rudder angle, which is the control input.

## 3 Controller Design

In this section, waves or other disturbances are not considered. For the six degree of freedom model (3), we propose the following control law which incorporates a wave filter and requires no velocity measurements

$$\boldsymbol{\tau} = \mathbf{J}^T(\boldsymbol{\eta}) \mathbf{K}_{P0}(\boldsymbol{\eta}_0 - \boldsymbol{\eta}) + \mathbf{g}(\boldsymbol{\eta}) \quad (5)$$

The controller matrix  $\mathbf{K}_{P0}$  is chosen positive definite and constant,  $\mathbf{g}(\boldsymbol{\eta})$  is the vector of restoring forces and moments, and  $\boldsymbol{\eta}$  is the measured position/attitude. The variable  $\boldsymbol{\eta}_0$  is included to obtain damping in the control law and is given by

$$\mathbf{K}_{D1}(\dot{\boldsymbol{\eta}}_0 - \dot{\boldsymbol{\eta}}_1) = \mathbf{K}_{P0}(\boldsymbol{\eta} - \boldsymbol{\eta}_0) + \mathbf{K}_{P1}(\boldsymbol{\eta}_1 - \boldsymbol{\eta}_0) \quad (6)$$

where  $\boldsymbol{\eta}_i$ ,  $i \in \{1, \dots, N\}$ , are given by the  $N$  equations

$$\begin{aligned} \mathbf{M}_i \ddot{\boldsymbol{\eta}}_i + \mathbf{K}_{Pi}(\boldsymbol{\eta}_i - \boldsymbol{\eta}_{i-1}) + \mathbf{K}_{Di}(\dot{\boldsymbol{\eta}}_i - \dot{\boldsymbol{\eta}}_{i-1}) \\ + \mathbf{K}_{P(i+1)}(\boldsymbol{\eta}_i - \boldsymbol{\eta}_{i+1}) + \mathbf{K}_{D(i+1)}(\dot{\boldsymbol{\eta}}_i - \dot{\boldsymbol{\eta}}_{i+1}) = \mathbf{0} \end{aligned} \quad (7)$$

Here  $\boldsymbol{\eta}_{N+1}$  is chosen to be the constant desired position/attitude  $\boldsymbol{\eta}_d$ . Hence,  $\dot{\boldsymbol{\eta}}_{N+1} = \mathbf{0}$ . The regulator design matrices  $\mathbf{K}_{Pi}$ ,  $\mathbf{K}_{Di}$ ,  $i \in \{1, \dots, N+1\}$ , and  $\mathbf{M}_i$ ,  $i \in \{1, \dots, N\}$ , are chosen positive definite and constant. The equations given by (7) can be thought of as the wave filter part of the control law. An interesting feature for the controller consisting of (5), (6), and (7), is that it has a mechanical interpretation. Figure 1 shows how the vehicle and control system can be interpreted as a mechanical system. The controller can be thought of as a virtual system of  $N$  masses,  $N+1$  dampers, and  $N+2$  springs. In addition, the controller consists of a compensator for the restoring forces and moments  $\mathbf{g}(\boldsymbol{\eta})$ , see (5).

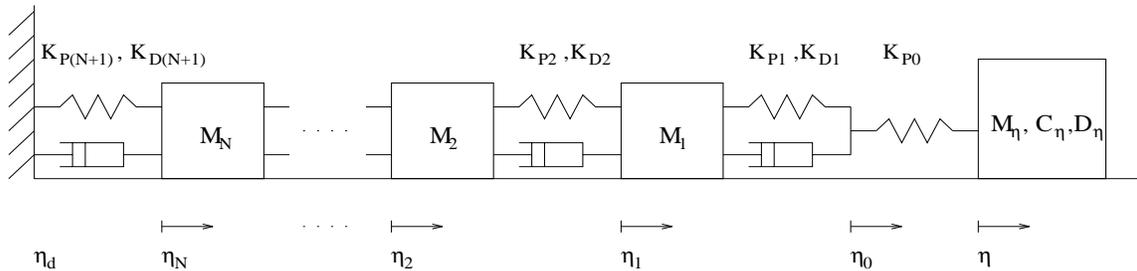


Figure 1: Mechanical interpretation of the control system and vehicle.

We select the state vector as

$$\begin{aligned} \mathbf{x} = [ & (\boldsymbol{\eta} - \boldsymbol{\eta}_0)^T, (\boldsymbol{\eta}_0 - \boldsymbol{\eta}_1)^T, \dots, (\boldsymbol{\eta}_i - \boldsymbol{\eta}_{i+1})^T, \\ & \dots, (\boldsymbol{\eta}_N - \boldsymbol{\eta}_{N+1})^T, \dot{\boldsymbol{\eta}}^T, \dots, \dot{\boldsymbol{\eta}}_i^T, \dots, \dot{\boldsymbol{\eta}}_N^T ]^T \end{aligned} \quad (8)$$

for  $i \in \{1, \dots, N\}$ . Note that  $\mathbf{x} \in R^{6(3+2N)}$ .

**Theorem 1** Consider the marine vehicle (3) with controller (5)–(7). This system is asymptotically stable about  $\mathbf{x} = \mathbf{0}$ , where  $\mathbf{x}$  is given in (8). Consequently, the position/attitude of the vehicle  $\boldsymbol{\eta}$  is asymptotically stable about the desired position/attitude  $\boldsymbol{\eta}_{N+1} = \boldsymbol{\eta}_d$ .

