Passivity-Based Designs for Synchronized Path Following

Ivar-André F. Ihle[1], Murat Arcak[2] and Thor I. Fossen[1,3]

Abstract— **We consider a formation control system where individual systems are controlled by a path-following design and the path variables are to be synchronized. We first show a passivity property for the path following system and, next, combine this with a passivity-based synchronization algorithm developed in [1]. The passivity approach expands the classes of synchronization schemes available to the designer. This generality offers the possibility to optimize controllers to, e.g., improve robustness and performance. Two designs are developed in the proposed passivity framework: The first employs the path error information in the synchronization loop, while the second only uses synchronization errors. A sampled-data design, where the path variables are updated in discrete-time and the path following controllers are updated in continuous time, is also developed.**

I. Introduction

Recent advances in control techniques for vehicles and communication capabilities have sparked an interest in formation control that has resulted in a number of publications with applications ranging from autonomous underwater vehicles (AUVs) to multiple satellite control [2], [3], [4]. Examples of marine cooperative control include autonomous oceanographic sampling networks and under-way replenishment operations between marine surface vessels.

In these applications, communication constraints and disturbances due to the environment pose challenges for control design [5]. Robustness with respect to disturbances are important for formations of vehicles, and especially in marine applications where environmental disturbances, such as waves, wind, and ocean currents, can influence vehicles motion. In addition, group coordination is highly dependent on the communication channels as information necessary for coordination might arrive delayed and destabilize an otherwise stable system.

The primary goal in *path-following* problems is to design control laws that force the output of a system to follow a desired *path*. The secondary goal for the system is to obey a dynamic assignment such as time, speed or acceleration along the path [6], [7], [8], [9]. In particular the formulation in [10] aims to develop control laws that force a system to follow a prescribed path, parameterized by a path variable θ, and that assign a desired speed to be achieved by $\dot{\theta}$ in the limit as $t \to \infty$. In the classical tracking problem the path variable θ is designed to follow a specific time function, such that the system output tracks a time-dependent signal. In the path following formulation studied in this paper, the priority is to converge to the path while the desired speed profile is to be achieved by $\dot{\theta}$ in the limit. Among other advantages, the path following approach offers flexibility to shape the transient behavior of $\dot{\theta}$.

In this paper, we exploit this flexibility to synchronize the path variables for a group of path-following systems. In particular, we make use of a passivity-based framework for formation control proposed in [1]. This framework allows us to obtain a broad class of synchronization schemes for a general communication topology, and encompasses earlier designs such as [11]. In this framework, we represent the closed-loop system as the feedback interconnection of a dynamic block for path variable synchronization and another block that incorporates the path following systems. We design each block to be *passive* and prove stability of their interconnection by using the Passivity Theorem which states that an interconnection of two passive blocks is passive and, thus, stable in the absence of exogenous inputs.

A major advantage of the passivity approach is that it allows the designer to construct filters that preserve passivity properties of the closed-loop system. This additional flexibility may be used to improve the performance and robustness of the design. We further consider a sampled-data framework where the synchronization scheme is implemented in discrete time while the path-following controllers are continuous-time systems. This formulation is meaningful because communication of path parameters between vehicles will likely occur over a digital network which introduces delays, while the path-following controllers are implemented locally in continuous-time or with fast sampling.

The rest of this paper is organized as follows: Section II gives an overview of the path-following problem. Section III presents passivation designs for synchronization and gives the main stability results. A sampled-data scheme for synchronization and path following is given in Section IV, followed by examples of filter design and simulations in Section V. Concluding remarks are given in Section VI. In abbreviations like GS, GAS, UGAS etc, U stands for for Uniform, G for Global, A for Asymptotic, and S for Stable.

II. Path-Following Design and Synchronization

We introduce the concept of a *parameterized path*, that is, a geometric curve

$$Y_d := \{y \in \mathbb{R}^m : \exists \theta \in \mathbb{R} \text{ such that } y = y_d(\theta)\}$$

where y_d is continuously parameterized by the path variable θ [10]. Consider a general system

$$\begin{aligned} \dot{x} &= f(x,u) \\ y &= h(x) \end{aligned} \tag{1}$$

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[1]*Centre for Ships and Ocean Structures, Norwegian University of Science and Technology (NTNU), NO-7491 Trondheim, Norway.*

[2]*Department of Electrical, Computer, and Systems Engineering, Rensselaer Polytechnic Institute, Troy NY 12180, USA.*

[3]*Department of Engineering Cybernetics, Norwegian University of Science and Technology, NO-7491 Trondheim, Norway.*

E-mails: ihle@ntnu.no, arcakm@rpi.edu, fossen@ieee.org.

where $x \in \mathbb{R}^n$ denotes the state vector, $y \in \mathbb{R}^m$ is the system output, and $u \in \mathbb{R}^n$ is the control. To force y to a prescribed feasible path $y_d(\theta)$, and to assign a feasible speed $\upsilon(t)$ to $\dot{\theta}$ on this path, [10] studies subclasses of (1) and develops maneuvering design procedures based on feedback linearization and backstepping techniques. Other techniques for path-following are considered in, e.g., [6], [12], [7], [13], [8] and [10]. The designs in [10] lead to a closed-loop system of the form

$$\begin{aligned} \dot{z} &= F(x)\,z - g(t, x, \theta)\,\omega \\ \dot{\theta} &= \upsilon(t) - \omega \end{aligned} \tag{2}$$

where z is a set of new parameters that include the tracking error $y - y_d(\theta)$ and its derivatives, and ω is a feedback term to be designed such that the desired speed $\upsilon(t)$ is recovered asymptotically; that is

$$\omega \to 0 \text{ as } t \to \infty. \tag{3}$$

$F(x) \in \mathbb{R}^{n\times n}$ and $g(t, x, \theta) \in \mathbb{R}^n$ depend on the control design and, in particular, $F(x)$ satisfies

$$PF(x) + F(x)^\top P \leq -I \tag{4}$$

for some matrix $P = P^\top > 0$. A path-following design for marine vehicles is presented in Section V.

Assumption A1: We assume that the function $g(t, x, \theta)$ is uniformly bounded in its arguments – see [10, Assumption 2.1].

