Trajectory Tracking and Ocean Current Estimation for Marine Underactuated Vehicles

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Abstract— **In this work, a guidance system for 2-D straight-path tracking applications of underactuated marine vessels exposed to unknown ocean currents is developed. A relative velocity kinematic model is considered, hence making the method suitable for underwater vehicles where absolute velocities might not be available. The total position error vector has two components, the along-track error (tangent to the path) and the cross-track error (normal to the path). Two adaptive nonlinear observers are designed in order to estimate the current components w.r.t. the path-fixed frame and the origin of the error dynamics is shown to be globally κ-exponentially stable. The guidance algorithm uses this information and generates appropriate relative surge speed reference trajectories for minimizing the along-track error, while an augmented version of the line-of-sight (LOS) guidance is designed for minimization of the cross-track error. Moreover, the estimates from the nonlinear observers can be used to compute the ocean current vector w.r.t. the inertial frame.**

I. INTRODUCTION

When it comes to the performance of unmanned vehicles, the importance of their ability to compensate for unknown environmental forces is very high and cannot be overemphasized. More specifically, the motion of marine vehicles is affected by forces due to ocean currents, wind and waves, which can induce large deviations from the desired position on the assigned path. To this end, it can be easily understood that suitable motion control algorithms, which will take into account such uncertain factors, need to be developed. The task of canceling the effects of unknown forces is often assigned to the guidance system, since this is the one responsible for generating reference trajectories to be fed to the control system.

Among the guidance methods used for marine craft applications, LOS guidance has been by far the most popular in the literature and its stability properties have been studied extensively [1]–[4]. However, LOS guidance, in its simplest form, is designed to generate heading reference trajectories in cases where no external disturbances act on the vehicle. In motion control scenarios where this is not the case though, the vehicle will fail to converge to the desired path, that is, the cross-track error will not converge to zero. Its value will depend on factors such as the size of the disturbance, the geometry of the desired path and the dynamics of the vehicle. For that reason, a few modified versions of LOS guidance have been presented in the literature. The most straightforward one is to augment the algorithm by adding integral action, see [5]. This variant can compensate for constant, or slow time-varying, disturbances but is plagued by problems which are well-known to control practitioners, such as windup etc. A much more efficient approach for path-following applications, which was designed to include anti-windup properties, was developed in [6].

Regarding motion control scenarios, *path following* refers to the case where the vehicle is assigned to converge to a path without imposing any time (hence, velocity) constraints, see [7]–[9]. In this case it is sufficient to require that the vehicle has a nonzero forward velocity and the guidance system generates only reference commands for the heading. Contrary to path following, *trajectory tracking* imposes both spatial and temporal constraints, meaning that a vehicle is required to be at a specific position on a path at a specific time. As a result, the guidance system is responsible for generating both heading and velocity reference trajectories. Note that in the literature sometimes the term path tracking is used instead of trajectory tracking. Finally, *path maneuvering* gives priority to the geometric task (that is, converging on the path) before solving the dynamic task (tracking an object on the path), this approach was introduced in [10].

An integrated approach to trajectory tracking for marine craft, based on linearizing the error dynamics, was developed in [11]. Nonlinear approaches with improved results were published in [12]–[15]. Many researchers have also considered the case where ocean currents act on a marine vehicle. The authors in [6], [16]–[18] presented solutions assuming the unknown ocean current is constant, whereas this assumption was relaxed in [19], [20]. In [21] we developed a guidance system capable of minimizing the cross-track and along-track errors for path-tracking applications of underactuated vehicles. The stability proof took into account the cascaded structure between the guidance system and the heading and surge speed autopilots. The guidance system was designed to generate appropriate surge speed reference trajectories for minimizing the along-track error, and the LOS guidance was employed for minimizing the cross-track error.

In this paper, the main contribution is the extension of the work done in [21] by developing a guidance system at a kinematics level suitable for 2-D path-tracking applications of marine vehicles under the influence of unknown constant ocean currents. Regarding the vehicle kinematics, the problem is studied in a *relative-velocity* context, as was

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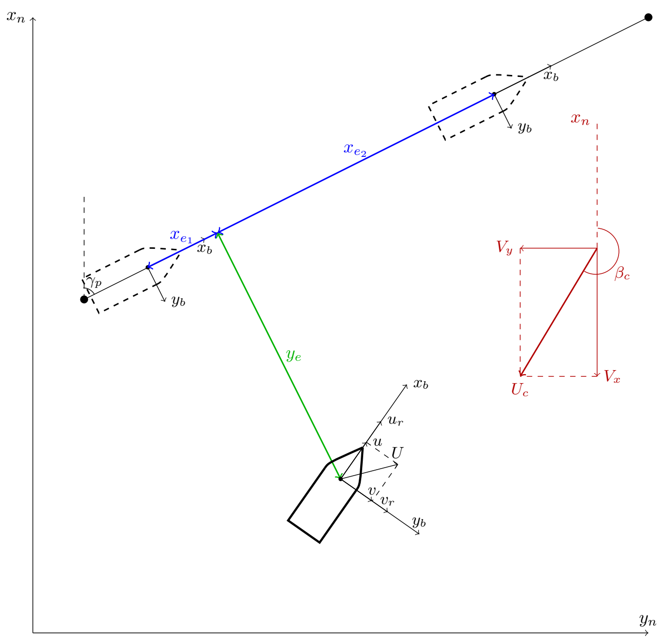