**Proof:** See Appendix A.

**Remark 1** Since the attitude is represented by the Euler angles, the transformation matrix  $\mathbf{J}(\boldsymbol{\eta})$  is not defined at  $\theta = \frac{\pi}{2} \pm 2\pi n, n \in \mathbb{N}$ . Thus, the system is asymptotically stable for all initial conditions such that a pitch angle of  $\theta = \frac{\pi}{2} \pm 2\pi n, n \in \mathbb{N}$ , is not encountered. Conditions on the initial conditions such that  $\theta$  never reaches  $\frac{\pi}{2} \pm 2\pi n, n \in \mathbb{N}$ , can for instance be found by studying the Lyapunov function (19). To avoid singularities, a 4-parameter representation of the attitude is needed, since all 3-parameter representations introduce singularities, see [7] for a thorough discussion on this matter.

**Remark 2** In the case of no wave disturbances, no filtering is required. Thus, the number of virtual masses  $N$  can be chosen to be zero. In this case, the controller reduces to:

$$\begin{aligned}\boldsymbol{\tau} &= \mathbf{J}^T(\boldsymbol{\eta})[\mathbf{K}_{P1}(\boldsymbol{\eta}_d - \boldsymbol{\eta}_0) - \mathbf{K}_{D1}\dot{\boldsymbol{\eta}}_0] + \mathbf{g}(\boldsymbol{\eta}) \\ \dot{\boldsymbol{\eta}}_0 &= \mathbf{K}_{D1}^{-1}[-(\mathbf{K}_{P0} + \mathbf{K}_{P1})\boldsymbol{\eta}_0 + \mathbf{K}_{P0}\boldsymbol{\eta} + \mathbf{K}_{P1}\boldsymbol{\eta}_d]\end{aligned}\quad (9)$$

This controller has a PD structure, and a linear observer provides estimates of the position/attitude  $\boldsymbol{\eta}_0$  and velocity  $\dot{\boldsymbol{\eta}}_0$ . Except for the transformation matrix  $\mathbf{J}$ , this controller has many similarities to robot controllers presented in [1], [4], and [12] and a controller in [5].

Several results on input-output stability exist for passive systems, see for instance [21]. Another interesting feature for systems consisting of feedback or parallel connections of passive blocks, is that the total system also will be passive. Thus, additional passive blocks can easily be added to the system without interfering with its passivity properties. Also, some robustness properties apply to passive systems. In the following we therefore investigate the passivity properties of the marine vehicle and the proposed controller.

The following Lemma 1 is taken from [11] and states output strictly passivity from the input to the velocity of the marine vehicle model (3). Similar results exist for other mechanical systems, see for instance [15].

**Lemma 1** Assuming there exists a constant  $d > 0$  such that  $\forall \mathbf{y}, \boldsymbol{\eta}, \boldsymbol{\nu}, \mathbf{y}^T d \mathbf{y} \leq \mathbf{y}^T \mathbf{D}_\eta \mathbf{y}$ . Then the mapping  $\mathbf{J}^{-T}(\boldsymbol{\eta})(\boldsymbol{\tau} - \mathbf{g}(\boldsymbol{\eta})) \mapsto \dot{\boldsymbol{\eta}}$  is output strictly passive as defined by [10].

**Proof:** See Appendix B.

Output strictly passivity of the monovisible ship model (4) from  $\delta$  to  $\dot{\psi}$  follows trivially.

Passivity of the controller is stated in the following lemma.

**Lemma 2** The controller given by (5)–(7) is passive from the vehicle's velocity to the control input. That is, the mapping  $\dot{\boldsymbol{\eta}} \mapsto -\mathbf{J}^{-T}(\boldsymbol{\eta})(\boldsymbol{\tau} - \mathbf{g}(\boldsymbol{\eta}))$  is passive.

**Proof:** See Appendix C.

The following is now valid.

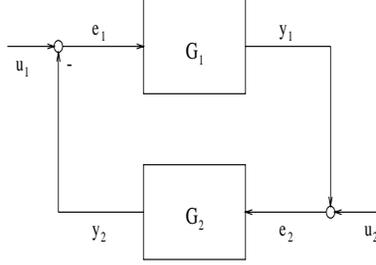


Figure 2: Feedback connection of two operators.

**Theorem 2** Consider the feedback connection of the output strictly passive marine vehicle and the passive controller as in Figure 2. If an external signal  $\mathbf{u}_1 \in L_2$  is acting additively with  $-\mathbf{J}^{-T}(\boldsymbol{\eta})(\boldsymbol{\tau} - \mathbf{g}(\boldsymbol{\eta}))$  on the input of the marine vehicle, and a signal  $\mathbf{u}_2 \in L_2$  is acting additively with  $\dot{\boldsymbol{\eta}}$  on the controller, then

$$\dot{\boldsymbol{\eta}} \in L_2, \quad -\mathbf{J}^{-T}(\boldsymbol{\eta})(\boldsymbol{\tau} - \mathbf{g}(\boldsymbol{\eta})) \in L_2. \quad (10)$$

