Global Output Tracking Control of a Class of Euler-Lagrange Systems[1]

Ole Morten Aamo[2], Murat Arcak[3], Thor I. Fossen[2] and Petar V. Kokotović[3]

Abstract

We address the problem of output feedback tracking control of a class of Euler-Lagrange systems subject to nonlinear dissipative loads. By imposing a monotone damping condition on the nonlinearities of the unmeasured states, the common restriction that the nonlinearities be globally Lipschitz is removed. The proposed observer-controller scheme renders the origin of the error dynamics uniformly globally asymptotically stable.

1 Introduction

In contrast to the tests for reachability and observability available for linear systems, there is no simple test that ensures that the output feedback tracking problem is solvable for general nonlinear systems. Existing results are restricted to classes of nonlinear systems for which certain structural properties can be exploited in order to obtain closed-loop stability when an observer is used in conjunction with a state feedback control law. Among the first papers on observer design for systems with nonlinearities in the unmeasured states was one by Thau [14], whose results were later generalized by Kou et al. [7]. Their main assumption was that the function of the unmeasured states be globally Lipschitz, and gave sufficient conditions in terms of the Lipschitz constant under which their observer would be convergent. Raghavan and Hedrick [12], and later Rajamani [13], provided systematic approaches to finding an observer gain matrix that satisfies this condition. In the recent papers [2, 3], Arcak and Kokotović remove the global Lipschitz condition, and instead require a monotonic damping property to hold. As we will demonstrate, this damping property is of interest in offshore applications.

The papers by Arcak and Kokotović ([2, 3]), and a recent paper by Loria and Panteley [9], are the main sources of motivation for this work. In [9], Loria and Panteley present a separation principle for fully actuated Euler-Lagrange systems that can be transformed, via a global change of coordinates, into the following form

$$\dot{q} = J(q)\nu \tag{1}$$
$$M\dot{\nu} + v(q) = \tau \tag{2}$$

where q and ν are n-dimensional vectors of generalized positions and velocities, respectively, M is a symmetric positive definite and constant matrix, τ is the control input, and J is an $n \times n$ matrix as a function of q. In contrast to the numerous references made to ship dynamics in [9], the class of systems described by equations (1) and (2), does not include systems where $\dot{\nu}$ depends on ν. Clearly, this fact excludes offshore structures and ships from the class, since they will be subject to coriolis and centripetal forces, as well as hydrodynamic drag. In general, these terms are nonlinear functions of the velocities. As already mentioned, nonlinearities in the unmeasured states are not trivial to include in the analysis, and the global output feedback tracking control problem has not been solved for general Euler-Lagrange systems. In the special case of one degree-of-freedom systems, Loria [8] presented the first explicit global solution to the problem. In [4], Besançon presents a more elegant solution to the one degree-of-freedom problem.

In this paper, we extend the class of systems considered in [9], by adding a term, nonlinear in the unmeasured states, that satisfies the monotone damping property employed for observer design in [2]. An observer-controller scheme is proposed, that renders the origin of the error dynamics uniformly globally asymptotically stable. In a separate result, positive definiteness of the linear damping term, or alternatively, an additional assumption on the matrix $J(q)$, is exploited to obtain UGAS by means of a simplified control law.

Notations and Definitions. We will denote the Euclidean space of dimension n by $\mathbb{R}^n$, the set of non-negative real numbers by $\mathbb{R}_+$, and the *closed* ball of radius r by B_r. The 2-norm of elements of $\mathbb{R}^n$, as well as the induced matrix 2-norm, is denoted by $\|\cdot\|$. Estimates will be denoted by a hat, that is, $\hat{x}$ and $\hat{\dot{x}}$ are estimates of x and $\dot{x}$, respectively. We refer the reader to

[1] This work was supported by the Norwegian Research Council, the National Science Foundation under grant ECS-9812346, and the Air Force Office of Scientific Research under grant F49620-95-1-0409.

[2] Department of Engineering Cybernetics, Norwegian University of Science and Technology, N-7491 Trondheim, Norway. E-mail: aamo@itk.ntnu.no, tif@itk.ntnu.no

[3] Center for Control Engineering and Computation, University of California, Santa Barbara, CA 93106-9560, USA. E-mail: murat@seidel.ece.ucsb.edu, petar@ece.ucsb.edu

[6] for definitions of the following notions: functions of class $\mathcal{K}$, $\mathcal{K}_\infty$, and $\mathcal{KL}$; *uniform global stability* (UGS), *uniform global asymptotic stability* (UGAS), *uniform local exponential stability* (ULES), and *uniform global exponential stability* (UGES) of equilibrium points; and *input-to-state stable* (ISS) systems.

