A 4-DOF SIMULINK MODEL OF A COASTAL PATROL VESSEL FOR MANOEUVRING IN WAVES

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Abstract: This paper presents a detailed simulation model of a Naval coastal patrol vessel. The vessel described is a 50m long, fast monohull coastal patrol vessel. The paper describes the complete model and its implementation in Matlab-Simulink®. In order to promote the use of this model, the Simulink files are openly available through a website.

Keywords: Ship model, autopilot, roll stabilisation

1. INTRODUCTION

Detailed mathematical models of marine vessels that are suitable for design and testing of guidance and control systems are difficult to find in the literature. If one finds a model, it is rarely freely and openly available: usually due confidentiality issues. In addition, many of the models suffer from incomplete modelling. Significant effort and engineering judgment is needed to fill in the missing information.

This paper presents a detailed model of a vessel in 4-DOF. This model has been implemented in Matlab-Simulink®, and the files can be accessed through the following website:

`www.cesos.ntnu.no/mss/vessels`.

The rest of the paper describes the model and its implementation.

2. THE VESSEL

The vessel model presented in this paper corresponds to a small relatively fast monohull—Figure A.1 shows the vessel profile. This vessel is a Navy coastal patrol vessel, of a design by ADI-Limited Australia, but was never put into production.

The mathematical model presented in this paper is based on the drawing lines and load conditions provided by ADI-Limited. The design evolved from a Danish vessel, for which a preliminary set of manoeuvring coefficients was published by Blanke and Christensen (1993). The vessel's main particulars and the loading condition are summarized in Table A.1.

The authors have modified the data of ADI-limited so as to match the load condition, such that the seakeeping model can be combined with the manoeuvring model to give a realistic simulation environment. All the seakeeping data were calculated using ShipX-VERES by MARINTEK AS (Fathi, 2004).

3. MATHEMATICAL MODEL IN 4-DOF

The mathematical model presented here consists of a manoeuvring model combined with the fist-order wave excitation forces. These forces are calculated using the vessel force frequency response functions (FRF) and a particular sea spectrum—this is further described in Section 3.4.

The equations of motion are formulated at the origin of the *body-fixed frame* (b-frame), which has coordinates

$$o_b = \begin{bmatrix} Lpp/2 \\ 0 \\ T \end{bmatrix},$$

with respect to the intersection of the aft perpendicular (AP), the centreline, and the base line (BL)—see Figure A.1.

The rigid-body equations of motion in 4-DOF are

$$\begin{aligned} m[\dot{u} - y_g^b \dot{r} - vr - x_g^b r^2 + z_g^b pr] &= \tau_1 \\ m[\dot{v} - z_g^b \dot{p} + x_g^b \dot{r} + ur - y_g^b (r^2 + p^2)] &= \tau_2 \\ I_{xx}\dot{p} - m z_g^b \dot{v} + m[y_g^b vp - z_g^b ur] &= \tau_4 \\ I_{zz}\dot{r} + m x_g^b \dot{v} - m y_g^b \dot{u} + m[x_g^b ur + y_g^b vr] &= \tau_6, \end{aligned} \tag{1}$$

where all the coordinates are body-fixed coordinates, and the position of the centre of gravity (CG) with respect to the body-fixed frame is denoted by

$$\mathbf{r}_g^b = \begin{bmatrix} x_g^b \\ y_g^b \\ x_g^b \end{bmatrix} = \begin{bmatrix} LCG - Lpp/2 \\ 0 \\ T - VCG \end{bmatrix}.$$

The lower script indicates that these are coordinates of the CG, with respect to the b-frame, and the upper script indicates that the vector is expressed in the b-frame.

The vector of forces on the right hand side of (1), can be separated into the following components:

$$\boldsymbol{\tau} = \boldsymbol{\tau}_{\mathrm{hyd}} + \boldsymbol{\tau}_{\mathrm{c}} + \boldsymbol{\tau}_{\mathrm{p}} + \boldsymbol{\tau}_{\mathrm{w}},$$

where subscripts stand for *hydrodynamic and hydrostatic* forces, *control device* forces, *propulsion* forces, and *wave excitation* forces. The mathematical models for the different components are discussed in the following subsections.

