Nonlinear Observer Design for Parametric Roll Resonance

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ABSTRACT

In this work a nonlinear two degrees-of-freedom (DOF) ship model is considered. For this model a state observer is designed in this paper. The type of state observer that is chosen is a Extended Kalman filter (EKF). First theory for the design of a EKF is presented. Using this theory an observer in the form of a EKF is then designed to estimate the wave frequency and wave encounter frequency for the ship experiencing parametric roll resonance. The performance of the proposed EKF is verified by a simulation study. In the simulation study the estimates of the observer are compared against fictive measurements generated by a measurement generation model. The model determines the necessary control input to obtain a desired velocity and contains a wave spectrum model to excite the ship model.

Keywords: Parametric roll, Nonlinear system, Observer Design, Extended Kalman Filter

1. INTRODUCTION

Parametric roll resonance is a dangerous nonlinear resonance phenomenon for ships that can cause roll oscillations up to $\pm 40^\circ$ (France et al., 2003). Certain ship types such as fishing vessels and container ships have been shown to be especially prone to parametric roll resonance in moderate to heavy head or stern seas, with long crested waves. Parametric roll is induced by variations in the ship's transverse stability during a wave passage. (Carmel, 2006; France et al., 2003)

A lot of research has been done to model parametric roll and the ship's dynamics during parametric roll, see for instance (Fossen and Nijmeijer, 2012) and (Neves and Rodríguez, 2005) and the references therein. Recent active control approaches are based on changing the ship's forward velocity (Holden et al., 2012), resulting in a Doppler shift of the frequency at which the ship is meeting the waves, the wave encounter frequency. Frequency detuning control can be used to alter the wave frequency out of a (ship-specific) frequency range where the ship is susceptible to parametric roll (Holden et al., 2012).

Assuming constant heading, the wave encounter frequency is dependent on the wave frequency and the ship's speed. Frequency detuning control generally assumes the knowledge of both. However, while the ship's speed is generally known the wave frequency is not and is nontrivial to obtain by measurements in a practical situation at sea. Consequently, a state observer to estimate the wave frequency and wave encounter frequency is crucial for the applicability of the frequency detuning control.

The 2-DOF ship model under investigation is a state-space model with five states. These states are the roll angle φ, the roll rate $\dot{\varphi}$, the velocity in surge direction u, the wave frequency ω_0 and the wave encounter frequency ω_e. This results in the state vector:

$$x = \left[\varphi, \dot{\varphi}, u, \omega_0, \omega_e\right]^\top \qquad (1)$$

The roll angle, roll rate, and the velocity in surge direction are assumed to be measured. The wave frequency and wave encounter frequency need to be estimated by the state observer.

The paper is organized as follows. In Section 2 the model of the ship under investigation is introduced. The theory for the design of a EKF is introduced in Section 3. Followed by a simulation study in Section 4 where the EKF is used to estimate the wave frequency and wave encounter frequency.

2. THE SHIP MODEL

In this section the ship model under investigation with state vector (1) is introduced.

Table 1: Model parameters

Sym.	Quantity
m_1	mass and added mass in surge
d_1	linear damping in surge
μ	control input in surge
m_2	added mass and inertia in roll
d_2	linear damping in roll
B_i	disturbance amplitude
w_i	velocity disturbance frequency
$\overline{GM}_a$	amplitude of change in meta-centric height
$\overline{GM}_m$	mean value of meta-centric height
ρ	density of water
∇	water displacement of the ship
g	gravitational acceleration

The roll dynamics for the ship in Table 1 is given by:

$$m_2\ddot{\varphi} + d_2\dot{\varphi} + k\left(1 + A\cos\left(\omega_e t\right)\right)\varphi = 0 \quad (2)$$

with $A = \overline{GM}_a/\overline{GM}_m$ and $k = \rho g \nabla \overline{GM}_m$. This roll dynamics is a simplified version of the models in (Fossen, 2011).