2 Problem statement

In the Lyapunov analysis that follows, we will require a monotone damping property to hold for the function of the unmeasured states (denoted d in (6) below). More precisely, we require that the function $d : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is locally Lipschitz and satisfies

$$(x-y)^T P(d(x)-d(y)) \geq 0, \qquad \forall x, y \in \mathbb{R}^n \qquad (3)$$

where $P = P^T > 0$. For the special case when d is continuously differentiable, the following lemma provides a test for this property (see [11]).

Lemma 1 (Monotone Damping Property) *Suppose $d : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ satisfies*

$$P\left[\frac{\partial d}{\partial x}\right] + \left[\frac{\partial d}{\partial x}\right]^T P \geq 0, \qquad \forall x \in \mathbb{R}^n \qquad (4)$$

where $P = P^T > 0$. Then, (3) holds for d.

We consider systems in the following form

$$\begin{aligned} \dot{q} &= J(q)\nu & (5)\\ M\dot{\nu} + D\nu + d(\nu) + v(q) &= \tau & (6) \end{aligned}$$

where $q, \nu \in \mathbb{R}^n$, M and D are constant matrices satisfying $M = M^T > 0$ and $D + D^T \geq 0$, respectively, and $d(\nu)$ satisfies (3) with $P = I$. As in [9] we will assume that $J : \mathbb{R}^n \longrightarrow \mathbb{R}^{n \times n}$ has the following properties

Property 1 *$J(q)$ is invertible and satisfies $0 < k_j \leq \|J(q)\| \leq k_J$ for all $q \in \mathbb{R}^n$.*

Property 2 *$\frac{d}{dt}J(q) = \dot{J}(q, \dot{q})$ is globally Lipschitz in $\dot{q}$, uniformly in q, with Lipschitz constant L_j.*

Given a desired trajectory, $q_d : \mathbb{R}_+ \longrightarrow \mathbb{R}^n$, that has continous first and second derivatives whose norms are bounded above by β_d, our objective is to find a controller such that $q \longrightarrow q_d$, and $\dot{q} \longrightarrow \dot{q}_d$, as $t \longrightarrow \infty$.

3 Main results

3.1 Observer design

Copying equations (5)–(6) and adding output injection terms yield the following observer

$$\begin{aligned} \dot{\hat{q}} &= J(q)\hat{\nu} + K_{o_1}\tilde{q} & (7)\\ M\dot{\hat{\nu}} + D\hat{\nu} + d(\hat{\nu}) + v(q) &= \tau + MK_{o_2}(q)\tilde{q} & (8) \end{aligned}$$

with error dynamics, in terms of $\tilde{q} \triangleq q - \hat{q}$ and $\tilde{\nu} \triangleq \nu - \hat{\nu}$, as follows

$$\begin{aligned} \dot{\tilde{q}} &= -K_{o_1}\tilde{q} + J(q)\tilde{\nu} & (9)\\ \dot{\tilde{\nu}} &= -K_{o_2}(q)\tilde{q} - M^{-1}\left[D\tilde{\nu} + d(\nu) - d(\hat{\nu})\right] & (10) \end{aligned}$$

Now, consider the Lyapunov function candidate given by

$$V_o(t, \tilde{q}, \tilde{\nu}) = \frac{1}{2}\left(\tilde{q}^T P_o \tilde{q} + \tilde{\nu}^T M \tilde{\nu}\right) \qquad (11)$$

where $P_o = P_o^T > 0$. Setting $K_{o_2}(q) = M^{-1}J(q)^T P_o$, the time derivative of V_o along the trajectories of (9)–(10) is

$$\dot{V}_o(t, \tilde{q}, \tilde{\nu}) = -\frac{1}{2}\tilde{q}^T(P_o K_{o_1} + K_{o_1}^T P_o)\tilde{q} + \tilde{q}^T P_o J(q)\tilde{\nu}$$
$$-\tilde{\nu}^T M K_{o_2}(q)\tilde{q} - \frac{1}{2}\tilde{\nu}^T\left(D + D^T\right)\tilde{\nu} - \tilde{\nu}^T\left[d(\nu) - d(\hat{\nu})\right]$$

Since d satisfies (3) with $P = I$, we get

$$\begin{aligned} \dot{V}_o(t, \tilde{q}, \tilde{\nu}) \leq -\frac{1}{2}\tilde{q}^T(P_o K_{o_1} + K_{o_1}^T P_o)\tilde{q} \\ -\frac{1}{2}\tilde{\nu}^T\left(D + D^T\right)\tilde{\nu} \qquad (12) \end{aligned}$$