In this paper, we consider a group of vehicles $i = 1, \ldots r$, each controlled by an individual path-following design with a prescribed velocity $\upsilon(t)$ assigned to the group, resulting in the closed loop system

$$\begin{aligned} \dot{z}_i &= F_i(x_i)\,z_i - g_i(t, x_i, \theta)\,\omega_i \\ \dot{\theta}_i &= \upsilon(t) - \omega_i. \end{aligned} \tag{5}$$

Our goal is to design ω_i to synchronize the path variables θ_i, $i = 1, \ldots, r$, while achieving (3). The design of ω_i depends on variables of the ith system and on the path parameters for the neighboring vehicles, so only one scalar variable needs to be transmitted from each vehicle. The communication topology between the members of the formation is described by a graph $\mathcal{G}$. Two members, i and j, are *neighbors* if they can access the synchronization error $\theta_i - \theta_j$. In this case, we let the ith and jth vertices of $\mathcal{G}$ be connected by an edge. The information flow is bidirectional, but to simplify the derivation we assign an orientation to the graph by considering one of the vertices to be the positive end of the edge. For a group of r members with p edges, the $r \times p$ incidence matrix $D(\mathcal{G})$ is defined as

$$d_{ik} = \begin{cases} +1 & \text{if } i\text{th vertex is the positive end of the } k\text{th edge} \\ -1 & \text{if } i\text{th vertex is the negative end of the } k\text{th edge} \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

Assumption A2: We assume that $\mathcal{G}$ is connected, i.e. a path exists between every two distinct vertices of $\mathcal{G}$.

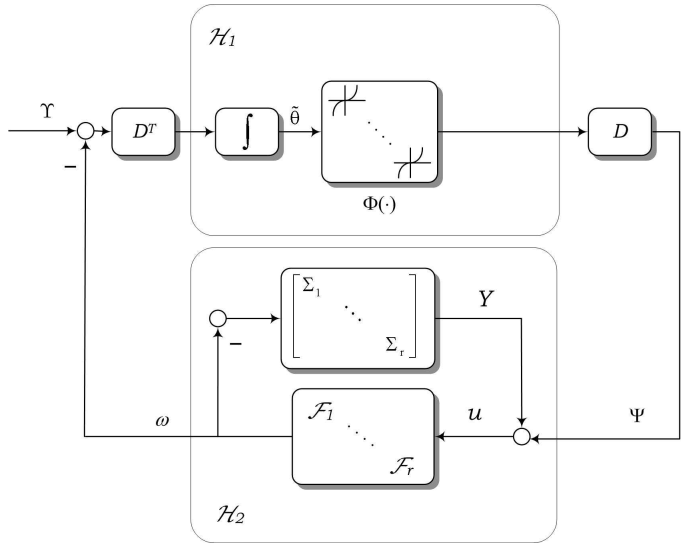

Fig. 1. Block diagram for the synchronized path following control system. Υ is a $r \times 1$ vector with each entry equal to $\upsilon(t)$.

III. Passivation Designs for Synchronization

A. Design 1: With Path Error Feedback

When the path error is available for feedback we design $\omega = [\omega_1, \ldots, \omega_r]^\top$ as

$$\omega_i = \mathcal{F}_i\left\{2z_i^\top P_i g_i + \psi_i(\theta)\right\} \tag{7}$$

where all path parameters are collected in the vector $\theta = [\theta_1, \ldots, \theta_r]^\top$, $\mathcal{F}_i\{\cdot\}$ denotes the output of a static or dynamic block, which will be specified. This $\mathcal{F}_i$ can be a filter added to enhance performance and robustness properties, as illustrated with examples in Section V. The input to this filter is

$$u_i := 2z_i^\top P_i g_i + \psi_i(\theta) \tag{8}$$

where the first component is the path error feedback $2z_i^\top P_i g_i$, which serves to improve convergence properties to the desired path [14]. The second component ψ_i is for the synchronization of the path parameters, and is designed as

$$\psi_i(\theta) = \sum_{k=1}^{p} d_{ik}\phi_k(\theta_i - \theta_k) \tag{9}$$

where ϕ_k is a sector nonlinearity, that is

$$x\phi_k(x) > 0 \quad \forall x \neq 0, \quad \text{and} \quad \lim_{|x|\to\infty} \int_0^x \phi_k(\sigma)d\sigma = +\infty. \tag{10}$$

The feedback law (7) is implementable with local information because it depends only on the neighbors of the ith member ($d_{ik} \neq 0$). With ω as in (7), the total closed-loop system can be represented as in Figure 1, where Σ_i is the z_i-subsystem as in (5),

$$\begin{aligned} &\tilde{\theta} := D^\top \theta,\ \mathcal{F} := \text{diag}(\mathcal{F}_1, \ldots, \mathcal{F}_r), \\ &\Phi(\cdot) = [\phi_1(\cdot), \ldots, \phi_p(\cdot)]^\top, \Psi(\theta) = [\psi_1, \ldots, \psi_r]^\top, \end{aligned} \tag{11}$$

and

$$Y := 2z^\top PG, \quad u = Y + \Psi \tag{12}$$

where $z := [z_1^\top, \ldots, z_r^\top]^\top, P := \mathrm{diag}(P_1, \ldots, P_r)$, for $P_i = P_i^\top > 0$, and $G := \mathrm{diag}(g_1, \ldots, g_r)$. In particular, note from (6) that $\tilde{\theta}$ in (11) is a vector that consists of the differences between the path parameters of neighboring vehicles. Because the graph $\mathcal{G}$ is connected, $\tilde{\theta} = 0$ is achieved if and only if all path parameters are synchronized.

We investigate stability properties of the closed-loop by separating it into two blocks, $\mathcal{H}_1$ and $\mathcal{H}_2$, as in Figure 1, and analyze passivity properties of each block. The Passivity Theorem guarantees that the negative feedback interconnection of two passive systems is passive and, thus, stable in the absence of exogenous inputs [15]. Although Υ appears as an exogenous input in Figure 1, it lies in the null space of the proceeding block D^T because its entries are identical. Following [1] we now characterize the properties that $\mathcal{F}_i$'s must possess to ensure passivity of $\mathcal{H}_2$ and appropriate detectability conditions that are used to guarantee asymptotic convergence. If $\mathcal{F}_i$ is a static block we restrict it to be of the form

$$\omega_i = h_i(u_i), \tag{13}$$

where $h_i : \mathbb{R}^p \to \mathbb{R}^p$ is a locally Lipschitz function satisfying the sector property

$$u_i h_i(u_i) > 0 \quad \forall u_i \neq 0. \tag{14}$$

If $\mathcal{F}_i$ is a dynamic block of the form

$$\begin{aligned} \dot{\xi}_i &= f_i(\xi_i) + g_i(\xi_i) u_i \quad \xi_i \in \mathbb{R}^{n_i} \\ \omega_i &= h_i(\xi_i) + j_i(\xi_i) u_i \end{aligned} \tag{15}$$

