Robust Combined Position and Formation Control for Marine Surface Craft

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***Abstract*— We consider the robustness properties of a formation control system for marine surface vessels. Inter-vessel constraint functions are stabilized to achieve the desired formation configuration. We show that the formation dynamics is Input-to-State Stable (ISS) to both environmental perturbations that affect each vessel and to disturbances that affect the inter-vessel relationships e.g. communication noise. Next, we prove ISS of a formation where at least one vessel is in closed loop with, a class of position control laws, in addition to the formation control law. This class encompass control laws for point stabilization or path following. Hence, the designer can utilize previously developed controllers for single vessels in a formation control setting. A formation of three tugboats where one is in closed loop with a path following controller is simulated to verify the theoretical results.**

***Keywords*— Robustness, Formation Control, Marine Control Systems.**

I. Introduction

Recent advances in control techniques for vehicles and communication capabilities have sparked an interest in formation control which has resulted in a number of publications with applications ranging from autonomous underwater vehicles (AUVs) to multiple satellite control [1], [2], [3], [4]. Utilizing several vehicles instead of just one has several benefits, such as robustness to vehicle loss, increased instrumentation range, and faster reconfigurability for different missions. An important application of marine cooperative control is autonomous oceanographic sampling networks [5], [6] which is used for exploration of natural undersea resources and gathering of scientific data. Other examples are cooperative control of surface ships and AUVs and underway replenishment operations for surface vessels.

In these applications, changes in the environment, unknown disturbances, communication and sensor noise, model uncertainties, and communication constraints between formation members pose challenges for control design [7] and robustness with respect to these disturbances are important. Previously, stability has been addressed for formations with changing topologies and time-delays. Formation feedback is used in [8] to prevent the vehicle to leave the formation when exposed to disturbances and control saturation. Robustness properties of leader/follower formations for nonholonomic systems have been investigated in [9], [10], with emphasis on formation errors with respect to disturbances and leader-to-formation stability gains, respectively. The authors of [11] show that consensus algorithms are Input-to-State Stable and apply the results in a cooperative control setting. However, many of the issues mentioned above has not yet been addressed–see the survey paper [4] for an overview. Previously, the authors have addressed robustness designs for apriori known environmental disturbances [12].

In this paper, we investigate robustness for a group of marine surface craft to communication and environmental disturbances. The formation configuration is described with the framework of [13] using inter-vessel functions describing a vessels behavior with respect to its neighbors. The functions are treated analogous to constraint functions in analytical mechanics and by treating the constraint forces that arise as control laws we formulate a formation control problem. Stability of the inter-vessel constraint functions implies that vessels assemble into the formation configuration. The group is studied in two cases: we first consider control laws for maintaining a given formation configuration in the presence of environmental loads and communication disturbances. Second, we extend this scheme in pursuit of a modularity approach where one or more of the vessels in the formation are individually controlled by a position control law for point stabilization, trajectory tracking or path following.

Even though the individual position controllers in closed-loop with the vessels and the formation control laws are stable with respect to their origins, an interconnection might not be stable unless it satisfies additional properties [14]. Motivated by results on passivity design for coordinated control [15], [16] and robustness for network flow control [17] we show that the proposed scheme can be modeled as a feedback interconnection of a block of vessels and position control laws and a block that maintains the formation configuration is maintained. Furthermore, we use a small-gain result [18], [19] to prove that the interconnection is Input-to-State Stable (ISS) [20] with respect to environmental and inter-vessel disturbances in both cases. Stability of the suggested modules enables the control designer to address motion control laws and formation maintenance separately instead of incorporating a motion control law in the formation control framework as in [13]. Thus, the literature on motion control for single

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vessels, see for example [21] and [22], is utilized in a formation setting.

The rest of this paper is organized as follows: Section II gives the notation and definitions used in this paper and an overview of the vessel dynamics. Undisturbed formation modeling and control is discussed in Section III while Input-to-State Stability with respect to disturbances are proved in Section IV. A case study in Section V verify the theoretical results and concluding remarks are given in Section VI.

II. Preliminaries

The notation in this paper is as follows. The vector norm of x is denoted by $|x|$, and the $\mathcal{L}_\infty$-norm of $x(t)$ is $|x|_{\mathcal{L}_\infty}$. For $d \in \mathcal{L}_\infty$, we define

$$|d|_a = \lim_{t\to\infty} \sup |d(t)| \quad (1)$$

and the induced matrix norm of A is denoted $\|A\|$. For a matrix $P = P^\top > 0$, let $p_m := \lambda_{\min}(P)$ and $p_M := \lambda_{\max}(P)$ be the minimum and maximum eigenvalue of P, respectively.

A function $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{K}$ if it is continuos, strictly increasing and $\alpha(0) = 0$. A function $\beta : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{KL}$ if, for each fixed s, the function $\beta(r,s)$ belongs to class $\mathcal{K}$ with respect to r and, for each fixed r, the function $\beta(r,s)$ is decreasing with respect to s and $\beta(r,s) \to 0$ as $s \to \infty$.

A system $\dot{x} = f(x,u)$ with state $x \in \mathbb{R}^n$ and input $u \in \mathbb{R}^m$ is said to be Input-to-State Stable (ISS) if there exist class $\mathcal{K}$ functions $\gamma_0(\cdot)$, $\gamma(\cdot)$, such that, for any input $u(\cdot) \in \mathcal{L}_\infty^m$ and $x_0 \in \mathbb{R}^n$, the response $x(t)$ in the initial state $x(0) = x_0$ satisfies

$$|x|_{\mathcal{L}_\infty} \leq \gamma_0(|x_0|) + \gamma\left(|u|_{\mathcal{L}_\infty}\right) \quad (2)$$

$$|x|_a \leq \gamma(|u|_a). \quad (3)$$

The derivation of the equations of motion for a marine surface vessel is based on [21] and [23]. We consider a ship model in surge, sway, and yaw

$$\dot{\eta} = R(\psi)\nu \quad (4a)$$

$$M\dot{\nu} + D(\nu)\nu = \tau \quad (4b)$$

where $\eta = [x, y, \psi]^\top$ is the Earth-fixed position vector, (x, y) is the position on the ocean surface and ψ is the heading angle (yaw), $\nu = [u, v, r]^\top$ is the body-fixed velocity vector, and τ is a vector of generalized control forces and moments. The model matrices M and D denote inertia and damping, respectively, and $R = R(\psi) \in SO(3)$, $\|R\| = 1 \ \forall\psi$, is the rotation matrix between the body and Earth coordinate frame. In addition, we have that

Assumption 1: The inertia matrix M is positive definite, i.e., $M = M^\top > 0$.