Clearly, for $D + D^T$ positive definite, the origin $(\tilde{q}, \tilde{\nu}) = (0, 0)$ of (9)–(10) is UGES, provided that K_{o_1} is chosen such that $P_o K_{o_1} + K_{o_1}^T P_o$ is positive definite. Now, what happens if the damping and friction forces cannot be represented by a positive definite $D + D^T$, that is, $D + D^T$ is only positive *semi*-definite? This is, for instance, the case for offshore vessels and structures for which the hydrodynamic drag is modelled by Morison's equation, and for ships not possessing straight-line stability. In [9], UGAS and ULES of the origin $(\tilde{q}, \tilde{\nu}) = (0, 0)$ of (9)–(10) is shown by using Theorem 1 in [5] (also in [10]), under the assumption that the trajectories of (5)-(6) are uniformly bounded. The idea of this theorem is to decompose the system into one part, for which the origin is ULES, plus a perturbation whose norm is bounded, on any compact subset of the state space, by a linear function of $\|\tilde{q}\|$. Direct application of this theorem requires a global Lipschitz condition to hold for d, but in our case, d is only locally Lipschitz. A small modification of the theorem, taking (3) into account, resolves this problem. The proof is given in [1].

Proposition 1 *Let $P_o = P_o^T > 0$, and suppose the observer gain matrices are chosen according to*

$$\begin{aligned} 0 &< P_o K_{o_1} + K_{o_1}^T P_o \\ K_{o_2}(q) &= M^{-1} J(q)^T P_o \end{aligned}$$

If the trajectories $\nu(t)$ are uniformly globally bounded, then the origin of (9)-(10) is uniformly globally asymptotically stable (UGAS) and uniformly locally exponentially stable (ULES). Moreover, if $D + D^T > 0$ then it is uniformly globally exponentially stable (UGES).

Under an additional assumption on $J(q)$, and by following the lines of Arcak and Kokotović [3], we can find an observer that renders the error dynamics UGES, even in the case of only positive semi-definite $D + D^T$.

Assumption 1 *A locally Lipschitz function, $\phi : \mathbb{R}^n \longrightarrow \mathbb{R}^n$, is known, such that*

$$\frac{\partial \phi}{\partial q}(q)\, J(q) + \left(\frac{\partial \phi}{\partial q}(q)\, J(q)\right)^T > \epsilon I, \quad \forall q \in \mathbb{R}^n$$

for some $\epsilon > 0$.

We will use the knowledge of such a function to perform a change of variables. Define the new variable as follows

$$y = \nu - \phi(q)$$

where ϕ satisfies Assumption 1. Writing the system (5)-(6) in terms of q and y, we obtain

$$\dot{q} = J(q)y + J(q)\phi(q)$$

$$\begin{aligned} \dot{y} = -M^{-1}\left(D(y + \phi(q)) + d(y + \phi(q)) + v(q) - \tau\right) \\ - \frac{\partial \phi}{\partial q}\left(J(q)y + J(q)\phi(q)\right) \end{aligned}$$

Now, consider the observer

$$\dot{\hat{q}} = J(q)\hat{y} + J(q)\phi(q) + K_{o_1}\tilde{q} \tag{13}$$

$$\begin{aligned} \dot{\hat{y}} = -M^{-1}\left(D(\hat{y} + \phi(q)) + d(\hat{y} + \phi(q)) + v(q) - \tau\right) \\ - \frac{\partial \phi}{\partial q}\left(J(q)\hat{y} + J(q)\phi(q)\right) + K_{o_2}(q)\tilde{q} \end{aligned} \tag{14}$$

with error dynamics, in terms of $\tilde{q} \triangleq q - \hat{q}$ and $\tilde{y} \triangleq y - \hat{y}$, as follows

$$\dot{\tilde{q}} = -K_{o_1}\tilde{q} + J(q)\tilde{y} \tag{15}$$

$$\begin{aligned} \dot{\tilde{y}} = -K_{o_2}(q)\tilde{q} - M^{-1}D\tilde{y} - \frac{\partial \phi}{\partial q} J(q)\tilde{y} \\ - M^{-1}\left[d(y + \phi(q)) - d(\hat{y} + \phi(q))\right] \end{aligned} \tag{16}$$

Proposition 2 *Suppose the observer gain matrices in (13)-(14) are chosen as in Proposition 1. If Assumption 1 holds, then the origin of (15)-(16) is uniformly globally exponentially stable (UGES).*