we assume $f_i(\cdot)$, $g_i(\cdot)$, $h_i(\cdot)$ and $j_i(\cdot)$ are locally Lipschitz functions such that $f_i(0) = 0$ and $h_i(0) = 0$. Our main restriction on (15) is that it be passive with a twice continuously differentiable, positive definite, radially unbounded storage function $S_i(\xi_i)$ satisfying

$$\dot{S}_i \leq -W_i(\xi_i) + u_i\omega_i - \nu_i u_i^2 \quad \nu_i \geq 0 \tag{16}$$

for some positive definite function $W_i(\xi_i)$. Inequality (16) with $\nu_i > 0$ is an *input-strict passivity* property which is possible only when the relative degree of (15) is zero. Our asymptotic stability proof below allows $\nu_i = 0$ provided that (15) have a well defined relative-degree-one at $\xi_i = 0$. According to [16, Proposition 2.44], this is indeed the case if

$$j_i(\xi_i) \equiv 0, \quad g_i(0) \neq 0, \quad \left.\frac{\partial h_i(\xi_i)}{\partial \xi_i}\right|_{\xi_i=0} \neq 0. \tag{17}$$

We thus make the following assumption:
Assumption A3: If $\nu_i = 0$ in (16) then (17) holds.

With $\mathcal{H}_1$ and $\mathcal{H}_2$ designed as above, we prove UGAS for $(\tilde{\theta}, z, \xi) = 0$ in Theorem 1 below. This UGAS property implies that in the limit as $t \to \infty$ the path parameters θ are synchronized ($\tilde{\theta} \to 0$) and that each system i follows its desired path ($z_i \to 0$). Furthermore $\omega \to 0$ which means that $\dot{\theta}$ in (2) recovers the speed assignment $v(t)$.

Theorem 1: Consider the feedback interconnection shown in Figure 1 where members $i = 1, \ldots, r$ are interconnected in a formation as described by (6), $\phi_k,\ k = 1, ..., p$ is as in (10), and $\mathcal{F}_i,\ i = 1, ..., r$ are designed as in (13)-(16). Under Assumptions A1-A3 the feedforward path $\mathcal{H}_1$ is passive from $\dot{\tilde{\theta}}$ to Φ, and the feedback path $\mathcal{H}_2$ is strictly passive from Ψ to ω. Furthermore the origin of the feedback interconnection $(\tilde{\theta}, z, \xi) = 0$ is UGAS.

Proof: We combine ideas from [11] and specific results for path following from [10]. To prove passivity from $\dot{\tilde{\theta}}$ to Φ, let

$$V_\psi := \sum_{k=1}^{p} \int_0^{\tilde{\theta}_k} \phi_k(\sigma)\, d\sigma. \tag{18}$$

Since ϕ_k is as in (10), V_ψ is a positive definite, radially unbounded storage function for $\mathcal{H}_1$. Differentiating (18) with respect to time yields

$$\dot{V}_\psi = \sum_{i=1}^{p} \phi_i(\theta_i - \theta_{i+1}) \cdot (\dot{\theta}_i - \dot{\theta}_{i+1}) = \Phi^\top \cdot \dot{\tilde{\theta}}, \tag{19}$$

which proves that the mapping from $\dot{\tilde{\theta}}$ to Φ is indeed passive. It follows that the path from $-\omega$ to Ψ is also passive. To see this substitute $\dot{\tilde{\theta}} = D(\Upsilon - \omega)$ in (19), and note that the sum of the rows of D being zero and the entries of Υ being equal imply $D\Upsilon = 0$. Thus,

$$\dot{V}_\psi = -\Phi^\top D^\top \omega = -\Psi^\top \omega. \tag{20}$$

To prove passivity of the feedback path, we first consider the Σ_i-blocks. The storage function

$$V_z = \sum_{i=1}^{r} z_i^\top P_i z_i \tag{21}$$

where P_i is as in (4) yields the following time-derivative along the trajectories of z

$$\dot{V}_z \leq -\left(\sum_{i=1}^{r} z_i^\top z_i\right) - Y^\top \omega \tag{22}$$

which proves that the Σ-block is strictly passive from $-\omega$ to Y. To establish passivity of $\mathcal{F}$, we let $\mathcal{I}$ denote the subset of indices $i = 1, \ldots, r$ for which $\mathcal{F}_i$ is a dynamic block as in (15), and employ the storage function

$$V_f := \sum_{i \in \mathcal{I}} S_i(\xi_i) \tag{23}$$

which yields

$$\dot{V}_f = \sum_{i \in \mathcal{I}} \dot{S}_i \leq \sum_{i \in \mathcal{I}} -(W_i(\xi_i) + u_i\omega_i) \tag{24}$$

$$\leq -\left(\sum_{i \in \mathcal{I}} W_i(\xi_i)\right) + u^\top \omega - \sum_{i \notin \mathcal{I}} u_i \omega_i. \tag{25}$$

Substitution of $u = Y + \Psi$ then yields

$$\dot{V}_f \leq -\left(\sum_{i \in \mathcal{I}} W_i(\xi_i)\right) + Y^\top \omega + \Psi^\top \omega - \sum_{i \notin \mathcal{I}} u_i \omega_i. \tag{26}$$

To conclude passivity of the feedback path we use the storage function

$$V_{fb}(z, \xi) := V_z(z) + V_f(\xi) \tag{27}$$

and, obtain by adding (22) and (26),

$$\dot{V}_{fb} \leq -\left(\sum_{i\in\mathcal{I}} W_i\left(\xi_i\right)\right) - \left(\sum_{i=1}^{r} z_i^\top z_i\right) + \Psi^\top \omega \\ -\sum_{i\notin\mathcal{I}} u_i\omega_i - \sum_{i\in\mathcal{I}} \nu_i u_i^2. \tag{28}$$

Finally, since the static blocks satisfy (14),

$$\sum_{i\notin\mathcal{I}} u_i\omega_i = \sum_{i\notin\mathcal{I}} u_i h_i(u_i) \geq 0. \tag{29}$$

We thus obtain

$$\dot{V}_{fb} \leq -\left(\sum_{i\in\mathcal{I}} W_i\left(\xi_i\right)\right) - \left(\sum_{i=1}^{r} z_i^\top z_i\right) + \Psi^\top \omega \tag{30}$$

and conclude that the feedback path is strictly passive from Ψ to ω.