Assumption 2: The damping matrix $D(\nu)$ consists of a linear and a nonlinear part, i.e., $D(\nu) = D + D_{nonlin}(\nu)$ where D is strictly positive, i.e., $(1/2)x^?(D + D^\top)x > 0, \quad \forall x = 0$ and $D_{nonlin}(\nu)$ is a matrix of nonlinear viscous damping terms, for instance quadratic drag.

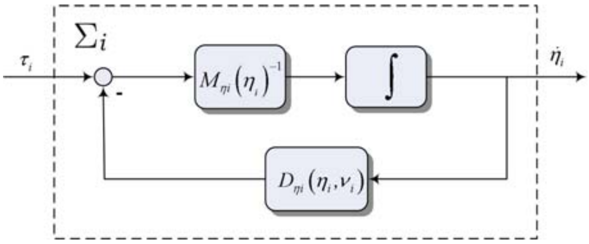

Fig. 1. Vessel Dynamics

We characterize the passivity properties of a vessel i without assumptions on the external control forces τ_i. Consider the positive definite, radially unbounded storage function

$$V_i(\nu_i) = \frac{1}{2}\nu_i^\top M_i \nu_i \quad (5)$$

with the following time-derivative

$$\dot{V}_i = -\nu_i^\top D_i(v_i)\nu_i + \nu_i^\top \tau_i \leq -\varepsilon_i |\nu_i|^2 + \nu_i^\top \tau_i \quad (6)$$

where $\varepsilon_i > 0$. Thus, (5) and (6) shows that the vessel dynamics is passive from τ_i to ν_i. For low-speed applications, such as dynamic positioning, the damping matrix is constant and the dissipative property of ships is then equivalent to consider the eigenvalues of $-M^{-1}D$: If they are nonpositive the ship is *course-stable*.

For notational convenience we rewrite the vessel dynamics (4) in Earth-fixed coordinates:

$$M_{\eta i}\ddot{\eta}_i + D_{\eta i}(\nu_i, \eta_i)\dot{\eta}_i = R(\psi_i)\tau_i \quad (7)$$

where

$$M_{\eta i}(\eta_i) = R(\psi_i) M_i R(\psi_i)^\top \quad (8)$$

$$D_{\eta i}(\nu_i, \eta_i) = R(\psi_i) D_i(\nu_i) R(\psi_i)^\top. \quad (9)$$

III. Formation Modeling and Control of Marine Surface Vessels

For the remaining part of this paper, we consider a group of r marine surface vessels

$$\Sigma_i : M_{\eta i}\ddot{\eta}_i + D_{\eta i}(\nu_i, \eta_i)\dot{\eta}_i = R(\psi_i)\tau_i, \quad i = 1, \ldots, r \quad (10)$$

where each vessel's dynamics is as in (5) and (6), depicted in Figure 1. The formation control scheme used in this paper is proposed in [13] which, motivated by multi-body dynamics and analytical mechanics, use constraint functions that describe the orientation or position of each vessel with respect to other members of the formation. In this paper, we will consider two functions for formation purposes, the *position-relative*

$$\mathcal{C}_r(\eta_i, \eta_j) = (x_i - x_j)^2 + (y_i - y_j)^2 - r_{ij}^2, \quad r_{ij} \in \mathbb{R} \quad (11)$$

and the *orientation-fixed*

$$\mathcal{C}_f(\eta_i, \eta_j) = \eta_i - \eta_j - o_{ij}, \quad o_{ij} \in \mathbb{R}^3. \quad (12)$$

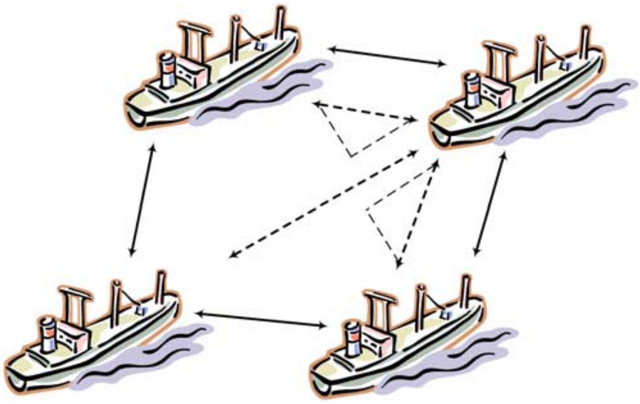

Fig. 2. Constraint functions acting between vessels determine the group's motion. Constraints are shown as position relative (–) and orientation-fixed (- -).

The scalar $r_{ij}(t)$ is the desired distance between vessel i and j while the column vector o_{ij} describes the offset between η_i and η_j in each degree of freedom (DOF)–see Figure 2. We say that two vessels are *neighbors* if they are connected by formation constraint functions such as (11) or (12), in which case they access each others information.

Given a set of l constraint functions as in (11) or (12). The desired formation structure is then characterized by the formation constraint function

$$\begin{aligned} \mathcal{C} &= \left[\mathcal{C}_1^\top, \ldots, \mathcal{C}_l^\top\right]^\top = 0 \\ \dim \mathcal{C} &= p \end{aligned} \tag{13}$$

subject to

Assumption 3: The constraints $\mathcal{C}_1, \ldots, \mathcal{C}_l$ are not redundant nor conflicting and dim$\mathcal{C} < 3(r-1)$.

Note that redundant or conflicting constraints arise when one, or more, row (column) in $\mathcal{C}$ is a linear combination of other rows (columns), or when the functions are contradicting. An example would be the same constraint function appearing twice in $\mathcal{C}(\eta)$.