Proof: Using the Lyapunov function candidate $V_o(t, \tilde{q}, \tilde{y}) = \frac{1}{2}\left(\tilde{q}^T P_o \tilde{q} + \tilde{y}^T M \tilde{y}\right)$, and setting $K_{o_2}(q) = M^{-1}J(q)^T P_o$, the time derivative of V_o along the trajectories of (15)-(16) is

$$\begin{aligned} \dot{V}_o(t, \tilde{q}, \tilde{y}) = -\frac{1}{2}\tilde{q}^T(P_o K_{o_1} + K_{o_1}^T P_o)\tilde{q} \\ + \tilde{q}^T P_o J(q)\tilde{y} - \tilde{y}^T J(q)^T P_o \tilde{q} - \frac{1}{2}\tilde{y}^T\left(D + D^T\right)\tilde{y} \\ - \tilde{y}^T\left[d(y + \phi(q)) - d(\hat{y} + \phi(q))\right] \\ - \frac{1}{2}\tilde{y}^T\left(\frac{\partial \phi}{\partial q}J(q) + \left(\frac{\partial \phi}{\partial q}J(q)\right)^T\right)\tilde{y} \end{aligned}$$

Since d satisfies (3) with $P = I$, we get

$$\begin{aligned} \dot{V}_o(t, \tilde{q}, \tilde{\nu}) \leq -\frac{1}{2}\tilde{q}^T(P_o K_{o_1} + K_{o_1}^T P_o)\tilde{q} \\ - \frac{1}{2}\tilde{y}^T\left(\frac{\partial \phi}{\partial q}J(q) + \left(\frac{\partial \phi}{\partial q}J(q)\right)^T\right)\tilde{y} \end{aligned}$$

In view of Assumption 1, UGES of the origin $(\tilde{q}, \tilde{y}) = 0$ of (15)-(16) follows from standard results [6, Corollary 3.4]. ■

3.2 Controller design

Now, moving on to controller design, we define the virtual tracking error as

$$\hat{z} \triangleq J(q)\hat{\nu} - \dot{q}_d + K_p q_e$$

where $q_e \triangleq q - q_d$ and $K_p + K_p^T > 0$. Then,

$$\dot{q}_e = -K_p q_e + \hat{z} + J(q)\tilde{\nu} \tag{17}$$

and

$$\begin{aligned} \dot{\hat{z}} = \dot{J}(q, \dot{q})\hat{\nu} - \ddot{q}_d + K_p\left(J(q)\nu - \dot{q}_d\right) \\ + J(q)M^{-1}\left(\tau + M K_{o_2}(q)\tilde{q} - D\hat{\nu} - d(\hat{\nu}) - v(q)\right) \end{aligned} \tag{18}$$

The theorem below covers the general case, that is, when $D + D^T$ is only positive semi-definite, and we cannot find a ϕ that satisfies Assumption 1. In this case, we achieve UGAS of the origin of the error dynamics.

Theorem 1 *Let $P_o = P_o^T > 0$, and set $K_{o_2} = M^{-1}J(q)P_o$. Suppose that the matrices K_{o_1}, K_p, and K_d, are chosen such that*

$$\begin{aligned} P_o K_{o_1} + K_{o_1}^T P_o &> 0 \\ K_p &= K_p^T > 0 \\ K_d &= K_d^T > 0 \end{aligned}$$

Then, the nonlinear output feedback tracking controller

$$\begin{aligned}\tau = MJ(q)^{-1}\Big\{&-q_e - \left(K_d + \|q_e\|^2 I + \|\hat{z}\|^2 I\right)\hat{z} \\ &+\ddot{q}_d + K_p\dot{q}_d - K_pJ(q)\hat{\nu} - \dot{J}(q,J(q)\hat{\nu})\hat{\nu}\Big\} \\ &+ v(q) + D\hat{\nu} + d(\hat{\nu}) \quad (19)\end{aligned}$$

renders the origin $\xi = (\tilde{q}, \tilde{\nu}, q_e, \hat{z}) = 0$ of the error dynamics (9)-(10) and (17)-(18) uniformly globally asymptotically stable (UGAS).

Proof: Inserting the control law (19) into (18), we obtain

$$\begin{aligned}\dot{\hat{z}} = &-q_e - \left(K_d + \|q_e\|^2 I + \|\hat{z}\|^2 I\right)\hat{z} \\ &+ J(q)M^{-1}J(q)^T P_0\tilde{q} \\ &+ \left[\dot{J}(q,J(q)\nu) - \dot{J}(q,J(q)\hat{\nu})\right]\hat{\nu} + K_pJ(q)\tilde{\nu} \quad (20)\end{aligned}$$