To prove stability of $(\tilde{\theta}, z, \xi) = 0$ we use the Lyapunov function

$$V(\tilde{\theta}, z, \xi) = V_\psi(\tilde{\theta}) + V_{fb}\left(z, \xi\right) \tag{31}$$

which from (20) and (28), gives the time-derivative:

$$\dot{V} \leq -\left(\sum_{i\in\mathcal{I}} W_i\left(\xi_i\right)\right) - \left(\sum_{i=1}^{r} z_i^\top z_i\right) \\ -\sum_{i\notin\mathcal{I}} u_i h_i(u_i) - \sum_{i\in\mathcal{I}} \nu_i u_i^2. \tag{32}$$

Since the right-hand side is negative semidefinite we conclude that the trajectories $(z(t), \xi(t), \tilde{\theta}(t))$ are uniformly bounded on the interval $t \in [t_0, t_0 + T]$, for any T within the maximal interval of existence. Due to the uniform boundedness of the speed assignment $\upsilon(t)$, it follows that $(\theta(t), x(t))$ is bounded by a continuous function of T and, thus, there are no finite escape times. This implies that $\tilde{\theta}(t)$ and $z(t)$ are well defined for all $t \geq t_0$ and, from (32), the equilibrium $(z, \xi, \tilde{\theta}) = 0$ is uniformly stable.

To prove uniform asymptotic stability we use the Nested Matrosov Theorem [17]. To this end we define the auxiliary function

$$V_2 = -\tilde{\theta}^\top D^+ \Lambda\omega \tag{33}$$

where D^+ denotes the pseudo-inverse of the incidence matrix D, and Λ is a diagonal matrix with entries

$$\Lambda_{ii} = \begin{cases} (L_{g_i} h_i(0))^{-1} & \text{if} \quad i \in \mathcal{I} \quad \text{and} \quad \nu_i = 0 \\ 0 & \text{if} \quad i \notin \mathcal{I} \quad \text{or} \quad \nu_i > 0. \end{cases} \tag{34}$$

In particular $L_{g_i} h_i(0) := \left.\frac{\partial h_i(\xi_i)}{\partial \xi_i}\right|_{\xi_i=0} g_i(0)$ is nonsingular and, thus, invertible because of the passivity of the ξ_i-subsystems and because of assumption (17) [16, Proposition 2.44]. To apply Matrosov's Theorem we denote by Y_1 the right-hand side of (32) and claim that

$$Y_1 = 0 \quad \Rightarrow \quad \dot{V}_2 =: Y_2 \leq 0. \tag{35}$$

To see this note that $Y_1 = 0$ implies $\xi = 0$ and $\omega = 0$, which mean that all terms in $\dot{V}_2$ vanish except

$$-\tilde{\theta}^\top D^+ \Lambda \left.\dot{\omega}\right|_{Y_1=0}. \tag{36}$$

Because $\left.\dot{\omega}_i\right|_{\xi=0} = L_{g_i} h_i(0) u_i$ when $i \in \mathcal{I}$ and $\nu_i = 0$, and because $Y_1 = 0$ implies $u_i = 0$ when $i \notin \mathcal{I}$ or $\nu_i > 0$, we conclude from (34) that $\Lambda \left.\dot{\omega}\right|_{Y_1=0} = u$ and rewrite (36) as

$$-\tilde{\theta}^\top D^+ u. \tag{37}$$

We then substitute $\tilde{\theta} = D^\top \theta$ in (37), and using the property $DD^+D = D$ of the pseudo-inverse, and noting that $Y_1 = 0$ means $z = 0$ which in turn implies $u = \Psi = D\Phi(\tilde{\theta})$, we conclude

$$Y_1 = 0 \quad \Rightarrow \\ Y_2 = -\theta^T DD^+D\Phi(\tilde{\theta}) = -\theta^\top D\Phi(\tilde{\theta}) = -\tilde{\theta}^\top \Phi(\tilde{\theta}). \tag{38}$$

Because $\tilde{\theta}^\top \Phi(\tilde{\theta})$ is positive definite in $\tilde{\theta}$ from (10), equation (38) proves the claim (35). It further follows from (32) and (38) that $Y_1 = 0$ and $Y_2 = 0$ together imply $(z, \xi, \tilde{\theta}) = 0$. All assumptions of [17, Theorem 1] being satisfied we conclude UGAS of $(z, \xi, \tilde{\theta}) = 0$. $\square$

Remark 2: A special case of (7), studied in [11], is when the neighbors for system i are system $i-1$ and system $i+1$, i.e., the synchronization error is $\tilde{\theta} = [\theta_1 - \theta_2, \ldots, \theta_{r-1} - \theta_r]^\top$. Reference [11] further assumes that $\mathcal{F}$ is a constant gain matrix and $\phi_i\left(r\right) = r^{2p-1}$, for $p = 1, 2, 3, \ldots$ In contrast, in this paper we have considered a general communication topology, and derived relaxed conditions on the feedback block and synchronization function. In particular, $\mathcal{F}_i$'s are not necessarily constant gains and ϕ_i's are not necessarily polynomials.

B. Design 2: Without Path Error Feedback

We next consider a design where ω_i only depends on the synchronization terms, and not on the path error. The update law for ω_i is now

$$\omega_i = \mathcal{F}_i\left\{\psi_i(\theta)\right\} \tag{39}$$

where $\mathcal{F}_i$ and ψ_i are as in Section III-A. Without the path error feedback, the closed-loop system becomes a cascade of $\mathcal{H}_{\text{sync}}$ and Σ as in Figure 2. The origin of $\mathcal{H}_{\text{sync}}$, $(\tilde{\theta}, \xi) = 0$, is proved to be GAS in [1] which means that $\omega \to 0$. In Theorem 3 below we prove that the Σ-block is Input-to-State Stable (ISS) [18] w.r.t. ω. Stability of the closed-loop system then follows because a cascade of an ISS and a UGAS system is UGAS [19], [18].

Theorem 3: Consider the cascaded system in Figure 2, where members $i = 1, \ldots, r$ are interconnected in a formation as described by (6), $\phi_k,\ k = 1, ..., p$ are as in (10), and $\mathcal{F}_i,\ i = 1, ..., r$ are designed as in (13)-(16). Then, the origin of $\mathcal{H}_{\text{sync}}$-block is GAS, the Σ block is ISS with respect to ω, and the origin $(\tilde{\theta}, \xi, z) = 0$ is UGAS.