Following the analytical approach in [24] the procedure in [13] stabilize the formation structure with the constraint force τ_{ci} acting on vessel i

$$\tau_{ci} = \sum_{k \in \mathcal{N}_i} -W_{k,i}^\top \lambda_k \tag{14}$$

where $\mathcal{N}_i$ is the set of indices of $\mathcal{C}$ where η_i appears, $W_{k,i}$ is the ith column of the Jacobian of the constraint , i.e., $W_{k,i} = \frac{\partial \mathcal{C}_k}{\partial \eta_i}$, and λ_k is the Lagrangian multiplier corresponding to $\mathcal{C}_k$. The Lagrangian multiplier is found by combining

$$\frac{d^2}{dt^2}\mathcal{C}_k = 0$$

with the vessel model (10) and solve for λ_k – see [13] for details.

We write the system matrices on block-diagonal form

$$\begin{aligned} M_\eta &: = \operatorname{diag}\{M_{\eta 1}, \ldots, M_{\eta r}\} \\ D_\eta &: = \operatorname{diag}\{D_{\eta 1}, \ldots, D_{\eta r}\} \\ R &: = \operatorname{diag}\{R(\psi_1), \ldots, R(\psi_r)\}, \end{aligned}$$

and the vectors as concatenated vectors

$$\eta := \left[\eta_1^\top, \ldots, \eta_r^\top\right]^\top, \quad \tau_c := \left[\tau_{c1}^\top, \ldots, \tau_{cr}^\top\right]^\top,$$

to obtain the the closed-loop dynamics for the entire formation

$$M_\eta \ddot{\eta} + D_\eta \dot{\eta} = -RW^\top \lambda, \tag{15}$$

where

$$W = \frac{\partial \mathcal{C}}{\partial \eta}, \text{ and } \lambda = [\lambda_1, \ldots, \lambda_p]^\top.$$

The formation members are now coupled together by the constraint force. The analytical expression for λ is, obtained from $\ddot{\mathcal{C}}$ and (15),

$$WM_\eta^{-1}RW^\top \lambda = -WM_\eta^{-1}D_\eta \dot{\eta} + \dot{W}\dot{\eta} + K_p\mathcal{C} + K_d\dot{\mathcal{C}} \tag{16}$$

where $K_p, K_d > 0$, and the two latter terms stabilize $\mathcal{C} = 0$ and thus force the formation to configure accordingly.

To obtain λ in (16) the term on the left-hand side must be invertible: The transformed inertia matrix is positive definite due to Assumption 1 and (8) and the Jacobian W has full row rank due to Assumption 3. Hence, the determinant of $WW^\top$ is nonzero and $WM_\eta^{-1}RW^\top$ is nonsingular.

Remark 1: The combination of ordinary-differential equations and constraint functions leads to a Differential Algebraic Equation (DAE). In our case, (10) and (13) yield a higher-order DAE which is inherently unstable. The formation control law (14) with λ as in (16) stabilize the constraint (13) and correspond to a numerical stabilization algorithm for multi-body systems [25].

Combining $\ddot{\mathcal{C}} = 0$ and (15) yields (16) and is equivalently written as the constraint-stabilization system

$$\ddot{\mathcal{C}} = -K_d\dot{\mathcal{C}} - K_p\mathcal{C}, \tag{17}$$

which has a stable origin, that is, the formation will be assembled in the desired configuration with parallel speed vectors:

Theorem 1: The constraint-stabilization system (17), or equivalently (15) and $\ddot{\mathcal{C}} = 0$, has a globally exponentially stable origin $(\mathcal{C}, \dot{\mathcal{C}}) = 0$ under Assumptions 1–3. Furthermore, the velocity vectors are aligned, i.e., $\dot{\eta}_i = \dot{\eta}_j$.

Proof: Let $\phi = [\mathcal{C}^\top, \dot{\mathcal{C}}^\top]^\top$ and consider (17) on state-space form

$$\dot{\phi} = \begin{bmatrix} \dot{\mathcal{C}} \\ \ddot{\mathcal{C}} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -K_p & -K_d \end{bmatrix} \begin{bmatrix} \mathcal{C} \\ \dot{\mathcal{C}} \end{bmatrix} = A\phi$$

and let the Lyapunov function be

$$V_c(\phi) = \phi^\top P\phi, \quad P = P^\top > 0 \tag{18}$$

When K_p, K_d are positive A is Hurwitz and there always exists a solution to $A^\top P + PA = -I$. Hence

$$\dot{V}_c(\phi) = -\phi^\top \phi$$

and the origin $\phi = 0$ is globally exponentially stable. Furthermore, the constraint derivative

$$\ddot{C} = W\dot{\eta} = 0$$

implies that $\dot{\eta}_i - \dot{\eta}_j = 0$. This is seen as follows: For both type of constraints the Jacobian W has zero-sum columns, thus $v = k[1, \ldots, 1]^\top,\ k \in \mathbb{R}$, lies in the null-space of W. Since W has full row rank v is the only null-space vector and the velocities η_i and η_j are equal, $\dot{\eta}_i = \dot{\eta}_j$. ∎

Before we proceed, we state the following lemma which is needed for the results in the remainder of this paper.

Lemma 2: The Jacobian W of (13) and its time-derivative $\dot{W}$ have bounded norms under the hypothesis of Theorem 1.

Proof: We know from matrix analysis that the matrix norm induced by the euclidean vector norm is $\|W\|_2 = \sqrt{\lambda_{\max}(W^\top W)}$. When the formation configuration is given by functions as in (12), the Jacobian is constant so the result is trivial. When the configuration is given by (11), the elements of the Jacobian are either zero or a relative displacement, $x_i - x_j$ or $y_i - y_j$. The norm of W is bounded when these elements are bounded, and stability of (13) follows from (17) which implies that the displacements are bounded, and the result follows. Boundedness of $||\dot{W}||$ follows from similar arguments as each element of $\dot{W}$ is bounded by Theorem 1. ∎

IV. Robustness to Disturbances

A ship is affected by unknown environmental loads due to wind, waves and currents. These loads can be represented by a force field where

- a slowly varying mean force which attacks the ship in a point in the body-fixed coordinates in
- a slowly varying mean direction relative to the Earth-fixed frame.

The slowly varying terms include model uncertainties, second-order wave-induced disturbances (wave drift), currents and mean wind forces. The first-order wave-induced forces (oscillatory wave-induced motion) is assumed to be filtered out by employing a *wave filter* [21].