3.1 Hydrodynamic forces

The hydrodynamic forces considered in this section are those due to the motion of the vessel in calm water. These forces are modelled as series expansions of the velocities and the roll angle for the restoring terms (Abkowitz, 1964). The following terms are considered:

Surge terms

$$\tau_{1\mathrm{hyd}} = X_{\dot{u}}\dot{u} + X_{vr}vr + X_{u|u|}u|u| \tag{2}$$

Sway terms

$$\begin{aligned} \tau_{2\mathrm{hyd}} = {} & Y_{\dot{v}}\dot{v} + Y_{\dot{r}}\dot{r} + Y_{\dot{p}}\dot{p} \\ & + Y_{|u|v}|u|v + Y_{ur}ur + Y_{v|v|}v|v| \\ & + Y_{v|r|}v|r| + Y_{r|v|}r|v| \\ & + Y_{\phi|uv|}\phi|uv| + Y_{\phi|ur|}\phi|ur| + Y_{\phi uu}\phi u^2. \end{aligned} \tag{3}$$

Roll terms

$$\begin{aligned} \tau_{4\mathrm{hyd}} = {} & K_{\dot{v}}\dot{v} + K_{\dot{p}}\dot{p} \\ & + K_{|u|v}|u|v + K_{ur}ur + K_{v|v|}v|v| + K_{v|r|}v|r| \\ & + K_{r|v|}r|v| + K_{\phi|uv|}\phi|uv| + K_{\phi|ur|}\phi|ur| \\ & + K_{\phi uu}\phi u^2 + K_{|u|p}|u|p \\ & + K_{p|p|}p|p| + K_p p + K_{\phi\phi\phi}\phi^3 - \rho g \nabla GMt\phi. \end{aligned} \tag{4}$$

Yaw terms

$$\begin{aligned} \tau_{6\mathrm{hyd}} = {} & N_{\dot{v}}\dot{v} + N_{\dot{r}}\dot{r} \\ & + N_{|u|v}|u|v + N_{|u|r}|u|r + N_{r|r|}r|r| + N_{r|v|}r|v| \\ & + N_{\phi|uv|}\phi|uv| + N_{\phi u|r|}\phi u|r| + N_p p + N_{|p|p}|p|p \\ & + N_{|u|p}|u|p + N_{\phi u|u|}\phi u|u|. \end{aligned} \tag{5}$$

The numerical values of the coefficients are included in Table A.2

3.2 Propulsion forces

The dynamics of the propulsion system are not included in the model. Rather, it is assumed that the propellers produce a constant thrust T that compensates for the calm water resistance:

$$T = -X_u u_{nom} - X_{u|u|}u_{nom}^2,$$

where the service speed is $u_{nom} = 7.71$ m/s (15kt). Then the motion of the rudders and fins produce drag forces that slow down the vessel. Similarly, the action of the waves produce small deviations from the service speed.

3.3 Control Forces: Rudder and Fins

The vessel model is equipped with two rudders and one pair of stabilizer fins. The lift and drag of the hydrofoils, from Lewis (1988), are calculated according to

$$\begin{aligned} L &= \frac{1}{2}\rho V_f^2 A_f \bar{C}_L \alpha_e \\ D &= \frac{1}{2}\rho V_f^2 A_f \left(C_{D0} + \frac{(\bar{C}_L \alpha_e)^2}{0.9\pi a} \right), \end{aligned} \tag{6}$$

where V_f is the local velocity at the foil, A_f is the area of the foil, α_e is the effective angle of attack in radians, and a is the effective aspect ratio. We use the linear approximation for the lift coefficient:

$$\bar{C}_L = \left. \frac{\partial C_L}{\partial \alpha_e} \right|_{\alpha_e = 0}.$$

Once the stall angle of the hydrofoils is reached, the lift saturates in value. Table A.3 shows the free-stream data for the rudder and fin profiles—this data was also adopted from Lewis (1988).