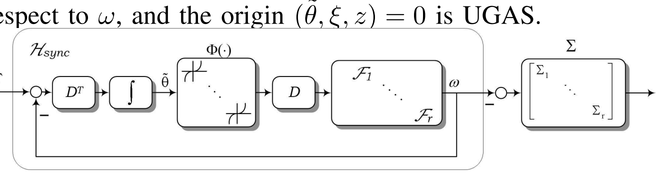

Fig. 2. Cascade interconnection for Design 2.

Proof: For completeness, the stability proof for $\mathcal{H}_{\text{sync}}$ is given here: The feedforward path of the $\mathcal{H}_{\text{sync}}$-block is an interconnection of a passive and a strictly passive block. Since pre- and post-multiplication of a matrix and its transpose does not change passivity properties the forward path is passive and with negative feedback the block is output strictly passive from ν to ω. Moreover, when $\Upsilon = 0$, due to (17):

$$\omega \equiv 0 \Rightarrow \mathcal{F}\left\{D\Phi(\tilde{\theta})\right\} \equiv 0 \Rightarrow D\Phi(\tilde{\theta}) \equiv 0 \qquad (40)$$

which implies that $\Phi(\tilde{\theta})$ lies in the nullspace $\mathcal{N}(D)$. When D has linearly independent columns $\mathcal{N}(D) = 0$ and hence $\Phi(\tilde{\theta}) \equiv 0 \Rightarrow \tilde{\theta} \equiv 0$ due to (10). When D has linearly dependent columns, the null space of D is nontrivial. However, a simultaneous solution to $\Phi(\tilde{\theta}) \in \mathcal{N}(D)$ and $\tilde{\theta} \in \mathcal{R}\left(D^\top\right)$ is possible only when $\tilde{\theta} = 0$. This is because $\mathcal{R}\left(D^\top\right)$ and $\mathcal{N}(D)$ are orthogonal to each other, which means $\tilde{\theta}^\top \Phi(\tilde{\theta}) = 0$, and we conclude from (10) that $\tilde{\theta} = 0$. Hence, the $\mathcal{H}_{\text{sync}}$-block is zero-state observable [15]. From [15, Lemma 6.7] we obtain GAS of the origin $\tilde{\theta} = 0$.

To prove the ISS-property we rewrite the time-derivative (22) as

$$\begin{aligned} \dot{V}_z &\leq \sum_{i=1}^{r} -z_i^\top z_i - 2z_i^\top P_i g_i \omega_i \\ &\leq \sum_{i=1}^{r} -|z_i|^2 + 2p_{iM}\delta_{gi}|z_i||\omega_i| \end{aligned}$$

where δ_{gi} is an upper bound on g_i due to Assumption A1. Furthermore, we get

$$|z_i| \geq \frac{2p_{iM}\delta_{gi}|\omega_i|}{\varepsilon} \Rightarrow \dot{V}_z \leq \sum_{i=1}^{r} -(1-\varepsilon)|z_i|^2 \qquad (41)$$

where $0 < \varepsilon < 1$. Thus, the system is ISS [15] from ω_i to z_i. Since the origin of $\mathcal{H}_{\text{sync}}$ is GAS and Σ is ISS with respect to ω, it follows from [19] that the origin $(\tilde{\theta}, \xi, z) = 0$ is UGAS. $\square$

Remark 4: Given the results on agreement protocols in [1], the results can be extended to a time-varying communication topology given by the incidence matrix $D(t)$ as long as $\mathcal{G}$ remains connected for all $t > 0$. A further result in [1] allows the graph to lose connectivity pointwise in time as long as it is established in a an integral sense – see [1] for details. This means that signal dropouts in the communication links can be tolerated if connectivity is eventually re-established.

IV. Sampled-Data Design with Discrete-Time Update for $\tilde{\theta}$

We now study the situation where the path parameters θ_i are updated in discrete-time. Such an implementation is practically relevant, because path parameters are to be exchanged over a communication network where the transmission occurs at discrete time-intervals. Since the path following controllers can be implemented locally by each vehicle with fast sampling, we consider the Σ-blocks in Figure 1 to be continuous time. This implementation thus results in the sampled-data closed-loop system:

$$\begin{aligned} \dot{z}_i &= F_i(x_i) z_i - g_i\left(t, x_i, \theta^{\text{zoh}}\right)\omega_i^{\text{zoh}} \\ \theta_i((n+1)T) &= \theta_i(nT) + \upsilon(nT) - \omega_i(nT) \end{aligned} \qquad (42)$$

where $n = 0, 1, 2, \ldots$ is the time index, and ω_i^{zoh} and θ^{zoh} denote the zero-order-hold equivalent continuous-time signals generated from the discrete-time signals θ and ω_i.

Because $\tilde{\theta}$ is now updated in discrete-time the integral block in the feedforward path of Figure 1 is replaced by a summation block, and $\dot{\tilde{\theta}}$ is replaced by

$$\delta\tilde{\theta} := \tilde{\theta}(n) - \tilde{\theta}(n-1). \qquad (43)$$

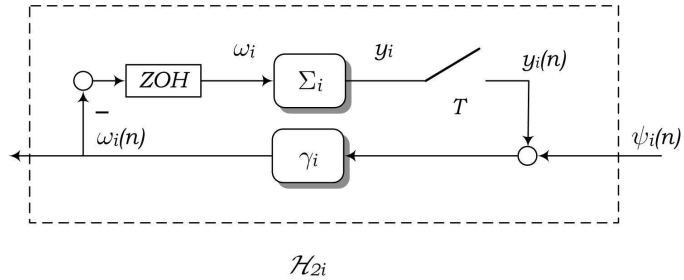

Fig. 3. A block diagram representation of the sampled-data dynamical block $\mathcal{H}_{2i}$ where *ZOH* stands for the "Zero-Order-Hold" function and T is the sampling period.

As discussed in [1], passivity of the feedforward path $\mathcal{H}_1$ cannot be achieved in discrete-time because the phase lag of a summation block exceeds 90°. In the feedforward system we restrict the slope of the nonlinearity $\phi_k(\tilde{\theta}_k)$ by

$$\phi_k'(\tilde{\theta}_k) \leq \mu \qquad (44)$$

for some constant $\mu > 0$. With this assumption it is shown in [1] that the storage function $V_\psi(\theta)$ in (18) satisfies

$$V_\psi\left(\tilde{\theta}((n+1)T)\right) - V_\psi\left(\tilde{\theta}(nT)\right) \leq -\Psi^\top\omega + \frac{\mu\lambda_N}{2}\omega^\top\omega \qquad (45)$$

where λ_N denotes the largest eigenvalue of the graph Laplacian matrix $DD^\top$ and the second term on the right hand side of (45) quantifies the shortage of passivity. Then, the $\mathcal{H}_2$-block must achieve an excess of passivity in the feedback path to guarantee stability for the interconnected system.