The ship control system should only counteract the slowly-varying motion components of the environmental to reduce wear and tear of actuators and propulsion system. In addition, there are no sensors to accurately measure the mean force and direction of the environmental loads. This motivates the assumption that the unknown mean environmental force and its direction are assumed to be constant (or at least slowly varying), and the unknown attack point is constant.

A. Robust Formation Control

We use the results in Section III to study system robustness with respect to disturbances. The closed-loop

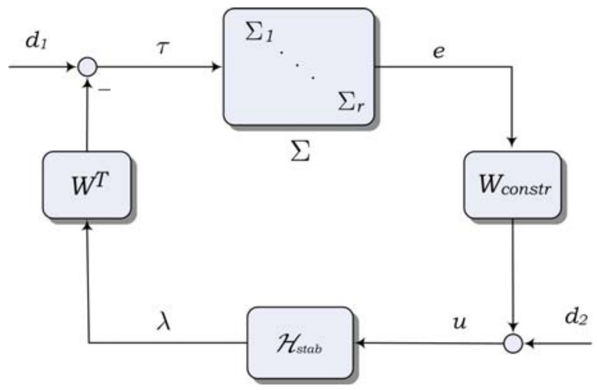

Fig. 3. A formation of vessels, Σ-block, with stabilized inter-vessel constraints, $\mathcal{H}_{\text{stab}}$, with disturbances d_1 and d_2.

system is represented as in Figure 3 where the Σ-block consists of vessels as in (10), shown in Figure 1, and we define the output of Σ as

$$e := \dot{\eta} = \left[\dot{\eta}_1^\top, \ldots, \dot{\eta}_r^\top\right]^\top.$$

The transformation of variables into constraint functions is done by multiplication with W_{constr} and corresponds to the right hand side of (16) without the stabilizing terms, that is,

$$W_{\text{constr}} = \dot{W} - WM_\eta^{-1}D_\eta.$$

Finally, the formation stabilization are handled by $\mathcal{H}_{\text{stab}}$ where the output λ is given in (16). The block Σ represent the vessels internal dynamics and stability of the feedback interconnection implies that, as in Theorem 1, the formation assembles in the desired configuration.

We assume that the disturbances act on the system as illustrated in Figure 3. Each vessel in the formation is affected by environmental disturbances d_1 while d_2 act on the stabilization of inter-vessel constraints and is interpreted as disturbances on the inter-vessel communication links. Other disturbances can be represented by using loop transformations. In Theorem 3 we first prove Input-to-State Stability of vessel dynamics in the Σ-block and the inter-vessel constraint dynamics in $\mathcal{H}_{\text{stab}}$-block. Furthermore, we show that the forward path from λ to e has gain

$$g_1 = \frac{m_M}{m_m \varepsilon_M}\|W\| \tag{19}$$

where $\epsilon_M = \max_i\{\varepsilon_i\}$ and m_m (m_M) is a lower (upper) bound for all mass matrices. The feedback path from e to λ has gain

$$g_2 = \delta w_e, \tag{20}$$

that is, g_2 is the gain from $\dot{\eta}$ to λ in (16). Then, if the small-gain condition

$$g_1 g_2 < 1 \tag{21}$$

holds the interconnection is Input-to-State Stable with respect to d_1 and d_2.

Theorem 3: Consider the feedback interconnected system (15), (17) as depicted in Figure 3 where Σ_i is as given in (5)-(6). Suppose $d_1, d_2 \in \mathcal{L}_\infty$ and all stated assumptions hold. Then, the Σ-block is Input-to-State Stable (ISS) with respect to τ, and the $\mathcal{H}_{\text{stab}}$-block is ISS with respect to u. Furthermore, if (21) holds the feedback-interconnection is ISS with respect to the disturbances d_1 and d_2.

Proof: To prove ISS from τ to e, we use (5) for each vessel so an ISS-Lyapunov function candidate is

$$V_\Sigma = \sum_{i=1}^{r} V_i(\dot{\eta}_i) \quad \text{where} \quad V_i = \frac{1}{2}\dot{\eta}_i^\top M_{\eta i}\dot{\eta}_i. \tag{22}$$

The function V_Σ in (22) is bounded by

$$m_m |\dot{\eta}|^2 \le V_\Sigma \le m_M |\dot{\eta}|^2 \tag{23}$$

where

$$m_m = \min_i \{\lambda_{\min}(M_i)\} \quad \text{and} \quad m_M = \max_i \{\lambda_{\max}(M_i)\}.$$

The time-derivative of (22) is, from (8),

$$\dot{V}_\Sigma = \sum_{i=1}^{r} \dot{\eta}_i^\top M_{\eta i}\ddot{\eta}_i$$

and it follows from (6) that

$$\begin{aligned}\dot{V}_\Sigma &\le \sum_{i=1}^{r} -\varepsilon_i |\dot{\eta}_i|^2 + \tau_i \dot{\eta}_i \\ &\le -\varepsilon_M |e|^2 + (\|W\| |\lambda| + |d_1|) |e|.\end{aligned}$$

From (23) we obtain

$$\begin{aligned}\dot{V}_\Sigma &\le -\frac{\varepsilon_M}{m_M} V_\Sigma + \frac{1}{\sqrt{m_m}} (\|W\| |\lambda| + |d_1|) \sqrt{V_\Sigma} \\ &\le -2\alpha_1 V_\Sigma + 2\beta_1 \sqrt{V_\Sigma},\end{aligned} \tag{24}$$

where

$$\alpha_1 = \frac{\varepsilon_M}{2m_M} \quad \text{and} \quad \beta_1 = \frac{1}{2\sqrt{m_m}} (\|W\| |\lambda| + |d_1|).$$

Setting $S_\Sigma := \sqrt{V_\Sigma}$ we obtain

$$D^\dagger S_\Sigma \le -\alpha_1 S_\Sigma + \beta_1, \tag{25}$$

where $D^\dagger$ denotes the upper Dini-derivative [26]. Equation (25) and Lemma 5 from the Appendix implies that

$$|S_\Sigma|_{\mathcal{L}_p} \le (\alpha_1 p)^{-1/p} |S_\Sigma(0)| + (\alpha_1 q)^{-1/q} |\beta_1|_{\mathcal{L}_p}$$

and

$$|S_\Sigma(t)| \le e^{-\alpha_1 t} |S_\Sigma(0)| + \alpha_1^{-1} |\beta_1|_{\mathcal{L}_\infty}. \tag{26}$$