To calculate the lift of the rudder, the effective angle of attack is approximated by the mechanical angle of the rudder: $\alpha_e \approx \alpha_r$, and the local flow velocity at the rudder is considered to be equal to the vessel's total speed, i.e., $V_f = \sqrt{u^2+v^2}$. Then, a global correction for the lift and drag is used to account for the rudder-propeller interaction (Bertram, 2004):

$$\begin{aligned}\Delta L &= T\left(1+\frac{1}{\sqrt{1+C_{Th}}}\right)\sin\alpha_e,\\ \Delta D &= T\left(1+\frac{1}{\sqrt{1+C_{Th}}}\right)(1-\cos\alpha_e),\end{aligned} \tag{7}$$

where T is the propeller thrust, and C_{Th} is the propeller loading coefficient:

$$C_{Th} = \frac{2T}{\rho V_f^2 A_p},$$

in which A_p is the propeller disc area. The propeller diameter is 1.6 m.

The forces generated by the rudder in body-fixed frames are then approximated by:

$$\begin{aligned}\tau_{1\text{rudder}} &\approx -D\\ \tau_{2\text{rudder}} &\approx \quad L\\ \tau_{4\text{rudder}} &\approx z^b_{CP}\,L\\ \tau_{6\text{rudder}} &\approx x^b_{CP}\,L,\end{aligned} \tag{8}$$

where x^b_{CP} and z^b_{CP} are the coordinates of the center of pressure of the rudder (CP) with respect to the b-frame. The center of pressure is assumed to be located at the rudder stock and half the rudder span. The rudder data is shown in Table A.4.

For the stabilizer fins, the center of pressure is located halfway along the span of the fin. The coordinates of the center of pressure with respect to the b-frame are given by the vector $\mathbf{r}^b_{CP}$—see Figure 1.

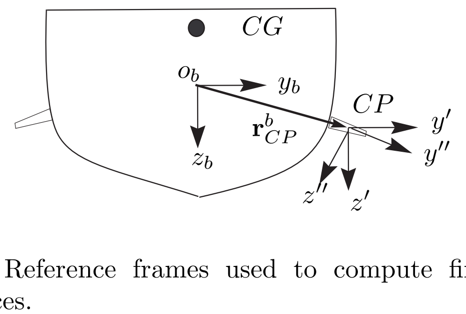

Fig. 1. Reference frames used to compute fin forces.

To calculate the forces of the fins, the velocities in the b-frame are expressed in the frame x', y', z', which is located at the CP for the fin. These velocities are then rotated by the tilt angle of the fin, γ, expressing them in the frame x'', y'', z''. This frame is used to calculate the angle of attack of the fin, and thus calculate the forces and moments generated. The mechanical angle of the fin is defined using the right hand screw rule along the y'' axis: a positive angle means leading edge up: trailing edge down.

3.4 Wave excitation forces

The wave excitation forces are simulated as a multisine time series. This uses the force frequency response functions (FRF) of the vessel in combination with the wave spectrum. The force-FRF were computed using a ShipX VERES (Fathi, 2004) for the service speed and at intervals of 10 deg of encounter angle.

The sea surface elevation is considered as a realization of a random process characterized in terms of a directional sea spectrum $\mathrm{S}_{\zeta\zeta}(\omega,\chi)$. The dominant wave propagating direction is defined in the North-East frame, with propagation angle positive clockwise; that is, if the dominant direction is 0 deg, the waves travel towards north, and if the dominant direction is 45 deg, the waves travel towards the N-E.

The wave dominant direction and the vessel heading are used to find the encounter angle χ between the vessel and the waves. The following convention is adopted:

- $\chi = 0$ deg following seas
- $\chi = 90$ deg beam seas from port
- $\chi = 180$ deg head seas.

The calculation of the forces uses interpolation with a smooth switching of the encounter angle and the speed. The the following formulae are the basis to calculate the forces in the different DOF (Perez, 2005):

$$\tau_{wi}(t) = \sum_{n=1}^{N}\sum_{m=1}^{M}\bar{\tau}_{inm}\cos\left[\omega_{enm}t+\varphi_{imn}\right],$$

for $i = 1, 2, 4, 6$, with

$$\begin{aligned}\omega_{enm} &= \omega_n - \frac{\omega_n^2 U}{g}\cos(\chi_m)\\ \varphi_{imn} &= \arg H_i(\omega_n^*, U, \chi_m^*) + \varepsilon_n\\ \bar{\tau}_{inm} &= \sqrt{2|H_i(\omega_n^*, U, \chi_m^*)|^2 \mathrm{S}_{\zeta\zeta}(\omega_n^*, \chi_m^*)\Delta\chi\Delta\omega},\end{aligned}$$

where H_i are the force FRF of the vessel, and ω_n^* and χ_m^* are chosen randomly in the intervals

$$\left[\omega_n - \frac{\Delta\omega}{2}, \omega_n + \frac{\Delta\omega}{2}\right], \quad \left[\chi_m - \frac{\Delta\chi}{2}, \chi_m + \frac{\Delta\chi}{2}\right].$$

For further details see Perez (2005) and MSS (2004).

