LOW ORDER POTENTIAL DAMPING MODELS FOR SURFACE VESSELS

Kari Unneland [*,1] **Thor I. Fossen** [*]
Paul Van Dooren [**] **Olav Egeland** [*]

[*] *Department of Engineering Cybernetics, Norwegian University of Science and Technology, NO-7491 Trondheim, Norway.*
[**] *Department of Mathematical Engineering, Universite Catholique de Louvain, B-1348 Louvain la Neuve, Belgium.*

Abstract: The modeling of the radiation forces in a 3 degrees of freedom (DOF) model of a surface vessel for dynamic positioning (DP) is addressed. The radiation forces consist of added mass and potential damping. These forces are passive, and they can be represented in state-space form before they are included in the overall DP model. The state-space models representing the potential damping are of high order and it is necessary to use model reduction techniques to make them efficient when designing marine control systems. Different reduction procedures are applied, with the aim of keeping the passivity properties of the potential damping. A novel passivity preserving reduction technique is proposed and compared with existing techniques. Previous approaches have relied on reducing SISO-systems representing the potential damping in every mode. Here the modes which have couplings are combined in MIMO-systems before reduction. This approach takes better care of the coupled dynamics and in addition the order of the model can be reduced significantly compared to the SISO-representation.

Keywords: Vessel Modeling, Radition Forces, Potential damping, Model reduction

1. INTRODUCTION

In this paper, special attention is given to the representation of radiation forces in a 3 DOF model of a surface vessel for DP. The radiation forces describes the interaction of the ship with the surrounding fluid. The radiation forces are an important part of the ship model, and recently is has been shown how they can be represented in state-space form and in this way included in the overall ship model (Fossen and Smogeli, 2004). The radiation forces can be split in one part due to added mass and one part due to potential damping. The state-space models representing potential damping are usually of high-order, and it is necessary to use model reduction techniques to make them efficient in models for control and simulation. It has been shown that the state-space models of the potential damping are positive real (Unneland and Egeland, 2006), this needs to be addressed when reducing the order of the models.

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In Section 2, the equations of motion for a seagoing vessel is presented, whereas special attention is given to the state-space representation of the potential damping. The potential damping terms have usually been represented as SISO-models for every mode, and it is suggested to combine the modes which have couplings in MIMO-systems. With this approach the dynamics of the overall radiation forces will be better kept during model reduction; the same dynamics can be expressed with lower order models. In Section 3, the different model reduction algorithms intended for use will be presented. A novel algorithm is proposed, with the aim of keeping the positive real properties of the potential damping. In Section 4, the different algorithms are applied to the state-space models of the potential damping. First the algorithms are compared in terms of accuracy. Subsequently order reduction of the SISO-systems representing the potential damping and the new proposed MIMO-representation will be compared. Finally, in Section 5 the conclusions are given.

2. EQUATIONS OF MOTION

By using the Newton-Euler equations of motion for rigid bodies the forces and moments working on the 3 DOF horizontal rigid body, which represents the DP vessel, can be incorporated into the model by means of force and moment superposition (Fossen, 2002):

$$\dot{\eta} = R(\psi)\nu \quad (1)$$

$$M_{RB}\dot{\nu} + C_{RB}(\nu)\nu = \tau_R + \tau_A + \tau_c + \tau_{FK+diff}. \quad (2)$$

Here M_{RB} is the rigid-body system inertia matrix, C_{RB} is the rigid-body Coriolis and centripetal matrix, τ_R represents the radiation forces, τ_A represents the control forces, τ_c represents the current forces and $\tau_{FK+diff}$ represents the Froude-Krylov and diffraction forces. The generalized position vector is defined in the North-East-Down frame (n-frame) as a combination of the positions in surge and sway and the yaw Euler angle:

$$\eta = [n, e, \psi]^T. \quad (3)$$

And the generalized velocity vector is defined in the Body-fixed frame (b-frame) as a combination of the linear velocities in surge and sway and the angular velocity in yaw, of the moving vessel in the b-frame with respect to the n-frame:

$$\nu = [u, v, r]^T. \quad (4)$$

The position vector η is related to the velocity vector ν through the rotation matrix in yaw $R(\psi)$:

$$\dot{\eta} = R(\psi)\nu = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \nu. \quad (5)$$