The derivative of $V_c(t, q_e, \hat{z}) = \frac{1}{2}\|q_e\|^2 + \frac{1}{2}\|\hat{z}\|^2$ along the trajectories of (17) and (20) is

$$\begin{aligned}\dot{V}_c(t,q_e,\hat{z}) = &-q_e^T K_p q_e - \hat{z}^T K_d \hat{z} - \|q_e\|^2\|\hat{z}\|^2 - \|\hat{z}\|^4 \\ &+ \hat{z}^T J(q)M^{-1}J(q)^T P_0\tilde{q} \\ &+ \hat{z}^T\left[\dot{J}(q,J(q)\nu) - \dot{J}(q,J(q)\hat{\nu})\right]\hat{\nu} \\ &+ \hat{z}^T K_pJ(q)\tilde{\nu} + q_e^T J(q)\tilde{\nu} \quad (21)\end{aligned}$$

Using Properties 1 and 2, we get

$$\hat{z}^T\left[\dot{J}(q,J(q)\nu) - \dot{J}(q,J(q)\hat{\nu})\right]\hat{\nu} \le L_J k_J \|\hat{z}\|\|\tilde{\nu}\|\|\hat{\nu}\| \quad (22)$$

and, since $J(q)\hat{\nu} = \hat{z} + \dot{q}_d - K_p q_e$, we get

$$\|\hat{\nu}\| \le k_{J^{-1}}\|\hat{z}\| + k_{J^{-1}}\|K_p\|\|q_e\| + k_{J^{-1}}\beta_d \quad (23)$$

where $k_{J^{-1}}$ is an upper bound on $\left\|J(q)^{-1}\right\|$ (which exists by Property 1). Inequalities (22) and (23) imply

$$\begin{aligned}&\hat{z}^T\left[\dot{J}(q,J(q)\nu) - \dot{J}(q,J(q)\hat{\nu})\right]\hat{\nu} \le \\ &\quad L_J k_J k_{J^{-1}}\left(\|\hat{z}\| + \|K_p\|\|q_e\| + \beta_d\right)\|\hat{z}\|\|\tilde{\nu}\| \quad (24)\end{aligned}$$

Substitution of inequality (24), along with the inequalities

$$\hat{z}^T J(q)M^{-1}J(q)^T P_0\tilde{q} \le k_J^2\left\|M^{-1}\right\|\|P_0\|\|\hat{z}\|\|\tilde{q}\| \quad (25)$$

$$\hat{z}^T K_pJ(q)\tilde{\nu} \le k_J\|K_p\|\|\hat{z}\|\|\tilde{\nu}\| \quad (26)$$

$$q_e^T J(q)\tilde{\nu} \le k_J\|q_e\|\|\tilde{\nu}\| \quad (27)$$

into equation (21), yields

$$\begin{aligned}\dot{V}_c \le &-\frac{1}{2}q_e^T K_p q_e - \frac{1}{2}\hat{z}^T K_d\hat{z} \\ &+\left(-\frac{1}{4}\lambda_m(K_d)\|\hat{z}\|^2 + k_J^2\left\|M^{-1}\right\|\|P_0\|\|\hat{z}\|\|\tilde{q}\|\right) \\ &+\left(-\|q_e\|^2\|\hat{z}\|^2 + L_J k_J k_{J^{-1}}\|K_p\|\|q_e\|\|\hat{z}\|\|\tilde{\nu}\|\right) \\ &+\left(-\frac{1}{2}\lambda_m(K_p)\|q_e\|^2 + k_J\|q_e\|\|\tilde{\nu}\|\right) \\ &+\left(-\|\hat{z}\|^4 + L_J k_J k_{J^{-1}}\|\hat{z}\|^2\|\tilde{\nu}\|\right) + \\ &\left(\left(L_J k_J k_{J^{-1}}\beta_d + k_J\|K_p\|\right)\|\hat{z}\|\|\tilde{\nu}\| - \frac{1}{4}\lambda_m(K_d)\|\hat{z}\|^2\right)\end{aligned}$$

where $\lambda_m(K_?)$ denotes the smallest eigenvalue of the matrix $K_?$. Upon completion of squares, one can find a constant c such that

$$\dot{V}_c(t,q_e,\hat{z}) \le -\frac{1}{2}q_e^T K_p q_e - \frac{1}{2}\hat{z}^T K_d\hat{z} + c\|(\tilde{q},\tilde{\nu})\|^2$$

which proves, using standard results [6, Theorem 5.2], that the $(q_e, \hat{z})$-subsystem is ISS with input $(\tilde{q}, \tilde{\nu})$. From the proof of Proposition 1 (see [1]), $\|(\tilde{q}, \tilde{\nu})\|$ is uniformly globally bounded, so it follows that q_e and $\hat{z}$ are uniformly globally bounded, and therefore, ν is uniformly globally bounded. The theorem now follows from [6, Lemma 5.6] along with Proposition 1. ■

In the control law (19) of Theorem 1, the variables q_e and z are raised to the third power in order to dominate the estimation error. This may be undesirable due to practical issues such as measurement noise, saturation in the actuators and unmodeled actuator dynamics. The next result covers the case when an exponentially convergent observer is available, in which case we can achieve UGAS of the origin of the overall system with less control effort. Moreover, we achieve exponential convergence to any ϵ-neighborhood of the origin. Before we state the theorem, we need the following lemma.