A. Design 1: With Path Error Feedback

When $z(t)$ is available for feedback we design ω_i in (42) as

$$\begin{aligned} &\omega_i(nT) = \\ &\gamma_i\left(2z_i(nT)^\top P_i g_i(nT, x_i(nT), \theta(nT)) + \psi_i(\theta(nT))\right) \end{aligned} \qquad (46)$$

where $\gamma_i > 0$ is an adaptation gain to be specified and $\psi_i(\cdot)$ is as in (9). With this design the $\mathcal{H}_2$ block is as in Figure 3 and, as we shall see, its excess of passivity compensates for the shortage in (45) when γ_i and T are sufficiently small. To make this claim precise we need the following lemma, proven in [20] using the techniques of [21]:

Lemma 5: [20] Consider members $i = 1, \ldots, r$ interconnected as described by the graph representation (6), and let $\tilde{\theta}_k$, $k = 1, 2, \ldots, p$ denote the differences between the variables θ_i of neighboring members. Let $\phi_k(\tilde{\theta}_k)$'s be designed as a first-third quadrant nonlinearity satisfying (44) for some constant $\mu > 0$. Suppose that $\mathcal{H}_i$'s are sampled-data dynamic blocks as in Figure 3, and satisfy the following two assumptions:

L1. For $\omega_i = 0$, the time-derivative of V_{zi} for Σ_i is upper bounded by $\dot{V}_{zi}(z_i) \leq -C|z_i|^2$ for some $C > 0$,

L2. Σ_i-subsystems, $i = 1, \ldots, r$ are Input-to-State Stable from ω_i to y_i, i.e., there exist class-$\mathcal{KL}$ and class-$\mathcal{K}$ functions $\beta(\cdot,\cdot)$ and $\rho(\cdot)$, respectively, such that

$$|z_i(t)| \leq \beta(|z_i(t_0)|, t-t_0) + \rho\left(\sup_{t_0\leq\tau\leq t} |\omega_i(\tau)|\right).$$

Then given compact set $\mathcal{D}_{\tilde{\theta}}$ and $\mathcal{D}_z$ there exist positive constants $\bar{T}$ and $\bar{\gamma}$ such that for all sampling periods $T < \bar{T}$ and $\gamma_i \leq \bar{\gamma}$, the feedback law (46) achieves asymptotic stability of the origin $(\tilde{\theta}, z) = 0$ with a region of attraction that includes $\mathcal{D}_{\tilde{\theta}} \times \mathcal{D}_z$. □

Lemma 5 proves a semiglobal asymptotic stability property in T and γ_i for the equilibrium $(\tilde{\theta}, z) = 0$, which means that any prescribed region of attraction can be achieved by sufficiently reducing the sampling period T and the adaptation gain γ_i. In particular, increasing the size of the prescribed region of attraction or increasing the parameters μ and λ_N in (45) dictate smaller values for $\bar{T}$ and $\bar{\gamma}$ (see [20] for formulas that estimate $\bar{T}$ and $\bar{\gamma}$). The proof in [20] (see also [**?**]) is also applicable to time-varying sampling periods that are upper-bounded by $\bar{T}$. We now apply Lemma 5 to our system and state the main result of this section:

Theorem 6: Consider members $i = 1, \ldots, r$ interconnected in a formation as described by (6). Let ϕ_k's be as in (10) and (44), and suppose that $\mathcal{H}_2$ consists of sampled-data dynamic blocks as in Figure 3 where the continuous-time Σ_i-block and the discrete-time updates for θ_i are as given in (42). Then, given compact sets $\mathcal{D}_z$ and $\mathcal{D}_\theta$ there exist positive constants $\bar{T}$ and $\bar{\gamma}$ such that for all sampling periods $T < \bar{T}$ and $\gamma_i \leq \bar{\gamma}$, the feedback law (46) achieves UAS of the origin $(z, \tilde{\theta}) = 0$ with a region of attraction that includes $\mathcal{D}_z \times \mathcal{D}_{\tilde{\theta}}$.

Proof: First, we know from the proof of Theorem 1 that each Σ_i-block is strictly passive with a positive definite storage function $V_{z_i}(z_i)$ such that

$$\dot{V}_{z_i} \leq -W(z_i) - y_i\omega_i$$

where $W(z_i) = z_i^\top z_i$. Hence, we find a lower bound on $W(z_i) \geq C|z_i|^2$ where $C = 1 > 0$ so L1 of Lemma 5 holds. Second, L2 holds since Input-to-State Stability from ω_i to z_i is proved in Theorem 3 using the same storage function. We thus conclude that the origin $(z, \tilde{\theta}) = 0$ is UAS with a region of attraction that includes the prescribed set $\mathcal{D}_z \times \mathcal{D}_{\tilde{\theta}}$. □

B. Design 2: Without Path Error Feedback

We next consider the case where $z_i(t)$ is not employed in discrete-time θ_i updates. In this case we have

$$\omega_i(nT) = \mathcal{F}_i\{\psi_i(\theta(nT))\}, \tag{47}$$

where $\mathcal{F}_i$ is a discrete-time dynamic or static block. In order to guarantee excess of passivity in the feedback path, we restrict static $\mathcal{F}_i$ blocks $y_i = h_i(t, u_i)$ by

$$u_i y_i - \tau_i y_i^2 \geq \varpi_i(u_i) \tag{48}$$

where $\varpi_i(u_i)$ is a positive definite function and $\tau_i > 0$ quantifies the excess of passivity. When $\mathcal{F}_i$ is a dynamic block of the form

$$\begin{aligned} \xi_i((n+1)T) &= f_i(\xi_i(nT), u_i(nT)) \quad \xi_i \in \mathbb{R}^{n_i} \\ y_i &= h_i(\xi_i, u_i) \end{aligned} \tag{49}$$

we assume that (17) holds and that there exists a positive definite and radially unbounded storage function $S_i(\xi_i)$ satisfying

$$S_i(\xi((n+1)T)) - S_i(\xi_i(nT)) \leq -W_i(\xi_i) + u_i y_i - \tau_i y_i^2 \tag{50}$$

for some positive definite function $W_i(\cdot)$. We then guarantee stability of the feedback system by choosing

$$\tau_i \geq \frac{\mu\lambda_N}{2} \quad i = 1, \cdots, r. \tag{51}$$

As before, GAS of $\tilde{\theta} = 0$ implies $\omega \to 0$ and stability of the path errors follows from the cascade structure and the ISS-property of the Σ_i-subsystems driven by ω:

Theorem 7: Consider members $i = 1, \ldots, r$ interconnected in a formation as described by (6). Let $\phi_k,\ k = 1, ..., p$ be as in (10) and (44), and suppose that $\mathcal{H}_2$ consists of sampled-data dynamic blocks where the continuous-time Σ_i-blocks are as given in (5) and $\mathcal{F}_i$'s are as in (48)-(50). Under these conditions if (51) holds then the update law (47) renders the the origin $(\tilde{\theta}, \xi, z) = 0$ UGAS.