4. RUDDER AND FIN HYDRAULIC MACHINERY

The hydraulic machinery moving the fins and rudder are implemented using the model of van

Amerongen (1982). That is, they have a maximum angle α_{max}, and a maximum rate $\dot{\alpha}_{max}$. When working in the unsaturated zone, behave like first order systems with time constant:

$$t = \frac{\alpha_{pb}}{\dot{\alpha}_{max}},$$

where α_{pb} is the so-called proportional band.

5. SIMULATION EXAMPLES

Figures 2, 3 and 4 show the simulation model during two manoeuvres. Figure 2 shows the vessel performing a 20-20 zig-zag test in calm water. Figure 3 shows the ship sailing northwards under the influence of waves, and 4 shows the wave excitation forces acting on the ship during this transit. A p-controller autopilot was applied to achieve this.

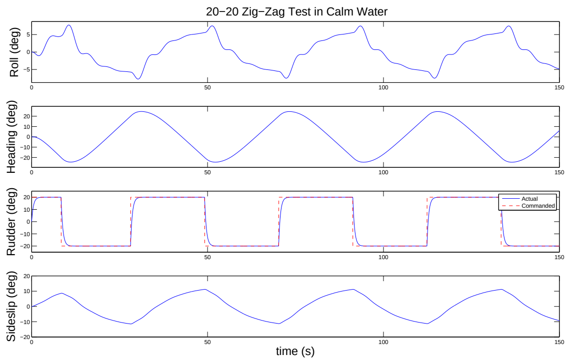

Fig. 2. 20-20 Zig Zag Test in Calm Water

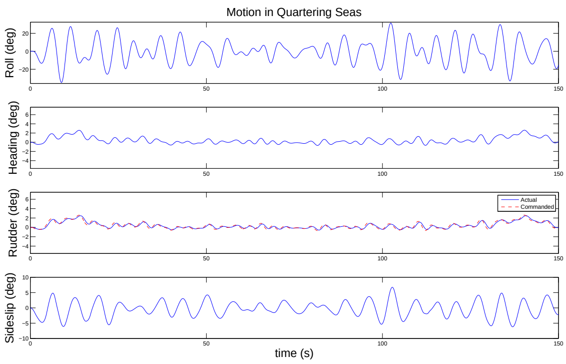

Fig. 3. Motion of the vessel in quartering seas (χ=60 deg). ITTC Spectrum: H_s=2m, t_0=9s

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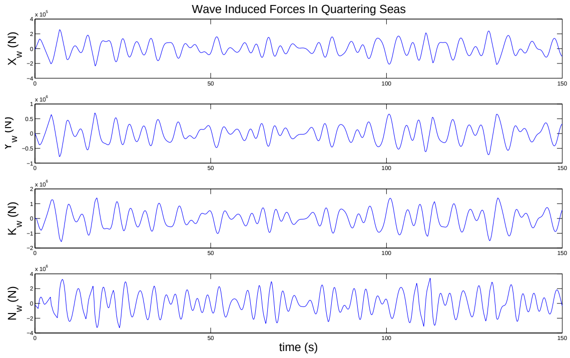

Fig. 4. Wave induced forces on the vessel in quartering seas (χ=60 deg). ITTC Spectrum: H_s=2 m, t_0=9 s

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Appendix A. VESSEL DATA

NOTE: Sea water density $\rho = 1025$ kg/m^3; acceleration due to gravity $g = 9.81$ m/s^2.