**Proof:** Using the results in Lemma 1 and Lemma 2, and the fact that the linear mapping  $\dot{\boldsymbol{\eta}} \mapsto -\mathbf{J}^{-T}(\boldsymbol{\eta})(\boldsymbol{\tau} - \mathbf{g}(\boldsymbol{\eta}))$  is bounded, the results follow straight forwardly from a theorem by [6]. The theorem is rendered in Appendix D.  $\square$

In fact, the results in Theorem 2 will be obtained if the mapping in Lemma 2 is interchanged by any other bounded and passive mapping. It should be pointed out that in the case that no damping is present in the marine vehicle model and the constant  $d$  from Lemma 1 is zero, the controller will have to be strictly passive instead of passive to obtain the results in Theorem 2. However, for the controller in this paper, we can still prove asymptotical stability for the total system even if  $d = 0$ , see Appendix A. More results on passive controllers for regulation of nonlinear mechanical systems can be found in [2].

The topic of the following section is the analysis of the wave filtering properties of the output feedback passive controller.

## 4 SISO Analysis of Wave Influence on the Control Input

As mentioned earlier, one of the control problems for marine vehicles is the disturbances caused by waves. In this section, the frequency response properties of a SISO linear model of a marine vehicle is analyzed to show that by proper selection of the controller parameters, the wave modulation on the control input can be reduced.

We want to determine the transfer function between the control input  $\delta(s)$  and the external disturbances  $w(s)$  to show that the control action may be attenuated in some frequency range around the dominant wave frequency.

Consider the linear ship model in [13]

$$m\ddot{\psi} + d_1\dot{\psi} = \delta \quad (11)$$

where  $\delta$  is the control input (rudder),  $m$  is the constant moment of inertia of the ship,  $d_1$  is a constant damping term, and  $\psi$  is the yaw angle (heading) of the ship.

For marine vehicles, it is common to consider the wave disturbances as an output disturbance, see for instance [17]. The measured yaw angle  $\psi_m$  is written as the sum

$$\psi_m = \psi + w \quad (12)$$

where  $w$  is the yaw angle induced by the wave disturbances.

For simplicity, the controller with only one virtual mass ( $N = 1$ ) derived by adjusting (5)–(7) to the ship model (11) and the ship model (11) is considered and given in the  $s$ -plane as

$$\begin{aligned} (ms^2 + d_1s)\psi(s) &= \delta(s) \\ \delta(s) &= K_{P0}(\psi_0(s) - \psi(s) - w(s)) \\ K_{P0}(\psi(s) + w(s) - \psi_0(s)) &= (K_{D1}s + K_{P1})(\psi_0(s) - \psi_1(s)) \\ (m_1s^2 + K_{D2}s + K_{P2})\psi_1(s) &= (K_{D1}s + K_{P1})(\psi_0(s) - \psi_1(s)) \end{aligned}$$

The desired yaw angle is chosen to be  $\psi_d = 0$ .

The transfer function between  $\delta$  and  $w$  becomes

$$\frac{\delta}{w}(s) = \frac{-K_{P0}h_1(s)h_2(s)h_s(s)}{[K_{P0}(h_1(s) + h_2(s)) + h_1(s)h_2(s)]h_s(s) + K_{P0}h_1(s)h_2(s)} \quad (13)$$

where  $h_1(s)$ ,  $h_2(s)$ , and  $h_s(s)$  are defined as

$$\begin{aligned} h_1(s) &= K_{D1}s + K_{P1} \\ h_2(s) &= m_1s^2 + K_{D2}s + K_{P2} \\ h_s(s) &= ms^2 + d_1s \end{aligned} \quad (14)$$

From (13) it can be seen that by proper choices of the transfer functions  $h_i(s)$ ,  $i \in \{1, 2\}$ , the wave influenced motion on the rudder can be reduced. This is possible by letting the transfer function  $h_2(s)$  have complex conjugated zeros with small damping to obtain a notch effect, and by ensuring that  $\frac{\delta}{w}(s)$  is not too large.

We rewrite (14) as

$$h_2(s) = m_1(s^2 + 2\xi\omega_n s + \omega_n^2) \quad (15)$$

which implies that the constants  $K_{D2}$  and  $K_{P2}$  should be chosen as

$$K_{D2} = 2\xi\omega_n m_1, \quad K_{P2} = \omega_n^2 m_1 \quad (16)$$

where  $\xi$  is the relevant damping, and  $\omega_n$  is the resonant frequency. If  $\omega_n$  is chosen equal to the dominant wave frequency, which is usually estimated with good accuracy, and  $\xi < 1$ , a notch effect will be obtained at  $\omega_n$  and attenuation of the wave influence on the control input will be achieved.

For simplicity, we have only analyzed the case with one virtual mass in this section. In the numerical example in Section 6, however, a comparison in the frequency domain between a controller with one ( $N = 1$ ) and with four virtual masses ( $N = 4$ ) is given. This is to investigate how the filtering properties of the controller depend on  $N$ .

## 5 Model Uncertainty

Knowledge about the model parameters is not needed in the design of the passive output feedback controller to ensure stability of the closed-loop system. This is an advantage compared to

LQG-control, where knowledge of the model parameters is necessary to guarantee stability. The only conditions on the parameters of the passive output feedback controller are that they must be chosen positive. However, to obtain good wave filtering, the dominant wave frequency of the wave disturbance should be known. This knowledge is necessary for all existing wave filters.