Thus, from (26) and

$$|e(t)| \le \frac{1}{\sqrt{m_m}} |S_\Sigma(t)|,$$

we find the $\mathcal{L}_\infty$-gain of the forward path

$$\begin{aligned}|e|_{\mathcal{L}_\infty} &\le \frac{1}{\sqrt{m_m}} e^{-\frac{\varepsilon_M}{2m_M}t} |S_\Sigma(0)| \\ &\quad + \frac{1}{\sqrt{m_m}} \left(\frac{\varepsilon_M}{2m_M}\right)^{-1} |\beta_1|_{\mathcal{L}_\infty}\end{aligned} \tag{27}$$

which shows that the Σ-block is ISS with respect to β_1 according to (2). For future reference, note that (27) is equivalent to

$$\begin{aligned}|e|_{\mathcal{L}_\infty} &\le \frac{1}{\sqrt{m_m}} e^{-\frac{\varepsilon_M}{2m_M}t} |S_\Sigma(0)| \\ &\quad + \frac{m_M}{m_m \varepsilon_M} \left(\|W\| |\lambda|_{\mathcal{L}_\infty} + |d_1|_{\mathcal{L}_\infty}\right)\end{aligned} \tag{28}$$

and the forward path has asymptotic gain

$$|e|_a \le \frac{m_M}{m_m \varepsilon_M} (\|W\| |\lambda|_a + |d_1|_a)$$

from which we find the gain of the forward path from λ to e as in (19).

Next, we prove ISS of $\mathcal{H}_{\text{stab}}$ with respect to u and find the gain from e to λ. The time-derivative of (18) in the presence of the disturbance d_2 is

$$\dot{V}_c = 2\phi^\top P (A\phi + d_2) = -\phi^\top \phi + 2\phi^\top P d_2.$$

From the bounds on the Lyapunov function, we obtain

$$\begin{aligned}\dot{V}_c &\le -|\phi|^2 + 2\|P\| |\phi| |d_2| \\ &\le -\frac{1}{p_M} V_c + \frac{2p_M}{\sqrt{p_m}} |d_2| \sqrt{V_c}\end{aligned}$$

which we rewrite as

$$\dot{V}_c \le -\frac{1}{p_M} V_c + 2\beta_2 \sqrt{V_c}, \quad \beta_2 = \frac{p_M}{\sqrt{p_m}} |d_2|.$$

Similar to (24)-(27), we invoke Lemma 5 and use

$$|\phi(t)| \le \frac{1}{\sqrt{p_m}} \sqrt{V_c(t)}$$

to obtain

$$\begin{aligned}|\phi(t)| &\le \frac{1}{\sqrt{p_m}} e^{-\frac{1}{2p_M}t} \sqrt{V_c(0)} + \frac{2p_M^2}{p_m} |d_2|_{\mathcal{L}_\infty} \quad (29) \\ |\phi(t)|_a &\le \frac{2p_M^2}{p_m} |d_2|_a\end{aligned}$$

which shows that the $\mathcal{H}_{\text{stab}}$-block is ISS with respect to d_2 according to (2) and (3).

Finally, we use a small-gain argument [18], [19] to prove ISS of the closed-loop system with respect to the disturbances d_1 and d_2. We find the gain of the feedback path from e to λ using (16) and (29)

$$|\lambda(t)| \le g_2 |e(t)| + \delta k_M |\phi(t)| \tag{30}$$

where the g_2-gain is given as in (20) and $w_e \ge \|\dot{W}\| + \|W\| \|D_\eta\|$, and $\delta = \|\left(W M_\eta^{-1} R W^\top\right)^{-1}\|$. Boundedness of δ and w_e follows from Assumption 3 and Lemma 2. We combine (29) and (30) to find the $\mathcal{L}_\infty$-gain of the feedback path from e to λ

$$\begin{aligned}|\lambda(t)| &\le g_2 |e(t)| + \frac{\delta k_M}{\sqrt{p_m}} e^{-\frac{1}{2p_M}t} \sqrt{V_c(0)} \\ &\quad + \frac{2\delta k_M p_M^2}{p_m} |d_2|_{\mathcal{L}_\infty}.\end{aligned} \tag{31}$$

By combining (28) and (31) and employing the small-gain condition (21) we find

$$|e|_{\mathcal{L}_\infty} \leq \frac{1}{1-g_1 g_2}\left\{\frac{1}{\sqrt{m_m}} e^{-\frac{\varepsilon_M}{2m_M}t}\left|S_\Sigma(0)\right| + \frac{g_1 \delta k_M}{\sqrt{p_m}} e^{-\frac{1}{2p_M}t}\sqrt{V_c(0)} + g_1 \frac{2\delta k_M p_M^2}{p_m}|d_2|_{\mathcal{L}_\infty} + \frac{2\sqrt{m_M}}{m_m \varepsilon_M}|d_1|_{\mathcal{L}_\infty}\right\} \quad (32)$$

which shows that e is ISS with respect to d_1 and d_2 as in (2) and (3). Similarly, from inserting (29) in (31) when (21) holds, we obtain

$$|\lambda|_{\mathcal{L}_\infty} \leq \frac{1}{1-g_1 g_2}\left\{\frac{g_2}{\sqrt{m_m}} e^{-\frac{\varepsilon_M}{2m_M}t}\left|S_\Sigma(0)\right| + \frac{\delta k_M}{\sqrt{p_m}} e^{-\frac{1}{2p_M}t}\sqrt{V_c(0)} + \frac{m_M}{m_m \varepsilon_M}|d_1|_{\mathcal{L}_\infty} + \frac{2\delta k_M p_M^2}{p_m}|d_2|_{\mathcal{L}_\infty}\right\} \quad (33)$$

We conclude from (32) and (33) that the interconnected system is ISS with respect to disturbances d_1 and d_2. ■

B. Robust Combined Control

As an extension to Theorem 3 we next consider the case where one or more vessels in Σ is in closed loop with individual control laws u_i, e.g., for dynamic positioning or path following. Disregarding the formation control laws, we assume that u_i renders equilibrium points of the closed-loop system $e_i = 0$ globally exponentially stable. Input-to-State Stability of the feedback interconnection implies that the vessels will behave according to the individual control laws while maintaining the formation configuration in the presence of disturbances.