2.1 Radiation forces and moments

The expression for the radiation forces, τ_R, can be found using potential theory programs like WAMIT, VERES or Octopus Seaway. The Hydrodynamic frame (h-frame) used in these programs do not correspond to the coordinate frames used in the equations of motion for control, observers and simulator for surface vessels (Fossen and Smogeli, 2004). Due to this, it is assumed for the rest of the paper, that the h-frame potential coefficients are computed in CG and the b-frame origin coincides with the h-frame origin. Hence for small rotations, we can assume that these frames coincide. Using numerical hydrodynamics we get an expression including the added mass, $A(\omega_e)$, and potential damping, $B(\omega_e)$, forces and moments. These are dependent on the frequency of encounter ω_e (Ogilvie, 1964):

$$\tau_R = -A(\omega_e)\dot{\nu} - B(\omega_e)\nu. \quad (6)$$

For a moving vessel, energy is supplied to the fluid through generated waves. This is represented by the potential damping forces, and these forces are proportional to the vessel velocities. When the vessel is accelerating, it gets an added mass due to the inertia of the fluid surrounding the hull. This is represented by the added mass forces, and they are are proportional to the vessel accelerations.

Several approaches have been proposed to represent the potential damping in state-space form e.g. (Yu and Falnes, 1995) and (Kristiansen *et al.*, 2005). In this paper an approach by (Kristiansen *et al.*, 2005) will be used, such that

$$\tau_R = -M_A\dot{\nu} - \mu_{pd} \quad (7)$$

$$\dot{\chi} = A_{pd}\chi + B_{pd}\nu \quad (8)$$

$$\mu_{pd} = C_{pd}\chi + D_{pd}\nu. \quad (9)$$

Here the added mass is represented by the matrix $M_A = A(\infty)$ and the potential damping is represented by μ_{pd}. In this setting the potential damping forces can be represented with up to 9 SISO systems depending on the symmetry properties of the vessel. Because of port-starboard symmetry of the vessel presented here, the resulting vessel model will consist of 5 SISO systems ($S_{ii} = (A_{ii}, B_{ii}, C_{ii}, D_{ii})$), where the off-diagonal terms are symmetric ($S_{jk} = S_{kj}$). Note that surge is decoupled from sway and yaw;

$$\mu_{pd}(s) = \left[\begin{array}{c|cc} S_{11} & 0 & 0 \\ \hline 0 & S_{22} & S_{23} \\ 0 & S_{32} & S_{33} \end{array}\right] \nu(s). \quad (10)$$

The state-space models describing the potential damping forces are of high order and it is necessary to use model reduction techniques to make them efficient for computer simulation and control synthesis. Model reduction is the process of converting a high order model to a low order approximation, which captures the main features of the original model.

Previous approaches looking at the model reduction of the state-space models representing the potential damping, e.g. (Kristiansen *et al.*, 2005), has represented every mode in the model as a SISO system, and they have been reduced independent of each other. This is illustrated in Fig. 1. Here, these results are further developed by

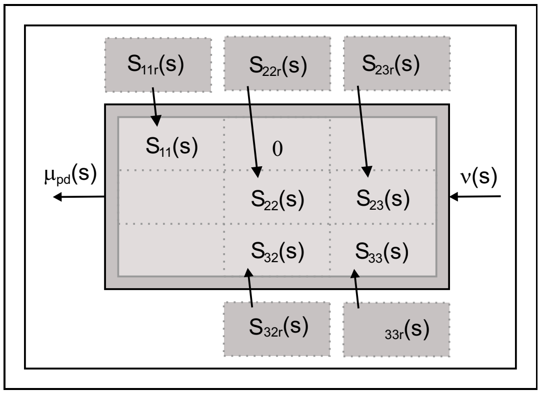

Fig. 1. SISO-reduction of the potential damping.

suggesting structures representing all the SISO systems, which have couplings, as one MIMO system. With this approach one gets 2 systems, one representing the surge dynamics and one system representing the coupled dynamics in sway and yaw. This new approach is illustrated in Fig. 2. By doing this, the order of the model can be

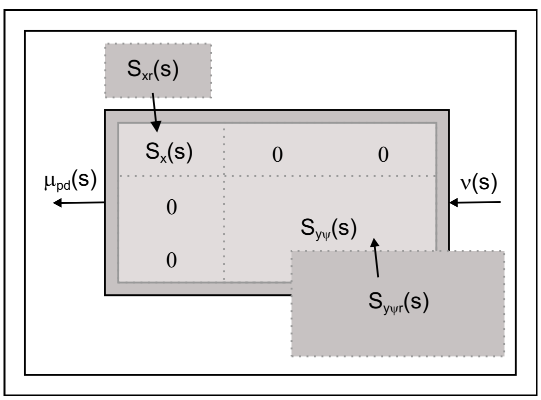

Fig. 2. Reducing the modes with couplings as MIMO-system.