Lemma 2 *Let $x = 0$ be an equilibrium point for the nonlinear system*

$$\dot{x} = f(t,x), \qquad x(t_0) = x_0 \quad (28)$$

where $f : [0,\infty) \times \mathbb{R}^n \to \mathbb{R}^n$ is piecewise continuous in t and locally Lipschitz in x. Let $V : [0,\infty) \times \mathbb{R}^n \to \mathbb{R}$ be a continuously differentiable function such that

$$k_1\|x\|^c \le V(t,x) \le k_2\|x\|^c \quad (29)$$

$$\begin{aligned}&\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}f(t,x) \le \\ &\quad -k_3\|x\|^c + g(\|x\|)\,\sigma(\|x(t_0)\|, t - t_0) \quad (30)\end{aligned}$$

$\forall t \geq t_0$, $\forall x \in \mathbb{R}^n$, where k_1, k_2, k_3, and c, are strictly positive constants, $g : \mathbb{R}_+ \rightarrow \mathbb{R}$ is continuous, and σ is a class $\mathcal{KL}$ function satisfying

$$\int_{t_0}^{\infty} \sigma(r, s)\, ds \leq \sigma_\infty r \tag{31}$$

for some constant σ_∞. Suppose that there exist constants $k > 0$, and $r \geq 0$ such that $k\,\|x\|^c \geq g(\|x\|)$, $\forall \|x\| \geq r$. Then, the equilibrium point $x = 0$ of (28) is uniformly globally asymptotically stable (UGAS).

Proof: First we show that solutions can be continued for all time. From (29) and (30), we get

$$\begin{aligned} \|x(t)\|^c &\leq \frac{V(t_0, x_0)}{k_1} + \frac{1}{k_1} \int_{t_0}^{t} \dot{V}(\tau, x) d\tau \\ &\leq \frac{k_2}{k_1} \|x_0\|^c + \frac{1}{k_1} \int_{t_0}^{t} g(\|x\|)\, \sigma(\|x_0\|, t - t_0)\, d\tau \end{aligned}$$

Define

$$\beta_r = \sup_{\|x\| < r} g(\|x\|)$$

By continuity of g on $\mathbb{R}_+$, β_r is finite. We then get

$$\|x(t)\|^c \leq \frac{k_2}{k_1} \|x_0\|^c + \frac{1}{k_1} \int_{t_0}^{t} (\beta_r + k\,\|x(\tau)\|^c)\, \sigma_0 d\tau$$

where we have set $\sigma_0 = \sigma(\|x_0\|, 0)$, and used the fact that $k\,\|x\|^c \geq g(\|x\|)$ for sufficiently large $\|x\|$. An application of Gronwall's inequality [6, Lemma 2.1] now shows that solutions exist for all $t \geq t_0$. We proceed to show that the solutions are in fact bounded. Since $\sigma \in \mathcal{KL}$ there exists a time $t_* \geq t_0$, dependent on the initial state, such that $\sigma(\|x_0\|, t_* - t_0) < \frac{k_3}{2k}$. Thus, we get for all $\|x\| \geq r$ and for all $t > t_*$, that

$$\dot{V} \leq -k_3 \|x\|^c + \frac{k_3 g(\|x\|)}{2k} \leq -\frac{k_3}{2} \|x\|^c$$

Again we have used the fact that $k\,\|x\|^c \geq g(\|x\|)$ for sufficiently large $\|x\|$. This shows that the set $\mathcal{S} = \{x : \|x\| \leq r\}$ is globally attractive, that is, the distance between x and $\mathcal{S}$ tends to 0 as $t \rightarrow \infty$, for all initial states x_0. Thus, boundedness of solutions follows. We finish the proof by showing uniform attractivity. Since the solutions are bounded, given β, there exists $\beta_b > 0$, such that for all $\|x_0\| < \beta$, $\|x(t)\| < \beta_b$ for all $t > t_0$. Now, define

$$\beta_g = \sup_{\|x\| < \beta_b} g(\|x\|)$$

By continuity of g on $\mathbb{R}_+$, β_g is finite. From (29)-(31), we get

$$\|x(t)\|^c \leq \frac{k_2}{k_1} \beta^c + \frac{\beta_g \sigma_\infty \beta}{k_1} + \int_{t_0}^{t} \left[-\frac{k_3}{k_1} \|x(\tau)\|^c \right] d\tau$$