Assumption 4: We assume that the individual control laws conflicting.

Assumption 5: Suppose (4) is in closed loop with a control law u_{ship_i} such that equilibrium points $e_i = 0$ are Input-to-State Stable respect to τ_i, that is, for Σ_i

$$M_{\eta i}\ddot{\eta}_i + D_{\eta i}(\nu_i, \eta_i)\dot{\eta}_i = R(\psi_i)(u_{ship\_i} + \tau_i), \quad (34)$$

which we rewrite as

$$\Sigma_i : \dot{e}_i = F_i(\eta_i, t) e_i + b_i(\eta_i)\tau_i \quad (35)$$

where $F_i(\eta_i, t) \in \mathbb{R}^{n\times n}$ and $b_i(\eta_i) \in \mathbb{R}^n$ depend on the control design and, we have the ISS-Lyapunov function

$$V_{ship\_i} = e_i^\top P_i e_i. \quad (36)$$

In particular, $F(x)$ satisfies

$$PF_i(\eta_i, t) + F_i(\eta_i, t)^\top P \leq -I \quad (37)$$

for some matrix $P = P^\top > 0$. so the time-derivative of (36) is

$$\dot{V}_i(t, e_i) \leq -\varepsilon_i |e_i|^2 + \rho_i |e_i| |\tau_i| \quad (38)$$

where $\varepsilon_i, \rho_i > 0$. □

By Assumption 5 the closed-loop system

$$M_{\eta i}\ddot{\eta}_i + D_{\eta i}(\nu_i, \eta_i)\dot{\eta}_i = R(\psi_i) u_{ship\_i}, \quad (39)$$

or equivalently

$$\Sigma_i : \dot{e}_i = F_i(\eta_i, t) e_i, \quad (40)$$

has uniformly globally exponentially stable equilibrium points $e_i = 0$. It further follows that for (36) we have

$$p_{i,m}|e_i|^2 \leq V_{ship\_i}(t, e_i) \leq p_{i,M}|e_i|^2 \quad (41)$$

$$\dot{V}_{ship\_i}(t, e_i) \leq -\varepsilon_i |e_i|^2 \quad (42)$$

$$\left|\frac{\partial V_{ship\_i}}{\partial e_i}\right| \leq p_{i,M}|e_i|. \quad (43)$$

The error vector e in Figure 3 consists now of equilibrium points e_i for the individually controlled vessels and velocities $\dot{\eta}_i$ for the remaining vessels in the formation. Similar to the proof of Theorem 3, we obtain a new gain from λ to e

$$g_{1c} = \frac{q_M \bar{\rho}}{q_m \bar{\varepsilon}}\|W\| \quad (44)$$

where q_m, q_M are bounds for the Lyapunov function of Σ, $\bar{\varepsilon} = \max_i\{\varepsilon_i\}$, $\bar{\rho} = \max_i\{\rho_i\}$, and prove that if the small-gain condition

$$g_{1c} g_2 < 1, \quad (45)$$

holds, the interconnection is ISS with respect to disturbances d_1 and d_2.

Theorem 4: Consider the feedback interconnected system (15), (17) as depicted in Figure 3 where $d_1, d_2 \in \mathcal{L}_\infty$. Σ_i is as given in (5)-(6) or as in (34)-(38). Then, if Assumptions 1–5 and the small-gain condition (45) hold, the feedback interconnection is ISS with respect to d_1 and d_2.

Proof: To establish ISS of the Σ-block, we let $\mathcal{I}$ denote the subset of indices $i = 1, \ldots, r$ for which Σ_i is the closed-loop of a vessel and an individual controller and define the ISS-Lyapunov function

$$V_\Sigma := \sum_{i\in\mathcal{I}} V_{ship\_i} + \sum_{i\notin\mathcal{I}} V_i \quad (46)$$

where V_{ship_i} and V_i are defined in (5) and (42). The function (46) is bounded by

$$q_m |\dot{\eta}|^2 \leq V_\Sigma \leq q_M |\dot{\eta}|^2 \quad (47)$$

where

$$q_m = \min_i\{\lambda_{\min}(P_i), \lambda_{\min}(M_i) : i \notin \mathcal{I}\} \text{ and}$$

$$q_M = \max_i\{\lambda_{\max}(P_i), \lambda_{\max}(M_i) : i \notin \mathcal{I}\}.$$

We first find the gain of the forward path: The time-derivative of V_Σ is, from (6) and (38),

$$\begin{aligned}\dot{V}_\Sigma &\leq \sum_{i\notin\mathcal{I}}\left\{-\varepsilon_i\left|\dot{\eta}_i\right|^2+\left|\tau_i\right|\left|\dot{\eta}_i\right|\right\}\\ &\quad+\sum_{i\in\mathcal{I}}\left\{-\varepsilon_i\left|e_i\right|^2+\rho_i\left|e_i\right|\left|\tau_i\right|\right\}\\ &\leq -\left(\sum_{i=1}^{r}\varepsilon_i\left|e_i\right|^2\right)+\bar{\rho}\left|e\right|\left|\tau\right|\leq-\bar{\varepsilon}\left|e\right|^2+\bar{\rho}\left|e\right|\left|\tau\right|\end{aligned}$$

which proves that Σ is ISS with respect to τ. From (47) we get

$$\begin{aligned}\dot{V}_\Sigma &\leq -\frac{\bar{\varepsilon}}{q_M}V_\Sigma+\frac{\bar{\rho}}{\sqrt{q_m}}\left|\tau\right|\sqrt{V_\Sigma}\\ &= -2\alpha_3V_\Sigma+2\beta_3\sqrt{V_\Sigma}\end{aligned}$$

where

$$\alpha_3=\frac{\bar{\varepsilon}}{2q_M}\quad\text{and}\quad\beta_3=\frac{\bar{\rho}}{2\sqrt{q_m}}\left|\tau\right|.$$

Setting $S_\Sigma:=\sqrt{V_\Sigma}$ we obtain

$$D^{\dagger}S_\Sigma\leq-\alpha_3S_\Sigma+\beta_3,$$

which, from Lemma 5, implies that

$$\left|S_\Sigma\left(t\right)\right|\leq e^{-\alpha_3t}\left|S_\Sigma\left(0\right)\right|+\alpha_3^{-1}\left|\beta_3\right|_{\mathcal{L}_\infty}. \tag{48}$$