Proof: When (51) holds asymptotic stability of $(\tilde{\theta}, \xi) = 0$ follows from [1]. Furthermore, from Theorem 3 we know that each z_i is Input-to-State Stable with respect to ω_i so $z(t)$ is bounded within each sampling period and constant between the sampling points. It then follows from arguments similar to those in Theorem 3 that the origin $(\tilde{\theta}, \xi, z) = 0$ is UGAS. □

V. EXAMPLES

A rendezvous maneuvering operation between marine surface vessels is considered to illustrate the proposed framework. The passivity framework is applied to obtain an extended class of feedback functions $\mathcal{F}_i$ that can address performance properties and increase robustness to disturbance and delays for a group of vessels. Some purposes of formation control for ships are underway replenishment operations, reduced drag forces, cooperative towing, etc.

We consider a model of a fully actuated tugboat in three degrees of freedom where the surge mode is decoupled from the sway and yaw mode due to port/starboard symmetry – see Figure 4. The body-fixed equations of motion for vessel $i = 1, \ldots, r$ are given as (see [22] for details)

$$\dot{\eta}_i = R\nu_i \tag{52a}$$

$$M_i\dot{\nu}_i + C_i(\nu_i)\nu_i + D_i(\nu_i)\nu_i = \tau_i \tag{52b}$$

where $\eta_i = [x_i, y_i, \psi_i]^\top$ is the earth-fixed position vector, (x_i, y_i) is the position on the ocean surface and ψ_i is the heading angle (yaw), and $\nu_i = [u_i, v_i, r_i]^\top$ is the body-fixed velocity vector. The model matrices M_i, C_i, and D_i denote inertia, Coriolis plus centrifugal, and damping, respectively, while τ_i is a vector of generalized control forces and moments, and R is the rotation matrix between the body and the Earth coordinate frame, dependent on the heading angle.

A nonlinear speed dependent formulation for station-keeping ($u = v = r = 0$) and maneuvering up to $u = 0.35\sqrt{gL_{pp}} = 6.3\,\mathrm{m/s}$ (Froude number 0.35) is derived in [23] where L_{pp} is the length between the perpendiculars. The 3 DOF horizontal plane vessel model linearized for cruise

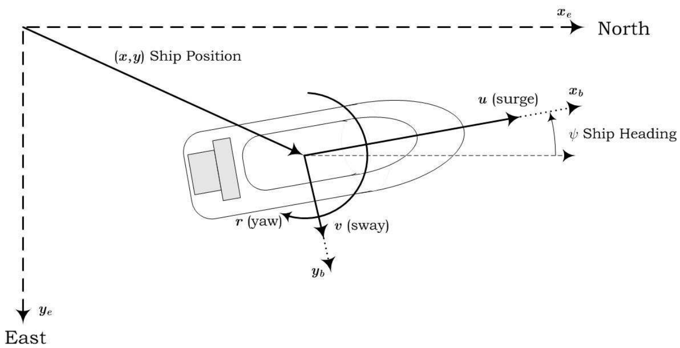

Fig. 4. Earth-fixed and body-fixed reference frame for a ship.

speeds in the neighborhood of $u = 5\ \mathrm{m}/\mathrm{s}$ with nonlinear viscous quadratic damping in surge, is taken from [24].

A group of r vessels will have r individual paths where the desired path for vessel i is given by $\eta_{di}(\theta_i) = [x_{di}(\theta_i),\ y_{di}(\theta_i),\psi_{di}(\theta_i)]^\top$, where ψ_{di} is the desired heading. Then, given a path $\eta_{di}(\theta_i)$ and a speed assignment $\upsilon(t)$ as in Assumption A1, the design in Section II gives a path following control law for (52)–see [11] for details.

The backstepping design for each ship model gives the static part of the control signal

$$\tau_i = -z_{1i} - K_{di} z_{2i} + N_i(\nu_i)\nu_i + M_i\left(\sigma_{1i} + \alpha_{1i}^{\theta_i}\upsilon\right)$$

where $N_i(\nu_i) = D(\nu_i) + C_i(\nu_i)$, $K_{di} = K_{di}^\top > 0$, α_{1i} is a virtual control determined by the backstepping procedure, and $\dot{\alpha}_{1i} =: \sigma_{1i} + \alpha_{1i}^{\theta_i}\dot{\theta}$ – see [10] for details. The resulting closed-loop system is given by (5) where

$$F_i(\nu_i) := \begin{bmatrix} -K_{pi} - r_i S & I \\ -M_i^{-1} & -M_i^{-1}(K_{di} + N_i(\nu_i)) \end{bmatrix}$$

$$g(\eta_i, \theta_i, t) := \begin{bmatrix} R(\psi_i)\,\eta_{di}^{\theta_i}(\theta_i) \\ \alpha_{1i}^{\theta_i}(\eta_i, \theta_i, t) \end{bmatrix}.$$

for $K_{pi} = K_{pi}^\top > 0$ and $S = -S^\top$.

A. Feedback Function Design

As mentioned in Remark 2 the passivity design in Section III encompasses the design in [11] where a *gradient update law* for ω_i is considered and $\mathcal{F}_i$ is a positive gain. The class of strictly positive real (SPR) systems include the *filtered gradient update law* considered in [10]: We design the dynamic block $\mathcal{F}_i$ with output ω_i and dynamics

$$\dot{\omega}_i = -\lambda_i \omega_i + \gamma_i u_i, \quad \lambda_i, \gamma_i > 0. \tag{53}$$

With the storage function $V_{\omega_i} = \frac{1}{2}\omega_i^2$ we obtain $\dot{V}_{\omega_i} = -\lambda_i \omega_i^2 + \gamma_i \omega_i u_i$ and since (15)-(16) are fulfilled we invoke Theorem 1 to conclude UGAS of $(\tilde{\theta}, \omega, z) = 0$. Equation (53) is essentially a low-pass filter where the cut-off frequency can be designed in a trade-off fashion of measurement noise attenuation versus bandwidth as determined by choice of λ_i and γ_i.