## 6 Ship Autopilot Design

In this section a numerical study is made of a controller applied to a SISO ship model. First, we investigate the controller applied to a linearized ship model (11) in the frequency domain, like in Section 4. Then the performance of the controller applied to the nonlinear model (4) is studied in the time domain.

The model parameters used in this example is adopted from [20]. In this reference, “the R.O.V. Zeefakkel”, a small training ship with length 42 m and typical cruising speed 6 m/s, is described by the following set of parameters:

$$m = 62, d_1 = 2, d_3 = 0.8. \quad (17)$$

For the linearized model (11), the parameter  $d_3$  is equal to zero.

### 6.1 Frequency Response Analysis

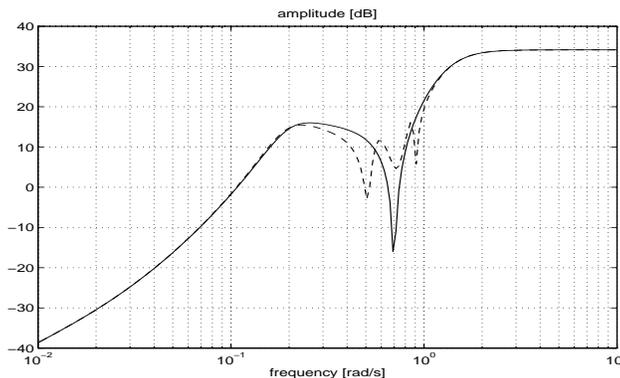


Figure 3: Amplitude  $|\frac{z}{w}(s)|$  in dB as a function of frequency,  $N = 1$  solid line, and  $N = 4$  dotted line.

In this section we want to establish what influence the number of virtual masses  $N$  has on the frequency range where good filtering properties can be achieved.

We study frequency plots of the transfer function  $\frac{z}{w}(s)$  for two controllers given by ( $N = 1$ ) and ( $N = 4$ ) applied to the ship model (11) with parameters given in (17), except that  $d_3 = 0$ . The parameters for both controllers are chosen as, see Section 6.3 for some comments on the parameter

tuning,

$$\begin{aligned}
 m_1 &= 0.25 m \\
 m_i &= 7 m_{i-1}, \quad (2 \leq i \leq 4) \\
 \omega_i &= 0.7, \quad (1 \leq i \leq 4) \\
 \xi_i &= 0.0001, \quad (1 \leq i \leq 3) \\
 \xi_4 &= 0.3 \\
 K_{P(i+1)} &= m_i \omega_i^2, \quad (1 \leq i \leq 4) \\
 K_{D(i+1)} &= 2 \xi_i \omega_i m_i, \quad (1 \leq i \leq 4) \\
 K_{D1} &= 7000 \cdot K_{D2} \\
 K_{P1} &= 0.2 \cdot K_{D1} \\
 K_{P0} &= 10 \cdot K_{P1}
 \end{aligned}$$

The amplitudes of the transfer functions,  $\frac{\delta}{w}(s)$ , for  $N = 1$  and  $N = 4$  are shown in Figure 3. A notch effect is obtained around  $\omega_n = 0.7$  rad/s for both controllers which means that the wave disturbances are attenuated around the dominant wave frequency. Note that filtering is obtained in a wider frequency range by increasing the number of virtual masses. However, the damping amplitude at the dominating wave frequency is smaller for the controller with  $N = 4$ .

Note that for high frequencies, the disturbances on the output are amplified by a factor  $K_{P0}$ . This implies that for practical purposes where high frequent measurement noise is present, a low pass filter should be included in the controller to remove the influence from the high frequent disturbance. In this case, the controller will no longer be passive. However, the closed-loop system will still be stable provided that the low pass filter is properly tuned.

## 6.2 Simulation Study

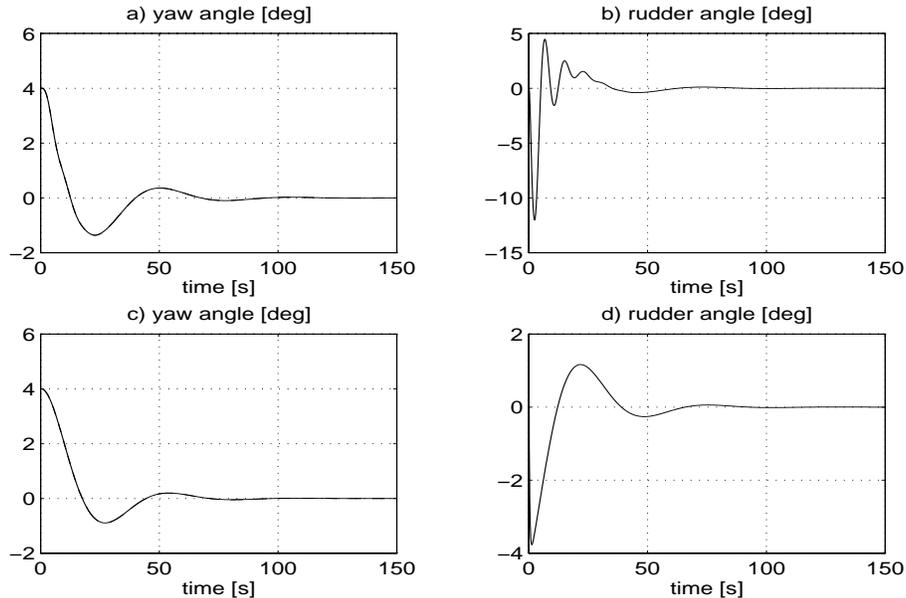


Figure 4: Yaw angle  $\psi$  and rudder angle  $\delta$  versus time for controller with wave filter, a) and b), and without wave filter, c) and d), ( $w = 0$ ).