Then, from (48) and

$$\left|e\left(t\right)\right|\leq\frac{1}{\sqrt{q_m}}\left|S_\Sigma\left(t\right)\right|$$

we find the gain of the forward path

$$\left|e\right|_{\mathcal{L}_\infty}\leq\frac{1}{\sqrt{q_m}}e^{-\alpha_3t}\left|S_\Sigma\left(0\right)\right|+g_{1c}\left|\lambda\right|_{\mathcal{L}_\infty}+\frac{q_M\bar{\rho}}{q_m\bar{\varepsilon}}\left|d_1\right|_{\mathcal{L}_\infty} \tag{49}$$

where g_{1c} are as in (44). Thus, $e\left(t\right)$ is ISS with respect to d_1. The gain of the feedback path is as in (20), and we insert (31) into (49) and use the small-gain condition (45) to obtain

$$\begin{aligned}\left|e\right|_{\mathcal{L}_\infty}\leq\frac{1}{1-g_{1c}g_2}&\left\{\frac{1}{\sqrt{q_m}}e^{-\alpha_3t}\left|S_\Sigma\left(0\right)\right|\right.\\ &+\frac{g_{1c}\delta k_M}{\sqrt{p_m}}e^{-\frac{1}{2p_M}t}\sqrt{V_c\left(0\right)}\\ &\left.+\frac{q_M\bar{\rho}}{q_m\bar{\varepsilon}}\left|d_1\right|_{\mathcal{L}_\infty}+g_{1c}\frac{2\delta k_Mp_M^2}{p_m}\left|d_2\right|_{\mathcal{L}_\infty}\right\}.\end{aligned} \tag{50}$$

Similarly, by inserting (49) into (31) and employing (45) we get

$$\begin{aligned}\left|\lambda\right|_{\mathcal{L}_\infty} &\leq \frac{1}{1-g_{1c}g_2}\left\{\frac{g_2}{\sqrt{q_m}}e^{-\alpha_3t}\left|S_\Sigma\left(0\right)\right|\right.\\ &\quad+\frac{\delta k_M}{\sqrt{p_m}}e^{-\frac{1}{2p_M}t}\sqrt{V_c\left(0\right)}\\ &\quad\left.+g_2\frac{q_M\bar{\rho}}{q_m\bar{\varepsilon}}\left|d_1\right|_{\mathcal{L}_\infty}+\frac{2\delta k_Mp_M^2}{p_m}\left|d_2\right|_{\mathcal{L}_\infty}\right\}.\end{aligned} \tag{51}$$

From (50) and (51) we conclude that the feedback interconnection is ISS w.r.t. disturbances d_1 and d_2. ■

V. Example

To illustrate the theoretical results we consider a formation control law for a group of vessels where one of the vessels are in closed-loop with a previously developed path following controller [27]. We briefly review the closed-loop properties for the controller and show that all assumptions of Theorem 4 are satisfied.

Consider a general system

$$\begin{aligned}\dot{x} &= f\left(x,u\right)\\ y &= h\left(x\right)\end{aligned} \tag{52}$$

where $x\in\mathbb{R}^n$ denotes the state vector, $y\in\mathbb{R}^m$ is the system output, and $u\in\mathbb{R}^n$ is the control. To steer y to a prescribed path $\xi(\theta)$, and to assign a speed $\upsilon\left(t\right)$ to $\dot{\theta}$ on this path, [27] studies subclasses of (52) and develops maneuvering design procedures based on backstepping techniques. Other techniques for path-following are considered in, e.g., [28], [29], [30], [31], [32] and [33]. The design are based on the Lyapunov function

$$V\left(z,\theta,t\right)=z^\top Pz,\quad P=P^\top>0 \tag{53}$$

with time-derivative

$$\dot{V}\left(z,\theta,t\right)\leq-z^\top Uz,\quad U=U^\top>0 \tag{54}$$

They lead to a closed-loop system of the form

$$\begin{aligned}\dot{z}&=F\left(x\right)z-g\left(t,x,\theta\right)\omega\\ \dot{\theta}&=\upsilon\left(\theta,t\right)-\omega\end{aligned} \tag{55}$$

where z is a set of new parameters that include the tracking error $y-\xi\left(\theta\right)$ and its derivatives, and ω is a feedback term to be designed such that the desired speed $\upsilon\left(\theta,t\right)$ is recovered asymptotically; that is

$$\omega\to0\text{ as }t\to\infty. \tag{56}$$

$F(x)\in\mathbb{R}^{n\times n}$ and $g\left(t,x,\theta\right)\in\mathbb{R}^n$ depend on the control design. For the systems considered in this paper, $F(x)$ and $g(t,x,\theta)$ are uniformly upper bounded for bounded path and speed derivatives and there are no finite escape times for (55). It then follows from (53) and (54) that

$$\mathcal{M}_c=\left\{(z,\theta,t):z=0\right\}$$

is a uniformly globally exponentially stable set of equilibrium points for (55). Assumption 5 is satisfied with $\varepsilon=\lambda_{\min}\left(U\right)$ and $\rho=\lambda_{\min}\left(P\right)$.

The case study considers fully actuated tugboats in three degrees of freedom (DOF), surge, sway, and yaw. The numerical values for the vessels have been developed using the results in [23]. The 3 DOF horizontal plane vessel model linearized for cruise speeds around $u=5$ $\mathrm{m/s}$ with nonlinear viscous quadratic damping in surge. Furthermore, the surge mode is decoupled from the sway and yaw mode due to port/starboard symmetry. The model is valid for maneuvering at cruise speed $5\,\mathrm{m/s}$ and the

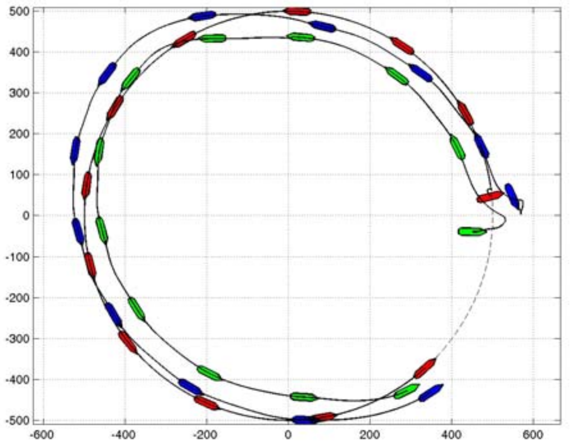

Fig. 4. Position snapshots of three surface vessels in a formation given by (58) where Vessel 1 (shown in red) follows the desired path (57) (dashed line).