extensively reduced with a satisfying accuracy. With this approach the radiation forces can be written as;

$$\tau_R = -M_A\dot{\nu} - \mu_{pd} \quad (11)$$

$$\dot{\chi} = A_{pd}\chi + B_{pd}\nu \quad (12)$$

$$\mu_{pd} = C_{pd}\chi + D_{pd}\nu. \quad (13)$$

Here $(A_{pd}, B_{pd}, C_{pd}, D_{pd})$ represents two systems; the dynamics in surge $(S_x{=}(A_x, B_x, C_x, D_x))$ and the coupled dynamics in sway and yaw $(S_{y\psi}{=}(A_{y\psi}, B_{y\psi}, C_{y\psi}, D_{y\psi}))$:

$$A_{pd} = diag[A_x, A_{y\psi}] = diag([A_{11}],[A_{22}, A_{23}, A_{32}, A_{33}]) \quad (14)$$

$$C_{pd} = diag[C_x, C_{y\psi}] = diag([C_{11}],[C_{22}, C_{23}, C_{32}, C_{33}]) \quad (15)$$

$$B_{pd} = \begin{bmatrix} B_x & 0 \\ 0 & B_{y\psi} \end{bmatrix} = \left[\begin{array}{c|cc} B_{11} & 0 & 0 \\ \hline 0 & B_{22} & 0 \\ 0 & 0 & B_{23} \\ 0 & B_{32} & 0 \\ 0 & 0 & B_{33} \end{array}\right] \quad (16)$$

$$D_{pd} = \begin{bmatrix} D_x & 0 \\ 0 & D_{y\psi} \end{bmatrix} = \left[\begin{array}{c|cc} D_{11} & 0 & 0 \\ \hline 0 & D_{22} & 0 \\ 0 & 0 & D_{23} \\ 0 & D_{32} & 0 \\ 0 & 0 & D_{33} \end{array}\right]. \quad (17)$$

2.2 Ship Kinetics

Combining (5), (2) and the equations for the radiation forces (11)-(13), the equations of motion for the vessel are as follows:

$$\dot{\eta} = R(\psi)\nu \quad (18)$$

$$\dot{\chi} = A_{pd}\chi + B_{pd}\nu \quad (19)$$

$$\mu_{pd} = C_{pd}\chi + D_{pd}\nu \quad (20)$$

$$M\dot{\nu} + C_{RB}(\nu)\nu + \mu_{pd} = \tau_c + \tau_A + \tau_{FK+diff}. \quad (21)$$

Here $M = M_{RB} + M_A$. In the next section we will look at different methods to do the reduction of the state-space models representing the potential damping in (19)-(20). In (Unneland and Egeland, 2006) it is shown that the mapping $\nu \mapsto \mu_{pd}$ representing the potential damping is positive real; this is a property which should be preserved during the reduction. For this model, this states that the systems S_x and $S_{y\psi}$ are positive real.

3. MODEL REDUCTION BY BALANCED TRUNCATION

Given a n^{th}-order LTI system in state space form,

$$\dot{x} = Ax + Bu \quad (22)$$

$$y = Cx + Du, \quad (23)$$

with associated transfer function,

$$G(s) = C(sI - A)^{-1}B + D. \quad (24)$$

The reduced order model, with order $r \ll n$, will be denoted as,

$$\dot{x}_r = A_r x_r + B_r u \quad (25)$$

$$y_r = C_r x_r + D_r u, \quad (26)$$

and the associated reduced order transfer function,

$$G_r(s) = C_r(sI - A_r)^{-1}B_r + D_r. \quad (27)$$

Model reduction based on balanced truncation is a commonly used scheme. The idea behind the balanced truncation is to transform the system to a balanced representation in terms of some physical measure, and discard the parts of the dynamics which are less important in terms of this measure. The most commonly used method is Lyapunov balancing, which balances the system in terms of the controllability and observability gramians.