This section examines the properties of our controller applied to the nonlinear ship model (4) with parameters given in (17), in the time domain. A controller with one virtual mass ( $N = 1$ ) is chosen.

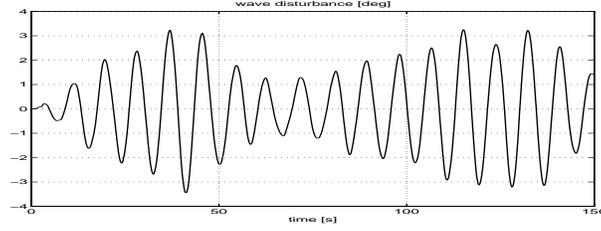


Figure 5: Wave disturbance  $w$  versus time.

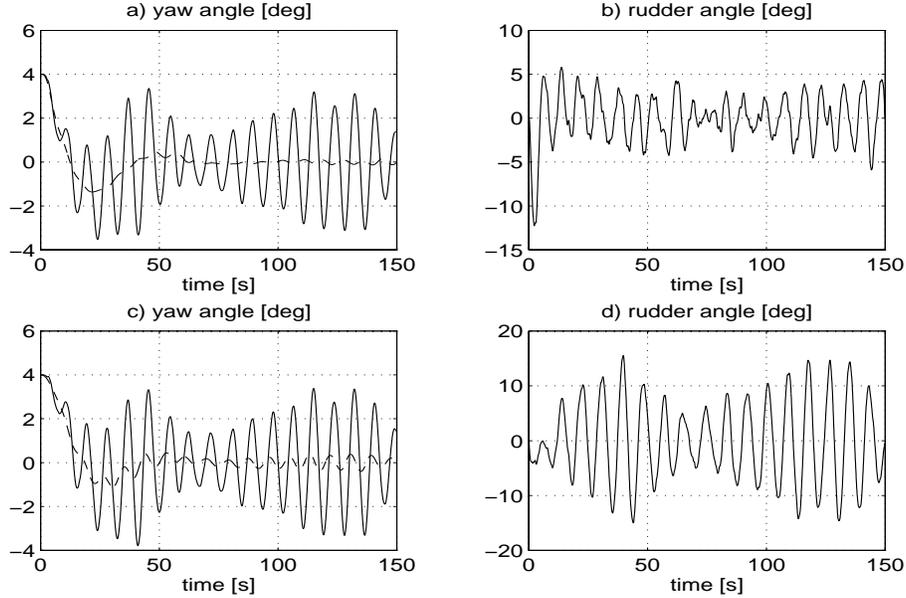


Figure 6: Measured yaw angle  $\psi + w$  (solid line), yaw angle  $\psi$  (dashed line) and rudder angle  $\delta$  versus time for controller with wave filter, a) and b), and without wave filter, c) and d), with wave disturbance  $w$ .

A linear approximation of the first order wave induced motion can be given in the s-plane as, [9],

$$w(s) = \frac{K_w s^2}{(s^2 + 2\xi\omega_n s + \omega_n^2)^2} \epsilon(s) \quad (18)$$

Here,  $\epsilon$  is white noise,  $\omega_n$  is the dominating wave frequency,  $\xi$  is the relative damping ratio of the waves,  $K_w$  is a gain that is dependent on the wave energy, and  $w$  is the wave induced yaw angle. The parameters of the wave model are chosen as:  $\omega_n = 0.7$ ,  $\xi = 0.1$ , and  $K_w = 0.01$ .

In the simulation study a sampling frequency of 10 Hz is used. The Runge-Kutta fourth order method is used for numerical integration. The desired yaw angle  $\psi_d = 0$ . The controller parameters are chosen to be

$$\begin{aligned} m_1 &= 0.25 \text{ m} & K_{D2} &= 2\xi_1\omega_1 m_1 \\ \omega_1 &= \omega_n & K_{P2} &= m_1\omega_1^2 \\ \xi_1 &= 0.0001 & K_{D1} &= 2500 \cdot K_{D2} \\ K_{P0} &= 15 \cdot K_{P1} & K_{P1} &= 0.2 \cdot K_{D1} \end{aligned}$$

This corresponds to a bandwidth of 0.15 rad/s. To illustrate the proposed controller's wave filtering properties, it is compared with a similar controller without a wave filter, that is  $N = 0$ , see Remark 2. The same values on the controller parameters are used for this controller as for the controller with wave filter.