The authors of [25] discuss how thruster saturation constraints in a single vessel cause steady-state errors in the path variables synchronization. This error is eliminated by employing integral feedback from the synchronization error. In the proposed framework of Section III the thruster saturation failure can be handled with a proportional-integral controller with limited integral effect, that is

$$\omega_i = \gamma_i(u_i(s)) + \gamma_i \frac{1 + \mu_i s}{1 + \gamma_i \mu_i s}(u_i(s)) \quad \gamma_i, \mu_i > 0 \tag{54}$$

where $u_i(s)$ is the input to $\mathcal{F}_i$. We consider the more general case where the dynamic $\mathcal{F}_i$ has a proportional-integral-derivative (PID) control structure with limited integral and derivative effect, also known as a lead-lag controller,

$$\omega_i(s) = H_{pid,i}(s)\, u_i(s)$$

given by

$$H_{pid,i}(s) = \nu_i \beta_i \frac{1 + \mu_i s}{1 + \beta_i \mu_i s} \frac{1 + T_{d,i} s}{1 + \alpha_i T_{d,i} s} \tag{55}$$

where $\nu_i > 0$, $0 \le T_{d,i} \le \mu_i$, $1 \le \beta_i < \infty$ and $0 < \alpha_i \le 1$. Then, (55) is Hurwitz and satisfies $\mathrm{Re}[H_{pid,i}(j\omega)] \ge \nu_i > 0$ for all $s = j\omega$ and it follows that the PID controller structure falls into the class of input strictly passive systems and stability of the interconnection follows from Theorem 1.

B. Simulation: Saturation in Thrust

We consider a simulation where one vessel's propellers saturate and is only able to move with a surge speed less than the speed assignment. We compare the synchronization error for the original control scheme [11], i.e. $T_{d,i} = 0 = \mu_i$ in (55) while $\gamma_i = 10$, with the PID structure with $T_{d,i} = 10$, $\mu_i = 1$, $\nu_i = 10$, $\alpha_i = 0.1$, and $\beta_i = 10$. The other control parameters are set as $P_i = \mathrm{diag}(0.2, 0.2, 1, 10, 10, 40)$ and $\phi_i(x) = x^3$. The desired speed is $\upsilon = 4$, and the desired path for Vessel 2 is given by $x_d(\theta_2) = \theta_2$, $y_d(\theta_2) = 1200 \sin \frac{2\pi}{4000}\theta_2$, and $\psi_d(\theta_2) = \arctan\left(\frac{y_{di}^{\theta_2}(\theta_2)}{x_{di}^{\theta_2}(\theta_2)}\right)$. The other paths are constructed such that the vessels will move parallel when $\tilde{\theta} = 0$. The initial conditions are $\eta_1(0) = [573, 222.5, 0]^\top$, $\eta_2(0) = [0, 0, 0]^\top$, $\eta_3(0) = [320, 420, 0]^\top$, and $\nu_i = 0$ for $i = 1, 2, 3$. The communication topology is given by the incidence matrix

$$D(\mathcal{G}) = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & -1 \end{bmatrix}$$

that is only vehicles 1 and 2, and 2 and 3 are exchanging their path parameters. The initial synchronization errors are $\tilde{\theta}_1(0) = 500$, $\tilde{\theta}_2(0) = -400$.

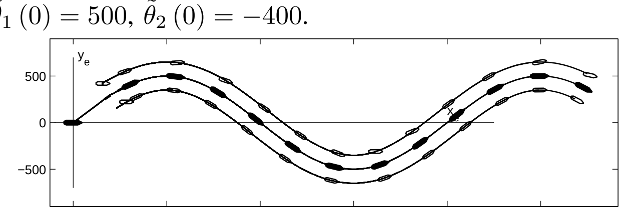

Fig. 5. Position snapshots of three tugboats where one vessel saturates. The feedback function $\mathcal{F}_i$ is as in (55).

Figure 5 shows that the formation follows the path as desired. The synchronization of $\tilde{\theta}_1$ for the two $\mathcal{F}_i$-structures are shown in Figure 6 and shows that the PID-structure yields a smaller error when one vessel saturates.

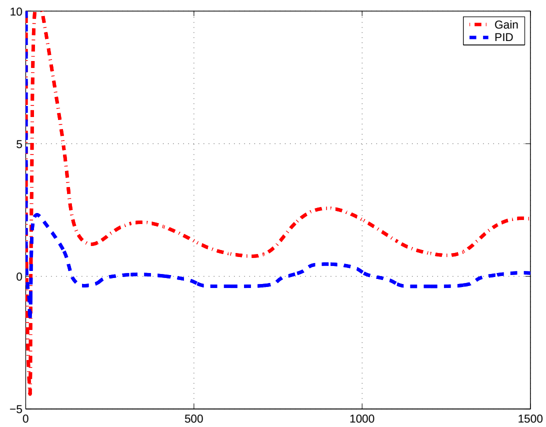

Fig. 6. Synchronization of $\tilde{\theta}_1$ when one vessel saturates. The result when $\mathcal{F}_i$ is a gain is shown in red (dashed-dot) and a PID structure for $\mathcal{F}_i$ results in the blue (dashed) plot.

VI. Discussion and Conclusions

This paper has used passivity properties to design a formation control scheme where path following systems are synchronized using a bidirectional communication structure. The first design used feedback from both the path error and the synchronization error in the update law for the path parameter, while the second only employed information about the synchronization error. The path error feedback in Design 1 emphasizes convergence to the path while the synchronization error feedback achieves the desired formation. This scheme thus enables the designer to prioritize path convergence or synchronization by choosing the relative gains of the two feedback terms. Furthermore, the system can employ its own path error information to handle situations where a trajectory tracking scheme has limitations [6]. However, our analysis for Design 1 is only valid for time-invariant graph structures.

However, the second design inherits the properties of the coordination scheme in [1] where a time-varying formation configuration is tolerated. In addition, since the incidence matrix D does not have to be pointwise connected for all times, communication dropouts are allowed, and similarly, vehicles can enter or leave the formation.

A sampled-data approach to synchronization, where the synchronization scheme is updated in discrete time and the path following systems in continuous time, is considered. The main motivation is that communication of path variables will likely occur over a digital network and a discrete-time system is more natural to address communication issues.

Formation control of multiple, independent, entities in an unknown environment is a non-trivial problem. Collisions that might occur before converging to the path has not been addressed in this paper but will be part of future work.

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