3.1 Lyapunov balancing

The Lyapunov balanced realization was introduced to the systems and control society by (Moore, 1981). The Lyapunov balancing procedure is based on information from two Lyapunov equations giving the controllability gramian, W_c, and the observability gramian, W_o,

$$AW_c + W_cA^T = -BB^T \tag{28}$$

$$A^TW_o + W_oA = -C^TC. \tag{29}$$

Notice that the gramians are positive definite if the system is minimal. The idea behind the Lyapunov balancing is to transform the mathematical model to a basis where the states which are difficult to control are also difficult to observe, and the reduced model is obtained by discarding the states which have this property. Below a general algorithm for balanced truncation is written out sequentially.

<u>Balanced truncation algoritm:</u>

(1) Choose a pair of positive definite matrices; M_c, M_o.
(2) Solve out Cholesky factors; $M_c = L_cL_c^T$, $M_o = L_oL_o^T$.
(3) Calculate SVD of Cholesky product; $U\Sigma V = L_o^TL_c$.
(4) Get the balancing transformations; $T = L_cV\Sigma^{-1/2}$, $T^{-1} = \Sigma^{-1/2}U^TL_o^T$.
(5) Compute the balanced realizations; $\hat{A} = T^{-1}AT, \hat{B} = T^{-1}B, \hat{C} = CT$
(6) Truncate $\hat{A}, \hat{B}, \hat{C}$ to form the reduced order system A_r, B_r, C_r

Lyapunov balancing can be done by choosing the positive definite matrix M_c equal to the controllability gramian W_c and the positive definite matrix M_o equal to the observability gramian W_o.

When applied to asymptotically stable systems the Lyapunov balancing preserves the stability of the system, but a property like passivity might not be preserved.

3.2 Riccati balancing

Riccati balancing (Desai and Pal, 1984) is used to reduce passive systems, and will keep the positive real properties of the system. The system $G(s)$ is positive real, with minimal realization, if it satisfies the positive real (PR) equations:

$$AX_c + X_cA^T = -B_lB_l^T \tag{30}$$

$$X_cC^T - B = -B_lD_l^T \tag{31}$$

$$-D - D^T = -D_lD_l^T \tag{32}$$

$$X_c = X_c^T > 0, \tag{33}$$

which can be solved for B_l and D_l. X_c can be seen as a matrix measuring the energy accumulated in the system, and in that sense analogous to the controllability gramian, W_c, in the balanced truncation algorithm. As an analogon to the observability gramian, W_o, the dual positive real (DPR) equations can be solved for C_r and D_r:

$$A^TX_o + X_oA = -C_r^TC_r \tag{34}$$

$$X_oB - C^T = -C_r^TD_r \tag{35}$$

$$-D - D^T = -D_r^TD_r \tag{36}$$

$$X_o = X_o^T > 0. \tag{37}$$

So-called Riccati balancing can now be done by choosing the positive definite matrix M_c equal to X_c and the positive definite matrix M_o equal to X_o in the balanced truncation algortihm. Notice that the minimal solutions to X_c and X_o are used. When applied to positive real systems the Riccati balancing preserves the positive real property of the system.

3.3 Mixed gramian balancing

In this section a new algorithm for achieving positive real reduced order systems is proposed. When using Riccati balancing the system is balanced based on the solution of two Riccati equations, and when the balanced system satisfies the PR lemma this gives postive real reduced order systems. The solution of two Riccati equations are computationally demanding. The idea behind the new approach is to solve one Riccati equation only. As long as the PR lemma is satisfied, this also holds for the balanced system and for the reduced system.

In this approach we balance the solution of one Riccati equation and one Lyapunov equation. Letting $G(s) = C(sI-A)^{-1}B+D$ denote the original system, it can be balanced by taking the controllability gramain W_c like in (29)

$$AW_c + W_cA^T = -BB^T, \tag{38}$$

and balance it with the minimal solution to X_o in (34)-(37)

$$A^TX_o+X_oA+(X_oB-C^T)(D+D^T)^{-1}(B^TX_o-C)=0. \tag{39}$$

Mixed gramian balancing can now be done by choosing the gramian M_c equal to the controllability gramian W_c and the gramian M_o equal to the output energy gramian X_o in the balanced truncation algorithm.

Definition: The positive real minimal system $G(s)$ is called mixed gramian balanced if,

$$W_c = X_o = \Sigma = diag(\sigma_1 I_{m_1}, \ldots, \sigma_q I_{m_q}), \quad (40)$$

where $\sigma_1 > \sigma_2 > \ldots > \sigma_q > 0$, m_i, $i = 1, \ldots, q$ are the multiplicities of σ_i and $m_1 + \ldots + m_q = n$.