The surge dynamics is given by:

$$m_1\dot{u}(t) + d_1 u(t) = \mu + \sum_i B_i\cos(\omega_i t) \quad (3)$$

where μ is the control input in surge and $\sum_i B_i\cos(\omega_i t)$ is a disturbance due to a sinusoidal wave spectrum. The wave frequency is assumed to be constant and the wave encounter frequency can be obtained by (Fossen, 2011):

$$\omega_e(u(t), \omega_0, \beta_\omega) = \left|\omega_0 - \frac{\omega_0^2}{g}U(t)\cos(\beta_w)\right| \quad (4)$$

The assumption is made that the velocity in sway is much smaller then the velocity in surge such that the speed of the ship is given by: $U(t) = \sqrt{u^2(t) + v^2(t)} \approx u(t)$. Moreover, it is assumed that the ship is sailing in head seas ($\beta_w = 180°$). Hence, (4) is reduced to:

$$\omega_e(u(t), \omega_0) = \left|\omega_0 + \frac{\omega_0^2}{g}u(t)\right| \quad (5)$$

Using (2)-(5) a state-space model for the system can be formulated as:

$$\dot{x} = f(x, \mu, t) = \begin{bmatrix} x_2 \\ \frac{-(d_2x_2 + k(1+A\cos(x_5 t))x_1)}{m_2} \\ \frac{\mu}{m_1} - \frac{d_1x_3}{m_1} + \sum_i B_i\cos(\omega_i t) \\ 0 \\ \frac{x_4^2\left(\frac{\mu}{m_1} - \frac{d_1x_3}{m_1} + \sum_i B_i\cos(\omega_i t)\right)}{g} \end{bmatrix}$$
$$y = h(x) = \left[x_1, x_2, x_3\right]^\top \quad (6)$$

3. DESIGN OF EKF

In this section an algorithm for a discrete-time EKF obtained from (Simon, 2006) will be presented. However, to implement the algorithm the model (6) needs to be discretized first. The discretization takes the form:

$$\begin{aligned} x_k &= F_{k-1}x_{k-1} + G_{k-1}\mu_{k-1} + w_{k-1} \\ y_k &= H_k x_k + v_k \end{aligned} \quad (7)$$

where

$$\begin{aligned} F_{k-1} &= \exp\left(\left.\frac{\partial f}{\partial x}\right|_{\hat{x}_{k-1}^+} T\right) \\ G_{k-1} &= \int_0^T \exp\left(\left.\frac{\partial f}{\partial x}\right|_{\hat{x}_{k-1}^+} \tau\right) \mathrm{d}\tau \frac{\partial f}{\partial \mu} \\ H_k &= \left.\frac{\partial y_k}{\partial x}\right|_{\hat{x}_k^-} \end{aligned} \quad (8)$$

Here T is the sampling time chosen as $0,1$ seconds and w_{k-1} and v_k are Gaussian white noise processes with zero mean and covariance matrices Q and R, respectively. The covariance matrix of the measurement noise R is chosen as a constant diagonal matrix with entries based on the variances of the sensors, that is,

$$R = \mathrm{diag}(\sigma_\varphi^2, \sigma_{\dot{\varphi}}^2, \sigma_u^2)$$

The estimation of the process noise covariance matrix Q is more complicated. For the measured states it should represent the uncertainty in the model equations. Note that for equations that are model exact, like the first equation of (6), process noise can be omitted or given a small positive value for numerical purposes. For the states to be estimated it should represent the covariance of the noise driving the state estimates. These driving noise terms for the wave frequency and wave encounter frequency should not be chosen too large or the filter might become too aggressive in its adjustments. Moreover the state estimates might converge to the wrong value if Q is too large. Especially the entry for the wave frequency should be chosen very small since estimation of the wave frequency is a parameter estimation problem. Making Q a constant diagonal matrix:

$$Q = \mathrm{diag}(Q_\varphi, Q_{\dot{\varphi}}, Q_u, Q_{\omega_0}, Q_{\omega_e})$$

Now the EKF can be initialized by:

$$\begin{aligned} \hat{x}_0^+ &= E(x_0) \\ P_0^+ &= E[(x_0 - \hat{x}_0^+)(x_0 - \hat{x}_0^+)^T] \end{aligned} \tag{9}$$

here $\hat{x}_0^+$ denotes the original estimate for the state and P_0^+ is the initial state error covariance.

Then with each discrete-time step of the Kalman filter the following has to be calculated. First the time update is done:

$$\begin{aligned} P_k^- &= F_{k-1} P_{k-1}^+ F_{k-1}^T + Q \\ \hat{x}_k^- &= f_{k-1}(\hat{x}_{k-1}^+, u_{k-1}, 0) \end{aligned} \tag{10}$$

with P_k^- the error covariance matrix of the error $\hat{x}_{k-1}^+ - \hat{x}_k^-$. In the time update the model is applied to the system to update the state estimate. The state error covariance is updated using the linearized system matrix form (8), the a posteriori state error covariance from the previous time step and the covariance of the process noise.