Using Gronwall's inequality, it follows that

$$\|x(t)\| \leq \left(\frac{k_2}{k_1} \beta^c + \frac{\beta_g \sigma_\infty \beta}{k_1} \right)^{\frac{1}{c}} e^{-\frac{k_3}{c k_1}(t - t_0)} \tag{32}$$

■

Theorem 2 *Let $P_o = P_o^T > 0$ be a symmetric, positive definite matrix, and set $K_{o_2} = M^{-1} J(q) P_o$. Suppose that the matrices K_{o_1}, K_p, and K_d, are chosen such that*

$$\begin{aligned} P_o K_{o_1} + K_{o_1}^T P_o &> 0 \\ K_p &= K_p^T > 0 \\ K_d &= K_d^T > 0 \end{aligned}$$

If Assumption 1 holds (respectively, $D + D^T$ is positive definite), then, the nonlinear output feedback tracking controller

$$\begin{aligned} \tau = &M J(q)^{-1} \{ -q_e - K_d z + \ddot{q}_d + K_p \dot{q}_d \\ &-K_p J(q) \hat{\nu} - \dot{J}(q, J(q)\hat{\nu}) \hat{\nu} \} + v(q) + D\hat{\nu} + d(\hat{\nu}) \end{aligned} \tag{33}$$

renders the origin $\xi = (\tilde{q}, \tilde{y}, q_e, z) = 0$ (respectively, $\xi = (\tilde{q}, \tilde{\nu}, q_e, z) = 0$) of the error dynamics (15)-(16) (respectively, (9)-(10)) and (17)-(18) uniformly globally asymptotically stable (UGAS).

Proof: Inserting the control law (33) into (18), we obtain

$$\begin{aligned} \dot{\hat{z}} = &-q_e - K_d \hat{z} + J(q) M^{-1} J(q)^T P_0 \tilde{q} \\ &+ K_p J(q) \tilde{\nu} + \left[\dot{J}(q, J(q)\nu) - \dot{J}(q, J(q)\hat{\nu}) \right] \hat{\nu} \end{aligned} \tag{34}$$

The derivative of $V_c(t, q_e, \hat{z}) = \frac{1}{2} \|q_e\|^2 + \frac{1}{2} \|\hat{z}\|^2$ along the trajectories of (17) and (34) is

$$\begin{aligned} \dot{V}_c(t, q_e, \hat{z}) = &-q_e^T K_p q_e - \hat{z}^T K_d \hat{z} \\ &+ \hat{z}^T J(q) M^{-1} J(q)^T P_0 \tilde{q} \\ &+ \hat{z}^T \left[\dot{J}(q, J(q)\nu) - \dot{J}(q, J(q)\hat{\nu}) \right] \hat{\nu} \\ &+ \hat{z}^T K_p J(q) \tilde{\nu} + q_e^T J(q) \tilde{\nu} \end{aligned} \tag{35}$$

As in the proof of Theorem 1, substitution of inequalities (24), and (25)-(27) into equation (35), and noticing that $\|\hat{z}\| \leq \|\xi\|$ and $\|q_e\| \leq \|\xi\|$, yields

$$\dot{V}_c \leq -q_e^T K_p q_e - \hat{z}^T K_d \hat{z} + \left(c_1 \|\xi\| + c_2 \|\xi\|^2 \right) \|(\tilde{q}, \tilde{\nu})\|$$

where we have defined

$$\begin{aligned} c_1 &\triangleq k_J \left(k_J \|M^{-1}\| \|P_0\| + L_j k_{J^{-1}} \beta_d + \|K_p\| + 1 \right) \\ c_2 &\triangleq L_j k_J k_{J^{-1}} (1 + \|K_p\|) \end{aligned}$$

In view of Proposition 2 (respectively, the UGES part of Proposition 1), the positive definiteness of K_p and K_d, ensures the existence of a strictly positive constant c, such that the time derivative of $V = V_o + V_c$ is bounded above as follows

$$\dot{V} \leq -c\,||\xi||^2 + \left(c_1\,||\xi|| + c_2\,||\xi||^2\right)||(\tilde{q},\tilde{\nu})||$$