We can now state the following theorem:

Theorem: Let the positive real and minimal system $G(s)$ have the mixed gramian balanced realization

$$G(s) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = \left[\begin{array}{cc|c} A_{11} & A_{12} & B_1 \\ A_{21} & A_{22} & B_2 \\ \hline C_1 & C_2 & D \end{array}\right] \quad (41)$$

with $W_c = X_o = \Sigma = diag(\Sigma_1, \Sigma_2)$ where $\Sigma_1 = diag(\sigma_1 I_{m_1}, \ldots, \sigma_k I_{m_k})$ and $\Sigma_2 = diag(\sigma_{k+1} I_{m_{k+1}}, \ldots, \sigma_q I_{m_q})$. Then the reduced order model,

$$G_r(s) = \left[\begin{array}{cc} A_{11} & B_1 \\ C_1 & D \end{array}\right], \quad (42)$$

obtained by truncation is positive real.

Proof: Since (A, B, C, D) is balanced, the two gramians, Σ, are equal and satisfy one Lyapunov equation (38) and one Riccati equation (39),

$$A\Sigma + \Sigma A^T + BB^T = 0 \quad (43)$$

$$A^T\Sigma + \Sigma A + (\Sigma B - C^T)(D + D^T)^{-1}(B^T\Sigma - C) = 0. \quad (44)$$

Writing out the second equation in terms of its partitioned matrices gives the following $(1,1)$ block;

$$A_{11}^T\Sigma_1 + \Sigma_1 A_{11} + (\Sigma_1 B_1 - C_1^T)(D + D^T)^{-1}(B_1^T\Sigma_1 - C_1) = 0. \quad (45)$$

Since $\Sigma_1 > 0$ the positive realness of the reduced order system (A_{11}, B_1, C_1, D) can be concluded. □

In the next section the different algorithms will be applied to the different representations of the model representing the potential damping in the vessel model, and they will be compared in terms of quality.

4. RESULTS

The numerical computation of the frequency dependent potential damping, $B(\omega_e)$, are done in SEAWAY Octopus, and positive real state space realizations are achieved in all modes. Below the sizes of the different state-space realizations in (10) are listed:

system	S_{11}	S_{22}	S_{23}	S_{32}	S_{33}
order	114	124	69	69	104

First in this section, the different reduction algorithms will be compared, special attention is given to the performance of the novel mixed balancing scheme compared to the Riccati balancing; the only algorithms ensuring positive real reduced systems. All of the SISO-systems S_{ii} are reduced to order 10, using the algorithms in Section 3. This gives an $\|\cdot\|_\infty$-error between the original system and reduced system around 0.01 times the maximum gain for each system. The $\|\cdot\|_\infty$-error between the original and reduced systems for the different systems and algorithms are listed below. The $\|\cdot\|_\infty$-norm gives a bound on the worst-case performance of the systems. Let S_{iir} denote the reduced system of S_{ii}. Notice that because of symmetry the results for S_{23} equals the results for S_{32}, and the latter is omitted.

Balancing :	Lyapunov	Riccati	Mixed
$\|S_{11} - S_{11r}\|_\infty$	$2.19 \cdot 10^3$	$2.30 \cdot 10^3$	$2.11 \cdot 10^3$
$\|S_{22} - S_{22r}\|_\infty$	$1.06 \cdot 10^4$	$1.71 \cdot 10^4$	$1.05 \cdot 10^4$
$\|S_{23} - S_{23r}\|_\infty$	$3.66 \cdot 10^4$	$7.21 \cdot 10^4$	$3.69 \cdot 10^4$
$\|S_{33} - S_{33r}\|_\infty$	$2.28 \cdot 10^6$	$2.63 \cdot 10^6$	$2.16 \cdot 10^6$

All the algorithms give reduced order systems which are positive real, even though the Lyapunov balancing gives no guaranty for this. The Lyapunov balancing and Mixed balancing have the best results. If one restricts oneself the algorithms which guarantee positive real systems, the Mixed balancing gives better results than the Riccati balancing. This result is reflected in Fig. 3, where the 114 states system S_{11} is reduced to different orders with the different algorithms. The model