At first we assume no disturbances, that is  $w = 0$ . The yaw and rudder angles for the two controllers are shown in Figure 4. We see that the yaw angles converge to the desired yaw angle. The controller without a wave filter has a slightly faster response than the controller with wave filter, and also introduces less rudder action. Therefore, when no wave disturbances are present, the controller without wave filter could be chosen.

We now add the disturbance given in (18), see Figure 5. In Figure 6 the plot of the measured output  $\psi + w$  is given for both controllers with a solid line, and  $\psi$  is given with a dashed line. The rudder angles  $\delta$  for both controllers are also presented in the same figure. As expected, the simulations show that the rudder action for the output feedback controller with wave filtering is significantly reduced compared to the controller without a wave filter.

A comparison between the passive output feedback controller with one virtual mass and an LQG-controller has been made in [16]. The comparison focuses on stability and wave filtering properties. The analysis shows that the two controllers have similar wave filtering properties, but the LQG-controller has a slightly stronger damping at some frequencies just above the dominant wave frequency. This indicates that the LQG-controller may be less sensitive to some changes in the dominant wave frequency. However, the paper shows that stability for the passive output feedback controller is easier to establish and analyze than for the LQG-controller, especially when they are applied to nonlinear marine vehicles.

### 6.3 Comments on Parameter Tuning

#### SISO Systems:

Section 4 and the numerical example suggest that some of the parameters of the controller can be chosen in a systematic manner. It seems reasonable to choose the parameter  $m_1$  smaller than the ship's moment of inertia  $m$ , (approximately  $0.1m - 0.3m$ ). The values of  $m_1$  and  $m_i$  in our case study are chosen by studying the frequency responses and simulation results. The  $\omega_i$ 's can be chosen equal to the dominant wave frequency  $\omega_n$ . The values of  $\xi_i$  determine the damping amplitude around the dominant wave frequency. We see that  $K_{P_i}$  and  $K_{D_i}$ ,  $i \in \{2..N\}$ , should be chosen as functions of  $m_i$ ,  $\xi_i$  and  $\omega_i$  according to (16). The other controller parameters  $K_{P_1}$ ,  $K_{D_1}$  and  $K_{P_0}$  should be tuned to obtain the desired bandwidth of the total system. The proportional parameter is  $K_{P_1}$ ,  $K_{D_1}$  is the derivative parameter, and  $K_{P_0}$  is the observer gain.

#### MIMO Systems:

The constant regulator design matrices,  $\mathbf{M}_i$ ,  $\mathbf{K}_{P_i}$ , and  $\mathbf{K}_{D_i}$  may be chosen diagonal, if desired. Then the tuning of the parameters will follow approximately the same procedure as for the SISO case for each degree of freedom.

## 7 Concluding Remarks

A marine vehicle output feedback controller with wave filter for regulation has been derived. The controller could be interpreted as a mechanical system of passive elements, like dampers, springs, and masses. Asymptotic stability for the position/attitude about the desired position/attitude was proven without measuring the velocity. The controller was also shown to be passive.

A frequency response analysis showed that the wave influence on the rudder action can be attenuated in the area around the dominant wave frequency by proper choices of the controller parameters. Nu-

merical examples showed that the frequency range where wave filtering can be obtained is dependent on the number of virtual masses (or the order of the controller).

The simulation study showed that the rudder action for our controller is significantly reduced compared to a controller without wave filter.

It is expected that the controller would perform well also if integral action for compensation of constant disturbances is included. In this case, stability could be ensured by simulation and frequency response studies and a careful selection of controller parameters. However, no theoretical justification for stability is given in this paper when integral action is included.

## Appendix A - Proof of Theorem 1

Consider the Lyapunov function candidate

$$\begin{aligned}
V(\mathbf{x}) = & \frac{1}{2}[\dot{\boldsymbol{\eta}}^T \mathbf{M}_\eta \dot{\boldsymbol{\eta}} + \dot{\boldsymbol{\eta}}_1^T \mathbf{M}_1 \dot{\boldsymbol{\eta}}_1 + \cdots \\
& + \dot{\boldsymbol{\eta}}_i^T \mathbf{M}_i \dot{\boldsymbol{\eta}}_i + \cdots + \dot{\boldsymbol{\eta}}_N^T \mathbf{M}_N \dot{\boldsymbol{\eta}}_N \\
& + (\boldsymbol{\eta} - \boldsymbol{\eta}_0)^T \mathbf{K}_{P0}(\boldsymbol{\eta} - \boldsymbol{\eta}_0) \\
& + (\boldsymbol{\eta}_0 - \boldsymbol{\eta}_1)^T \mathbf{K}_{P1}(\boldsymbol{\eta}_0 - \boldsymbol{\eta}_1) + \cdots \\
& + (\boldsymbol{\eta}_i - \boldsymbol{\eta}_{i+1})^T \mathbf{K}_{P(i+1)}(\boldsymbol{\eta}_i - \boldsymbol{\eta}_{i+1}) + \cdots \\
& + (\boldsymbol{\eta}_N - \boldsymbol{\eta}_{N+1})^T \mathbf{K}_{P(N+1)}(\boldsymbol{\eta}_N - \boldsymbol{\eta}_{N+1})] \tag{19}
\end{aligned}$$

which can be thought of as a sum of the kinetic and potential energy of the system. Differentiating  $V(\mathbf{x})$  with respect to time, substituting (3), (5), (6), and (7) into  $\dot{V}$ , and using Property 6 gives