The measurement update can then be done as in (Simon, 2006):

$$\begin{aligned} K_k &= P_k^- H_k^T (H_k P_k^- H_k^T + R)^{-1} \\ \hat{x}_k^+ &= \hat{x}_k^- + K_k(y_k - h_k(\hat{x}_k^-, 0)) \\ P_k^+ &= (I - K_k H_k) P_k^- (I - K_k H_k)^T + K_k R K_k^T \end{aligned} \tag{11}$$

where K_k is the Kalman gain, $\hat{x}_k^+$ is the a posteriori state estimate after the k-th time step and P_k^+ is the error covariance matrix of the error $\hat{x}_k^+ - \hat{x}_k^-$. The Kalman gain is calculated using the linearized measurements of (8) the a priori state error covariance and the covariance of the measurement noise. The estimate of the states is adapted by multiplying the error in the measured variables with the Kalman gain. Finally the a posteriori state error covariance is calculated using the Kalman gain and the covariance of the measurement noise.

As mentioned in (Simon, 2006) alternate formulations for K_k and P_k^+ can be chosen. However the formulations chosen here are to guarantee that P_k^+ is symmetric and positive definite as long as P_k^- is positive definite.

4. FREQUENCY ESTIMATION

The EKF of Section 3 will now be applied to the ship model (6) to create a EKF for the ship model. The EKF will be analyzed in a simulation study. However, to preform the measurement update, the Kalman filter needs a measurement of the roll angle, roll rate, and the surge velocity. Hence, a model to generate a fictive measurement is created first. This model includes a second-order wave model and velocity controller. The wave model is based on the second-order wave model found in (Fossen, 2011) and takes the form:

$$h(s) = \frac{2\lambda\omega_0\sigma s}{s^2 + 2\lambda\omega_0 s + \omega_0^2} \tag{12}$$

with λ as a damping coefficient and σ is a constant describing the wave intensity. Both of these parameters are determined by a linearization of the wave spectrum as in (Fossen, 2011). The velocity controller is designed such that it keeps the velocity at a pre-described reference value, r. The desired velocity u_d is generated by second-order reference model:

$$\ddot{u}_d + 2\xi\omega_{lp}\dot{u}_d + \omega_{lp}^2 u_d = \omega_{lp}^2 r \tag{13}$$

with $\omega_{lp} = 0,1\,\mathrm{rad/s}$ and relative damping $\xi = 1$.

Combining this with the surge model of (3) and this yields:

$$\begin{aligned} \dot{u} &= -\frac{d_1}{m_1}u + \frac{1}{m_1}\mu \\ \mu &= m_1\left(\dot{u}_d + \frac{d_1}{m_1}u_d\right) \end{aligned} \tag{14}$$

the following acceleration profile is obtained:

$$\dot{u} = \dot{u}_d + \frac{d_1}{m_1}(u - u_d) \tag{15}$$

The wave disturbance and the measurement noise are added to the velocity after numerical integration to create a disturbed velocity profile. In addition to the wave model and the velocity controller the measurement model also has dynamical equations for the roll angle and roll rate. The EKF tracks these profiles.

For the unmeasured variables a confidence interval for the estimates is also given. This confidence interval is calculated using the diagonal elements of the state error covariance matrix resulting in:

$$\hat{x}_{99\%} = \hat{\bar{x}} \pm 3\sqrt{P_{ii}}$$

A simulation study is now preformed in which the EKF needs to estimate a wave frequency of $0,4684\ \mathrm{rad/s}$. In the simulation study the ship accelerates to a constant velocity of $5,66\ \mathrm{m/s}$. This velocity is chosen such that the wave encounter frequency is about twice the ship's natural roll frequency given the specified wave frequency. As a consequence, the ship is experiencing parametric rolling which is apparent from the roll angle and roll rate plots in Figure 1. The initial conditions for the simulation study are chosen as:

$$\begin{bmatrix}\varphi & \dot{\varphi} & u & \omega_0 & \omega_e\end{bmatrix}^\top = \begin{bmatrix}\frac{2\pi}{90} & \frac{\pi}{90} & 5 & \frac{1}{2} & 0.62\end{bmatrix}^\top$$

The simulation results for the measured variables can be seen in Figure 1. It can clearly be seen from Figure 1 that estimation of the measured variables roll angle, roll rate, and surge velocity is trivial and they are estimated nearly perfect. For the velocity the EKF is tuned such that the filter does not follow all the disturbances on the velocity. This improves the estimate of the wave encounter frequency, which strongly dependents on the velocity. Note that the maximum roll angle that is achieved in the simulation is about $0,5$ rad which corresponds to about $29°$ and that the ship rolls with a angular speed of $0,15$ rad/s which corresponds to about 9 deg/s. This is quite a heavy resonance. The resonance is lost after about 2400 seconds. The effect of this on the estimation of the unknown frequencies will be demonstrated later.