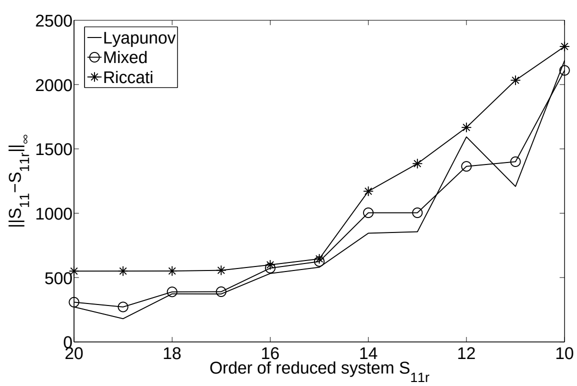

Fig. 3. Reduction error $\|S_{11} - S_{11r}\|_\infty$ for different order of the reduced system S_{11}.

representing the potential damping consists now of 5 SISO-systems of order 10 (the systems obtained by the Lyapunov balancing will be used). We would like to further reduce the order of the system representing the potential damping. Individually reducing the order of the SISO-systems representing the potential damping and the new proposed MIMO-representation will be compared. The latter approach consists of reducing the two systems S_x and $S_{y\psi}$ in (14)-(17) instead of the

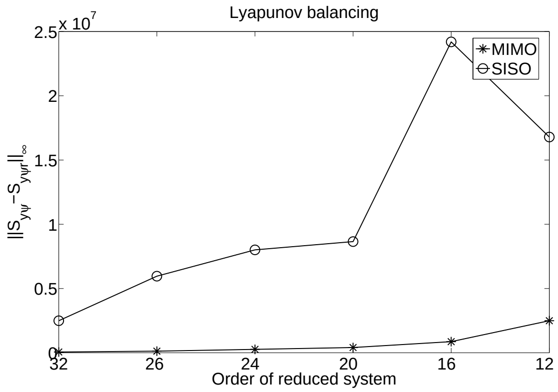

Fig. 4. Comparison of the reduction error when reducing $S_{y\psi}$ as MIMO-system or individually the SISO-systems.

SISO-systems $S_{11}, S_{22}, S_{23}, S_{32}$ and S_{33}, as illustrated in Fig. 1 and 2.

Since S_x only represents the dynamics in surge, this will be the same as reducing the system S_{11}. The interesting point is to see how the reduction of the MIMO-system $S_{y\psi}$ compares to reducing the SISO-systems S_{22}, S_{23}, S_{32} and S_{33} individually, and then let them represent the dynamics in sway and yaw. Below the $\| \cdot \|_\infty$-error is listed for the two different approaches and for the different algorithms when reducing the system from an order of 40 to 24;

$\|S_{y\psi} - S_{y\psi r}\|_\infty$

Balancing method :	SISO− representation	MIMO− representation
Lyapunov	$8.00 \cdot 10^6$	$2.66 \cdot 10^5$
Riccati	$9.04 \cdot 10^6$	$8.99 \cdot 10^6$
Mixed	$8.52 \cdot 10^6$	$2.25 \cdot 10^6$

One can see that for all algorithms the new MIMO formulation gives better performance. The new approach reflects that by reducing the overall systems as SISO systems one might cancel dynamics which are important for the modes which are coupled. In the new formulation the dynamics are better kept in lower order models. In Fig. 4 the difference between the SISO-reduction and MIMO-reduction using Lyapunov balancing is illustrated. All the reduced systems were positive real. As long as the Lyapunov algoritm gives positive real reduced systems, this is a good algoritm to use in terms of cost. The Mixed balancing shows better performance than the Riccati balancing for this type of system, but further investigation with other systems is needed before any conclusions can be drawn. It is a cheaper algorithm than the Riccati balancing; due to this it is preferred if it gives better results.

5. CONCLUSIONS

A 3 DOF model of a surface vessel for DP has been presented; special attention was given to the representation of potential damping forces in state-space form. A new formulation which includes modes which have couplings in MIMO-systems has been suggested, compared to the classical approach where all modes are represented as SISO-systems. This formulation has shown to be effective when the order of the potential damping is reduced; the same dynamics can be expressed with lower order models than in the SISO-approach. The potential damping forces are positive real, and a novel reduction algorithm keeping the positive real properties has been proposed; mixed gramian balancing. This approach is computationally more efficient than Riccati balancing which is usually used for positive real systems. Compared to the Riccati balancing, the new algorithm produces reduced order model with lower $\| \cdot \|_\infty$-error for the potential damping. Further investigation with other systems is needed before any general conclusion can be drawn.

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