$$\begin{aligned}
\dot{V} = & -\dot{\boldsymbol{\eta}}^T \mathbf{D}_\eta(\boldsymbol{\nu}, \boldsymbol{\eta}) \dot{\boldsymbol{\eta}} - (\dot{\boldsymbol{\eta}}_1 - \dot{\boldsymbol{\eta}}_0)^T \mathbf{K}_{D1}(\dot{\boldsymbol{\eta}}_1 - \dot{\boldsymbol{\eta}}_0) \\
& - (\dot{\boldsymbol{\eta}}_1 - \dot{\boldsymbol{\eta}}_2)^T \mathbf{K}_{D2}(\dot{\boldsymbol{\eta}}_1 - \dot{\boldsymbol{\eta}}_2) - \cdots \\
& - (\dot{\boldsymbol{\eta}}_{i-1} - \dot{\boldsymbol{\eta}}_i)^T \mathbf{K}_{Di}(\dot{\boldsymbol{\eta}}_{i-1} - \dot{\boldsymbol{\eta}}_i) - \cdots \\
& - (\dot{\boldsymbol{\eta}}_{N-1} - \dot{\boldsymbol{\eta}}_N)^T \mathbf{K}_{Dn}(\dot{\boldsymbol{\eta}}_{N-1} - \dot{\boldsymbol{\eta}}_N) \\
& - \dot{\boldsymbol{\eta}}_N^T \mathbf{K}_{D(N+1)} \dot{\boldsymbol{\eta}}_N \leq 0
\end{aligned}$$

Finally, by applying La Salle's invariant set theorem [21], we find that the equilibrium  $\mathbf{x} = \mathbf{0}$  is asymptotically stable.

Note that the system will be asymptotically stable even when  $\mathbf{D}_\eta(\boldsymbol{\nu}, \boldsymbol{\eta})$  is equal to zero. This is because it can be concluded that  $\boldsymbol{\eta}$  is constant and thus that  $\dot{\boldsymbol{\eta}}$  is zero from  $\dot{V} = 0$ .  $\square$

## Appendix B - Proof of Lemma 1

Taking the inner product  $\langle \mathbf{J}^{-T}(\boldsymbol{\eta})(\boldsymbol{\tau} - \mathbf{g}(\boldsymbol{\eta})), \dot{\boldsymbol{\eta}} \rangle_T$ , using (1), and Property 4 and 6 yields

$$\begin{aligned}
\langle \mathbf{J}^{-T}(\boldsymbol{\eta})(\boldsymbol{\tau} - \mathbf{g}(\boldsymbol{\eta})), \dot{\boldsymbol{\eta}} \rangle_T & = \langle \boldsymbol{\tau} - \mathbf{g}(\boldsymbol{\eta}), \mathbf{J}^{-1}(\boldsymbol{\eta}) \dot{\boldsymbol{\eta}} \rangle_T \\
& = \langle \boldsymbol{\tau} - \mathbf{g}(\boldsymbol{\eta}), \boldsymbol{\nu} \rangle_T \\
& = \langle \mathbf{M}\dot{\boldsymbol{\nu}} + \mathbf{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{D}(\boldsymbol{\nu})\boldsymbol{\nu}, \boldsymbol{\nu} \rangle_T
\end{aligned}$$

$$\begin{aligned}
&= \int_0^T \boldsymbol{\nu}^T [M\dot{\boldsymbol{\nu}} + \mathbf{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{D}(\boldsymbol{\nu})\boldsymbol{\nu}] dt \\
&= \int_0^T \left\{ \frac{d}{dt} \left( \frac{1}{2} \boldsymbol{\nu}^T M \boldsymbol{\nu} \right) + \boldsymbol{\nu}^T \mathbf{D}(\boldsymbol{\nu}) \boldsymbol{\nu} \right\} dt \\
&= \frac{1}{2} \boldsymbol{\nu}^T(T) M \boldsymbol{\nu}(T) - \frac{1}{2} \boldsymbol{\nu}^T(0) M \boldsymbol{\nu}(0) \\
&\quad + \int_0^T \boldsymbol{\nu}^T \mathbf{D}(\boldsymbol{\nu}) \boldsymbol{\nu} dt \\
&\geq \beta_1 + d \|\dot{\boldsymbol{\eta}}_T\|_2^2
\end{aligned} \tag{20}$$

where

$$\beta_1 = -\frac{1}{2} \boldsymbol{\nu}^T(0) M \boldsymbol{\nu}(0) \tag{21}$$

and  $\dot{\boldsymbol{\eta}}_T$  denotes the truncation of  $\dot{\boldsymbol{\eta}}$ .  $\square$