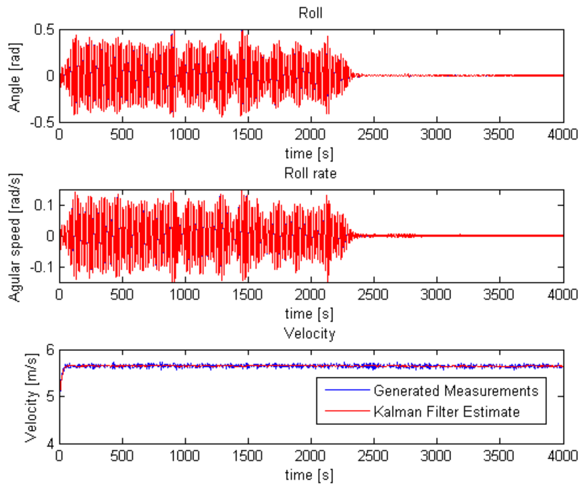

Figure 1: EKF frequency estimation of the measured states for the ship in resonance

The results of the estimation of the wave frequency can be seen in Figure 2. Figure 2 has a plot of the wave frequency estimate including the confidence interval and a plot with a detail of the mean of the wave frequency estimate.

From Figure 2 it can be seen that the mean is estimated quite well until the resonance is lost after this the estimate retains a bias. This is due to the fact that the estimation of the wave frequency is a parameter estimation process. This means that the estimate for the wave frequency is not adjusted in the time update (10) since $\dot{\omega}_0 = 0$. Hence, the state estimate is only ad-

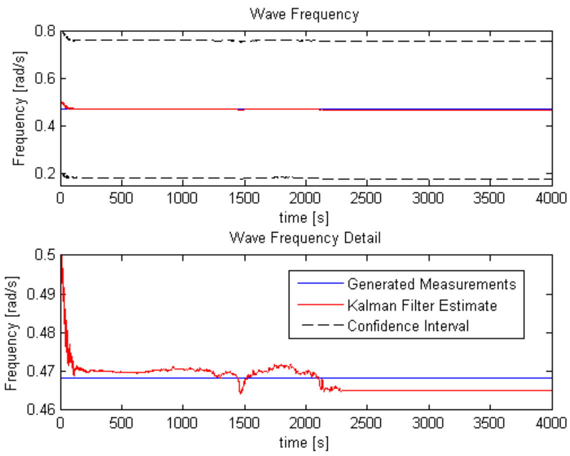

Figure 2: EKF frequency estimation of the wave frequency with detail of the mean estimate

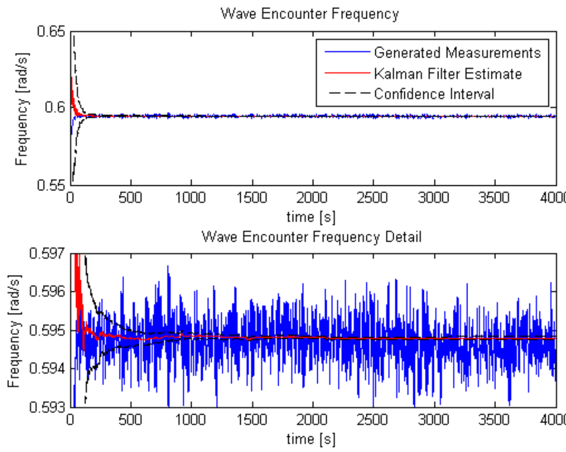

Figure 3: EKF frequency estimation of the wave encounter frequency with a detail of the mean estimate

justed by the measurement update (11). This makes the adjustment of the state estimate dependent on the size of the Kalman gain and the error in the measured variables. When the resonance is lost at about 2400 seconds, the error in the state estimate of the roll angle also reduces significantly and the wave frequency state estimate is hardly adjusted.