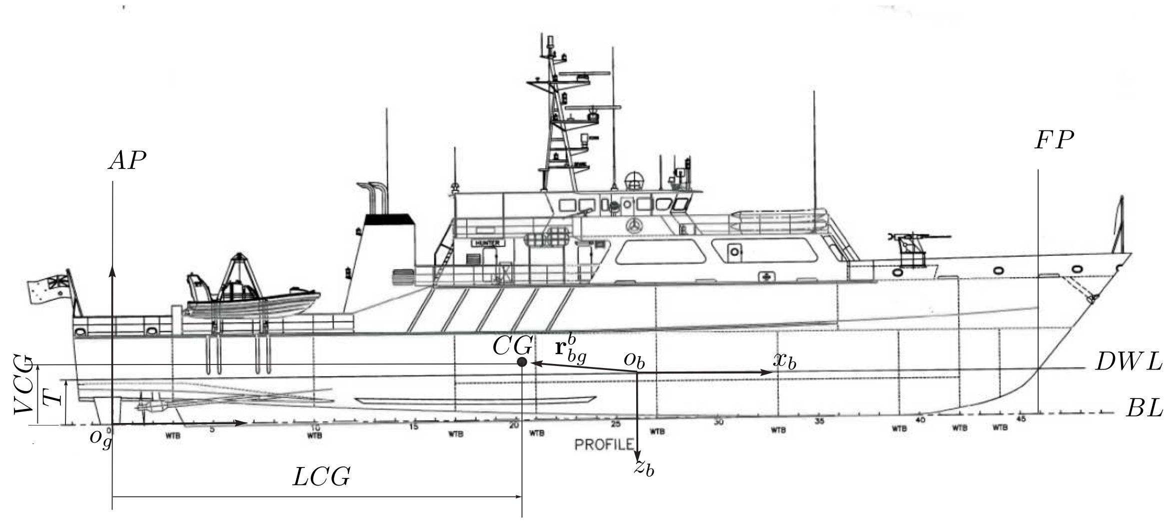

Fig. A.1. Vessel profile and reference frames. Reproduced with permission of ADI-limited Australia.

Table A.1. Main particulars and loading conditions.

Quantity	**Symbol**	**Full Load**	**Unit**
Length between perpendiculars	L_{pp}	51.5	m
Length over all	LOA	52.5	m
Beam over all	BOA	8.6	m
Nominal speed	U	15	kt
Draft at $Lpp/2$ at Design Waterline	T	2.29	m
Length Water Line	L_{WL}	47.702	m
Breadth Water Line	B_{WL}	7.726	m
Trim +ve aft	t	-0.06	m
Displacement mass	Δ	364.78 $\times 10^3$	kg
Displacement volume	∇	355.88 $\times 10^3$	m^3
Lateral Centre of Gravity from AP	LCG	19.82	m
Vertical Centre of Gravity from BL	VCG	3.36	m
Lateral Centre of Buoyancy from AP	LCB	19.82	m
Vertical Centre of Buoyancy from BL	VCB	1.549	m
Lateral Centre of Flotation from AP	LCF	18.27	m
Transverse Metacenter above keel	KMt	4.828	m
Longitudinal Metacenter above keel	KMl	124.15	m
Transverse Metacentric Height	GMt	1.0	m
Longitudinal Metacentric Height	GMl	113.99	m
Transverse Buoyancy to Metacentre	BMt	3.34	m
Longitudinal Buoyancy to Metacentre	BMl	114.49	m
origin of body-fixed frame	o_b	$[Lpp/2, 0, T]$	measured from AP and BL
Roll gyradius (about o_b)	k_4	3.053	m
yaw gyradius (about o_b)	k_6	9.591	m
Inertia Roll (about o_b)	I_{44}	3.4263×10^6	kg m^2
Inertia yaw (about o_b)	I_{66}	3.3818×10^7	kg m^2
Natural Roll Freq.	ω_ϕ	0.93	rad/s
Natural Roll Period	T_ϕ	6.76	s

Table A.2. Manoeuvring coefficients. Adapted from Blanke and Christensen (1993).