## Appendix C - Proof of Lemma 2

Consider the non-negative function

$$W(\mathbf{x}) = V(\mathbf{x}) - \dot{\boldsymbol{\eta}}^T M_\eta \dot{\boldsymbol{\eta}}$$

where  $V(\mathbf{x})$  is defined in (19). Differentiating  $W(\mathbf{x})$  with respect to time yields:

$$\begin{aligned}
\dot{W}(\mathbf{x}) &= \dot{\boldsymbol{\eta}}_1^T M_1 \ddot{\boldsymbol{\eta}}_1 + \cdots + \dot{\boldsymbol{\eta}}_i^T M_i \ddot{\boldsymbol{\eta}}_i + \cdots \\
&\quad + \dot{\boldsymbol{\eta}}_N^T M_N \ddot{\boldsymbol{\eta}}_N + (\boldsymbol{\eta} - \boldsymbol{\eta}_0)^T \mathbf{K}_{P0} (\dot{\boldsymbol{\eta}} - \dot{\boldsymbol{\eta}}_0) \\
&\quad + (\boldsymbol{\eta}_0 - \boldsymbol{\eta}_1)^T \mathbf{K}_{P1} (\dot{\boldsymbol{\eta}}_0 - \dot{\boldsymbol{\eta}}_1) + \cdots \\
&\quad + (\boldsymbol{\eta}_i - \boldsymbol{\eta}_{i+1})^T \mathbf{K}_{P(i+1)} (\dot{\boldsymbol{\eta}}_i - \dot{\boldsymbol{\eta}}_{i+1}) + \cdots \\
&\quad + (\boldsymbol{\eta}_N - \boldsymbol{\eta}_{N+1})^T \mathbf{K}_{P(N+1)} \dot{\boldsymbol{\eta}}_N
\end{aligned}$$

Substituting (3), (5), (6) and (7) into  $\dot{W}$  and using Property 6 yields

$$\begin{aligned}
\dot{W} &= -\dot{\boldsymbol{\eta}}^T \mathbf{K}_{P0} (\boldsymbol{\eta}_0 - \boldsymbol{\eta}) - (\dot{\boldsymbol{\eta}}_1 - \dot{\boldsymbol{\eta}}_0)^T \mathbf{K}_{D1} (\dot{\boldsymbol{\eta}}_1 - \dot{\boldsymbol{\eta}}_0) \\
&\quad - (\dot{\boldsymbol{\eta}}_1 - \dot{\boldsymbol{\eta}}_2)^T \mathbf{K}_{D2} (\dot{\boldsymbol{\eta}}_1 - \dot{\boldsymbol{\eta}}_2) - \cdots \\
&\quad - (\dot{\boldsymbol{\eta}}_{i-1} - \dot{\boldsymbol{\eta}}_i)^T \mathbf{K}_{Di} (\dot{\boldsymbol{\eta}}_{i-1} - \dot{\boldsymbol{\eta}}_i) - \cdots \\
&\quad - (\dot{\boldsymbol{\eta}}_{N-1} - \dot{\boldsymbol{\eta}}_N)^T \mathbf{K}_{Dn} (\dot{\boldsymbol{\eta}}_{N-1} - \dot{\boldsymbol{\eta}}_N) \\
&\leq -\dot{\boldsymbol{\eta}}^T \mathbf{K}_{P0} (\boldsymbol{\eta}_0 - \boldsymbol{\eta})
\end{aligned} \tag{22}$$

Integrating (22) yields

$$W(T) - W(0) \leq \int_0^T -\dot{\boldsymbol{\eta}}^T \mathbf{K}_{P0} (\boldsymbol{\eta}_0 - \boldsymbol{\eta}) dt \tag{23}$$

which finally yields

$$\begin{aligned}
&\langle \dot{\boldsymbol{\eta}}, -\mathbf{K}_{P0} (\boldsymbol{\eta}_0 - \boldsymbol{\eta}) \rangle \\
&= \langle \dot{\boldsymbol{\eta}}, -\mathbf{J}^{-T}(\boldsymbol{\eta})(\boldsymbol{\tau} - \mathbf{g}(\boldsymbol{\eta})) \rangle \geq -W(0) = \beta_2
\end{aligned}$$

According to [21],  $\dot{\boldsymbol{\eta}} \mapsto -\mathbf{J}^{-T}(\boldsymbol{\eta})(\boldsymbol{\tau} - \mathbf{g}(\boldsymbol{\eta}))$  is passive.  $\square$

## Appendix D

This theorem is adapted from [6].

**Theorem 3** *Given two operators  $G_1 : L_{2\epsilon} \rightarrow L_{2\epsilon}$  and  $G_2 : L_{2\epsilon} \rightarrow L_{2\epsilon}$ , where  $L_{2\epsilon}$  denotes the extended  $L_2$ -space. Assume that they are connected such that, Figure 2,*

$$\begin{aligned}e_1 &= u_1 - y_2 \\e_2 &= u_2 + y_1 \\y_1 &= G_1 e_1 \\y_2 &= G_2 e_2\end{aligned}$$

where  $u_1$  and  $u_2$  are two external signals. Assume there exist constants  $\beta_1$ ,  $\beta_2$ ,  $a$ , and  $b$ , and a positive constant  $c$  such that

$$\begin{aligned}\langle u, G_2 u \rangle_T &\geq a \|u\|_T^2 + \beta_2 \\ \|G_2 u\| &\leq c \|u\|_T \\ \langle u, G_1 u \rangle_T &\geq b \|G_1 u\|_T^2 + \beta_1\end{aligned}$$

for all  $T > 0$  and  $u \in L_{2\epsilon}$ . If  $a + b > 0$  and  $u_1, u_2 \in L_2$ , then  $y_1, y_2 \in L_2$ .

The proof can be found in [6].

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