Proposition 2 (respectively, the UGES part of Proposition 1) provides the following bound on $||(\tilde{q},\tilde{\nu})||$

$$||(\tilde{q},\tilde{\nu})|| \leq k_1\,||\xi(t_0)||\,e^{-\gamma(t-t_0)}$$

where k_1 and γ are strictly positive constants. Thus, we get

$$\dot{V} \leq -c\,||\xi||^2 + g(||\xi||)\sigma(||\xi(t_0)||\,, t - t_0)$$

where we have defined

$$g(\rho) \triangleq c_1\rho + c_2\rho^2$$

and

$$\sigma(\rho, \tau) \triangleq k_1\rho e^{-\gamma\tau}$$

Clearly, σ is integrable in its second argument on $\mathbb{R}_+$. Also, if we pick $r = 1$ and $k_2 = c_1 + c_2$, we have $g(\rho) \leq k_2\rho^2$, for all $\rho \geq r$. Thus, we can apply Lemma 2 to conclude that the origin $\xi = 0$ is UGAS. ■

Remark 1 *Notice from (32), that for any $\epsilon > 0$, we can find a constant k_ϵ, such that*

$$||x(t)|| \leq k_\epsilon\,||x(t_0)||\,e^{-\frac{k_3}{ck_1}(t-t_0)},\ \ \forall\,||x(t)|| > \epsilon$$

In other words, we have exponential convergence of the trajectories of (15)-(16) (respectively, (9)-(10)) and (17)-(18) to any ϵ-neighborhood of the origin.

Remark 2 *Notice that in both theorems above, a separation principle holds in the sense that the proposed controller, which clearly stabilizes the system when the full state is available, in conjunction with the proposed observer, renders the origin of the overall system UGAS. In addition, the controller gains and the observer gains can be chosen independently of each other.*

4 Conclusions

We have addressed the problem of output feedback tracking control of a class of Euler-Lagrange systems subject to nonlinear dissipative loads. By imposing a monotone damping condition on the nonlinearities of the unmeasured states, the common restriction that the nonlinearities be globally Lipschitz is removed. The proposed observer-controller scheme renders the origin of the error dynamics uniformly globally asymptotically stable, in the general case. Under certain additional assumptions, the result continue to hold for a simplified control law that gives less control effort.

References

[1] O.M. Aamo, M. Arcak, T.I. Fossen and P.V. Kokotović. Global Output Tracking Control of a Class of Euler-Lagrange Systems With Monotonic Nonlinearities in the Velocities. Preprint, 2000.

[2] M. Arcak and P.V. Kokotović. Observer-based stabilization of systems with monotonic nonlinearites. *Asian Journal of Control,* Vol. 1, pp. 42-48, 1999.

[3] M. Arcak and P.V. Kokotović. Noninear Observers: A Circle Criterion Design. In Proceedings of the 38th IEEE Conference on Decision and Control, Phoenix, Arizona, 1999.

[4] G. Besançon. Simple global output feedback tracking control for one-degree-of-freedom Euler-Lagrange systems. In Proceedings of the IFAC Conference on Systems Structure and Control, 1998.

[5] T.I. Fossen, A. Loria, and A. Teel. A Theorem for UGAS and ULES of Nonautonomous Systems: Robust Control of Mechanical Systems and Ships. Submitted to: Int. J. Rob. & Nonlinear Contr.

[6] H.K. Khalil. *Nonlinear Systems, 2nd Ed.* Prentice-Hall, 1996.

[7] S.R. Kou, D.L. Elliott, and T.J. Tarn. Exponential observers for nonlinear dynamic systems. Information and Control, Vol. 29, pp. 204-216, 1975.

[8] A. Loria. Global tracking control of one degree of freedom Euler-Lagrange systems without velocity measurements. European Journal of Control, Vol. 2, pp. 144-151, 1996.

[9] A. Loria and E. Panteley. A Separation Principle for a Class of Euler-Lagrange Systems. In H. Nijmeijer and T.I. Fossen (Eds.): *New Directions in Nonlinear Observer Design,* Springer-Verlag London, 1999.

[10] A. Loria, T.I. Fossen, and A. Teel. UGAS and ULES of Nonautonomous Systems: Applications to Integral Control of Ships and Manipulators. In Proceedings of the European Control Conference, Karlsruhe, Germany, 1999.

[11] J.M. Ortega and W.C. Rheinboldt. *Iterative solution of nonlinear equations in several variables.* Academic Press, New York, 1970.

[12] S. Raghavan and J.K. Hedrick. Observer design for a class of nonlinear systems. International Journal of Control, Vol. 59, pp. 515-528, 1994.

[13] R. Rajamani. Observers for Lipschitz nonlinear systems. IEEE Transactions on Automatic Control, Vol. 43, pp. 397-401, 1998.

[14] F.E. Thau. Observing the state of nonlinear dynamic systems. International Journal of Control, Vol. 17, pp. 471-479, 1973.