X-Coefficients	N-Coefficients	K-Coefficients	Y-Coefficients
$X_{\dot{u}} = -17400$	$N_{\dot{v}}$=538000	$K_{\dot{u}}$=296000	$Y_{\dot{v}} = -1.9022 \times 10^6$
$X_{u\vert u\vert} = -1960$	$N_{\dot{r}} = -4.3958 \times 10^7$	$K_{\dot{r}}$= 0.0	$Y_{\dot{r}} = -1.4 \times 10^6$
X_{vr}= $0.33 \times m$	$N_{\dot{p}}$=0.0	$K_{\dot{p}} = -0.674 \times 10^6$	$Y_{\dot{p}} = -0.296 \times 10^6$
	$N_{\vert u\vert v} = -92000$	$K_{\vert u\vert v} = 9260$	$Y_{\vert u\vert v} = -11800$
	$N_{\vert u\vert r} = -4.71 \times 10^6$	$K_{ur} = -102000$	$Y_{ur} = 131000$
	$N_{v\vert v\vert} = 0.0$	$K_{v\vert v\vert} = 29300$	$Y_{v\vert v\vert} = -3700$
	$N_{r\vert r\vert} = -202 \times 10^6$	$K_{r\vert r\vert} = 0.0$	$Y_{r\vert r\vert} = 0.0$
	$N_{v\vert r\vert} = 0.0$	$K_{v\vert r\vert} = 0.621 \times 10^6$	$Y_{v\vert r\vert} = -0.794 \times 10^6$
	$N_{r\vert v\vert} = -15.6 \times 10^6$	$K_{r\vert v\vert} = 0.142 \times 10^6$	$Y_{r\vert v\vert} = -0.182 \times 10^6$
	$N_{\phi\vert uv\vert} = -0.214 \times 10^6$	$K_{\phi\vert uv\vert} = -8400$	$Y_{\phi\vert uv\vert} = 10800$
	$N_{\phi u\vert r\vert} = -4.98 \times 10^6$	$K_{\phi\vert ur\vert} = -0.196 \times 10^6$	$Y_{\phi\vert ur\vert} = 0.251 \times 10^6$
	$N_{\phi u\vert u\vert} = -8000$	$K_{\phi uu} = -1180$	$Y_{\phi uu} = -74$
	$N_{\vert u\vert p} = 0.0$	$K_{\vert u\vert p} = -15500$	$Y_{\vert u\vert p} = 0.0$
	$N_{p\vert p\vert} = 0.0$	$K_{p\vert p\vert} = -0.416 \times 10^6$	$Y_{p\vert p\vert} = 0.0$
	$N_p = 0.0$	$K_p = -0.5 \times 10^6$	$Y_p = 0.0$
	$N_\phi = 0.0$	$K_{\phi\phi\phi} = -0.325\rho g\nabla$	$Y_\phi = 0.0$
	$N_{\phi\phi\phi} = 0.0$		$Y_{\phi\phi\phi} = 0.0$
			$Y_{\delta uu} = 2 \times 3.5044 \times 10^3$

Table A.3. Free stream data for rudder and fin profiles. Adopted according to data from Lewis (1988).

Surface	**Profile**	**Tip**	a **(eff.)**	$\frac{\partial C_L}{\partial \alpha_e}$ (per deg)	C_{Lmax}	C_{D0}	α_{stall}
Rudder	NACA15	Square	3	0.054	1.25	0.0065	23.0
Fin	NACA15	Square	2	0.046	1.33	0.0065	28.8

Table A.4. Rudder data.

Quantity	**Symbol**	**Measure**	**Unit**
Area	A_R	1.5	m^2
Span	sp	1.5	m
Mean cord	$\bar{c}$	1	m
Eff. aspect ratio	a	3	-
Max. angle	α_{max}	40	deg
Max. rate	$\dot{\alpha}_{max}$	20	deg/s
Hydra. Prop. band	α_{pb}	4	deg
Long. dist. to o_b	x^p_{CP}	25.75	m
Offset port	y^p_{CP}	-2	m
Offset Stbd	y^p_{CP}	2	m
Vert. Dist. to o_b	z^b_{CP}	1.54	m
Dist. CP–propeller	l_{prop}	1.5	m

Table A.5. Fin data

Quantity	**Symbol**	**Measure**	**Unit**
Area	A_f	1.7	m^2
Span	sp	1.3	m
Mean cord	$\bar{c}$	1.3	m
Eff. aspect ratio	a	2	-
Max. angle	α_{max}	35	deg
Max. rate	$\dot{\alpha}$	25	deg/s
Hydra. Prop. band	α_{pb}	10	deg
Tilt angle	γ	65	deg
Position Stbd	$\mathbf{r}^b_{CPs}$	[-5.75,-2.97,1.74]	m
Position Port	$\mathbf{r}^b_{CPp}$	[-5.75,2.97,1.74]	m