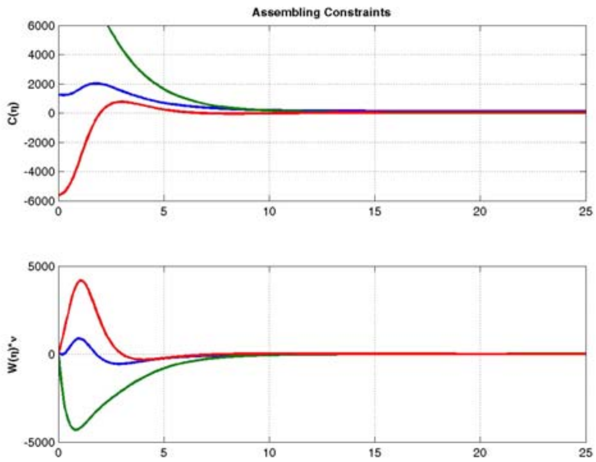

Fig. 5. Time-response of formation constraint functions.

model matrices are

$$M_i = \begin{bmatrix} 180.3 & 0 & 0 \\ 0 & 2.436 & 1.3095 \\ 0 & 1.3095 & 172.2 \end{bmatrix} \times 10^6,$$

$$D_i = \begin{bmatrix} 3.883 \times 10^{-9} & 0 & 0 \\ 0 & 0.2181 & -3.434 \\ 0 & 3.706 & 26.54 \end{bmatrix} \times 10^6$$

and $D_{n,i}(\nu_i) = \text{diag}\left\{-2.393 \times 10^3 |u_i|, 0, 0\right\}$ for $i = 1, 2, 3$.

The goal for Vessel 1 is to follow the desired path, a circle with radius $r = 500$,

$$\xi(\theta) = \begin{bmatrix} x_d(\theta) \\ y_d(\theta) \\ \psi_d(\theta) \end{bmatrix} = \begin{bmatrix} r\cos\left(\frac{\theta}{r}\right) \\ r\sin\left(\frac{\theta}{r}\right) \\ \text{atan2}\left(\frac{y_d^\theta(\theta)}{x_d^\theta(\theta)}\right) \end{bmatrix} \tag{57}$$

while all vessels should remain in the formation configuration

$$\mathcal{C}(\eta) = \begin{bmatrix} (x_1 - x_2)^2 + (y_1 - y_2)^2 - r_{12}^2 \\ (x_2 - x_3)^2 + (y_2 - y_3)^2 - r_{23}^2 \\ (x_3 - x_1)^2 + (y_3 - y_1)^2 - r_{31}^2 \end{bmatrix} = 0 \tag{58}$$

where $r_{12} = 90$, $r_{23} = 60$, $r_{31} = 90$. The formation is exposed to the following disturbances

$$d_1 = \begin{bmatrix} 10^5 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \cdot 10^4 \\ 2 \cdot 10^4 \\ 2 \cdot 10^4 \end{bmatrix} \sin(0.05t), \; d_2 = 2\sin(0.1t) \tag{59}$$

The control parameters in the maneuvering design are set as $P = \text{diag}(0.6, 0.6, 0.6, 10, 10, 40)$ and $U = \text{diag}(-0.6, -0.6, -0.6, -40, -40, -1600)$ while the formation constraint functions are stabilized with $K_p = I$ and $K_d = 2I$. The initial conditions for the vessels are $\eta_1(0) = [498, 43.5, 0]^\top$, $\eta_2(0) = [448, -40, \pi/2]^\top$, $\eta_3(0) = [548, 48, 0, \pi/3]$, $\dot{\eta}_1(0) = [1, 0, 0]$, $\dot{\eta}_{2,3}(0) = 0$ and $\theta(0) = 0$. The speed assignment for Vessel 1, υ_s, is chosen corresponding to a desired surge speed of 2 m/s along the path.

It follows from (20) and (44) that the small-gain condition (45) is satisfied and by Theorem 4 the formation with one path following controller is robust to disturbances. The position response is shown in Figure 4 while the constraint functions for the first 25 seconds are plotted in Figure 5. The plots verify that as Vessel 1 follows the desired path $\xi(\theta)$ the vessels converge to and remain in the desired formation configuration $\mathcal{C}(\eta) = 0$.

VI. Concluding Remarks

We have proved robustness of a closed-loop formation control scheme with respect to environmental perturbations and inter-vessel communication disturbances. The analysis is performed by considering control of the formation as an interconnection of vessels and formation constraint functions and robustness is proved by employing an ISS property. This approach is also valid when some of the vessels are subject to a position control law. For example, when one vessel is in closed-loop with a path following controller, the vessel tracks its desired path while the rest of the group follows according to the formation configuration. The combined control approach enables the control designer to use the available literature of marine control systems and apply it in a formation control setting.

Future research includes an extended analysis to time-delays and other constraints on the communication channels. Furthermore, we will look into other control issues, for example adaptive control, that can be applied for formation control purposes in a similar manner, and try to relax the assumptions on the position control law.

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Appendix

The following lemma is needed for our proofs and is proved in [26, Thm. 5.1]:

Lemma 5: Suppose that $S : [0, \infty) \to \mathbb{R}$ satisfies

$$D^{\dagger} S \leq -\alpha S(t) + \beta(t),$$

where $D^{\dagger}$ denotes the upper Dini derivative, α is a positive constant, and $\beta \in \mathcal{L}_p,\ p \in [1, \infty)$. Then

$$|S|_{\mathcal{L}_p} \leq (\alpha p)^{-1/p} |S(0)| + (\alpha q)^{-1/q} |\beta|_{\mathcal{L}_p},$$

where q is the complementary index of p, i.e., $1/p + 1/q = 1$. When $p = \infty$, the following holds

$$|S(t)| \leq e^{-\alpha t} |S(0)| + \alpha^{-1} |\beta|_{\mathcal{L}_\infty}$$

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