From Figure 2 it can also be seen that the confidence interval for the wave frequency estimate only converges very slowly. This can also be attributed to the fact that the estimation of the wave frequency is a parameter estimation process. Since the adaptation of the state error covariance of the wave frequency in (10) is governed by the small process noise term driving the estimate of the wave frequency.

Figure 3 shows the results of the estimation of the wave encounter frequency. The top plot in Figure 3 shows the wave encounter frequency estimate and its confidence interval. The bottom plot in Figure 3 shows a detail of the estimate of the wave encounter frequency and its confidence interval.

From Figure 3 it can be seen that the estimate for the wave encounter frequency converges to the correct value quite fast. Moreover the confidence interval also converges to within acceptable bounds quite fast. Note here that just like with the estimation of the velocity the filter is tuned such that the estimate tracks the mean of the wave encounter frequency and does not try to track all the disturbances. This would cause a filter that is too aggressive and is more likely to diverge. A close inspection of the wave encounter frequency estimate shows that when the resonance is lost, the estimate of the wave encounter frequency is adjusted less and the confidence interval increases a little. This is because the noise terms become more dominant for small values of the states. When the noise is larger with respect to the states the uncertainty in the state estimates becomes larger. Hence, the state error covariance increases.

Good initial performance of the filter is obtained by a suitable choice of the initial state error covariance. For the measured states this is not an issue, however for the unmeasured states it is. If the initial error covariance is chosen too small the filter will be sluggish and it will take longer for the estimate to converge or it might not converge at all. If the initial state error covariance is chosen too large the filter will react very aggressive in the beginning which can cause the state estimate to overshoot the true value and oscillate around it for a while. Both under tuning and over tuning of the initial state error covariance can cause the wave frequency to diverge. Hence, it is important to have a good initial estimate of the wave frequency when initializing the filter. Figure 4 shows the wave frequency and wave encounter frequency for a filter with large initial error covariance to illustrate the mentioned problems.

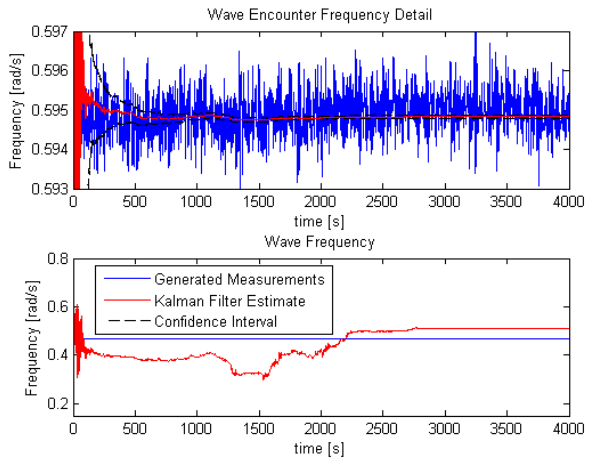

Figure 4: Wave encounter frequency and wave frequency estimate with large initial error covariance

For convergence the choice of the process noise covariance is also important. Large values for the process noise covariance will allow the filter to track the noisy signals of the velocity in Figure 1 and the wave encounter frequency in Figure 3 more closely. However, large values for the process noise covariance will also cause the filter to be more sensitive to e.g. a loss of the resonance which can cause the state estimate of the wave frequency and wave encounter frequency to diverge and their confidence interval to increase. However the filter will always need some process noise to adjust the estimates.

5. CONCLUSIONS

An EKF used as a state observer has been designed to estimate the wave frequency and wave encounter frequency of a ship experiencing parametric roll. This observer is designed on the basis of a ship model. The estimation of the wave frequency proved to be quite difficult due to the fact that the estimation of the wave frequency is a parameter estimation process. It has been shown that this parameter estimation process needs the parametric resonance to readily adjust the state estimate. Moreover the confidence interval only converges very slowly since it is only adapted by the small process noise term driving the filter. The estimate for the mean of the wave encounter frequency is good and converges fast. The confidence interval for the wave encounter frequency also converges to an acceptable value quite fast. It has to be noted though that estimation of the wave encounter frequency also becomes more difficult when the resonance is lost. The filter is sensitive to tuning of the initial state error covariance matrix and to the tuning of the process noise covariance matrix. This can cause a long time for convergence of especially the wave frequency. Hence, for good performance of the filter and convergence it is important to make a suitable choice for the process noise error covariance and the initial state error covariance.

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