Fig. 1. Illustration of the path-tracking problem solved in this paper. The vehicle (solid line) is tracking a virtual vehicle (dashed line) moving on a straight line under the influence of an unknown constant disturbance.

the case in [17]. A straight-line path is considered and two adaptive nonlinear observers are designed for estimating the two unknown current components w.r.t. the path-tangential frame. It is proved that the estimation error has a globally κ-exponentially stable equilibrium point at zero. The guidance system uses this information to correct the reference trajectories in order to minimize the position error caused by the ocean current. Moreover, a simple transformation is used so as to compute the ocean current parameters (magnitude and orientation) w.r.t. the inertial frame.

II. Vehicle and Virtual Vehicle Models

A. Vehicle Kinematics

The vehicle kinematic equations for horizontal plane motion can be expressed in terms of the relative *surge* and *sway* velocities $u_r = u - u_c$ and $v_r = v - v_c$ according to [7]:

$$\dot{x} = u_r \cos(\psi) - v_r \sin(\psi) + V_x \tag{1}$$
$$\dot{y} = u_r \sin(\psi) + v_r \cos(\psi) + V_y \tag{2}$$
$$\dot{\psi} = r \tag{3}$$

where ψ and r are the yaw angle and rate, respectively. The body-fixed ocean current velocities (u_c, v_c) and North-East current velocities (V_x, V_y) satisfy:

$$[u_c, v_c]^\top = \mathbf{R}^\top(\psi)[V_x, V_y]^\top \tag{4}$$

Notice that the pair (V_x, V_y) is constant in NED, while the body-fixed current velocities (u_c, v_c) depend on the heading angle ψ.

B. Virtual Vehicle Kinematics and Tracking Error

We consider a 2-D continuous straight path that connects two successive waypoints (x_k, y_k) for $k = 1, 2, ..., N$. In this case, the path-tangential angle is constant between the waypoints and can be computed as:

$$\gamma_p = \text{atan2}(y_{k+1} - y_k, x_{k+1} - x_k). \tag{5}$$

For the path-tracking scenario it is reasonable to assume that a virtual particle is navigating with a total speed $U_t > 0$ on the desired path, therefore its position $p_t^n = (x_t, y_t)$ is computed by integrating the inertial velocities:

$$\dot{x}_t = U_t \cos(\gamma_p), \tag{6}$$
$$\dot{y}_t = U_t \sin(\gamma_p). \tag{7}$$

The task is to track the virtual particle, consequently, the position error for a given vehicle position (x, y) is given by:

$$\begin{bmatrix} x_e \\ y_e \end{bmatrix} = \mathbf{R}^\top(\gamma_p) \begin{bmatrix} x - x_t \\ y - y_t \end{bmatrix}, \tag{8}$$

therefore, the along-track and the cross-track error become:

$$x_e = (x - x_t)\cos(\gamma_p) + (y - y_t)\sin(\gamma_p), \tag{9}$$
$$y_e = -(x - x_t)\sin(\gamma_p) + (y - y_t)\cos(\gamma_p). \tag{10}$$

The objective of the vehicle in this case is to track the virtual particle, that is $p - p_t \to 0$. An illustration of the problem can be seen in Fig. 1. Note that using the virtual vehicle approach not only makes it easier and more intuitive to generate desired position references but also helps to avoid singularities when defining the cross-track error, more details are given in [22].

III. Cross-Track Error Minimization

A. Formulating the Cross-track Error Equation

Time differentiation of (10) gives:

$$\begin{aligned}\dot{y}_e = &-(\dot{x} - \dot{x}_t)\sin(\gamma_p) + (\dot{y} - \dot{y}_t)\cos(\gamma_p) \\ &- [(x - x_t)\cos(\gamma_p) + (y - y_t)\sin(\gamma_p)]\,\dot{\gamma}_p.\end{aligned} \tag{11}$$

For a straight line we have that $\dot{\gamma}_p = 0$, therefore the bracket in (11) is zero. Consequently, (1), (2) and (11) give:

$$\begin{aligned}\dot{y}_e &= -\,\dot{x}\sin(\gamma_p) + \dot{y}\cos(\gamma_p), \\ &= -\,(u_r\cos(\psi) - v_r\sin(\psi) + V_x)\sin(\gamma_p) \\ &\quad + (u_r\sin(\psi) + v_r\cos(\psi) + V_y)\cos(\gamma_p),\end{aligned} \tag{12}$$

This can be written in *amplitude-phase form*:

$$\dot{y}_e = U_r \sin(\psi + \beta_r - \gamma_p) + \underbrace{U_c \sin(\beta_c - \gamma_p)}_{\theta_y} \tag{13}$$

where the relative speed and direction are recognized as $U_r = \sqrt{u_r^2 + v_r^2}$ and $\beta_r = \text{atan2}(v_r, u_r)$, respectively. Similarly, for the ocean current parameters we have $U_c = \sqrt{u_c^2 + v_c^2} = \sqrt{V_x^2 + V_y^2}$ and $\beta_c = \text{atan2}(V_y, V_x)$.

B. Conventional LOS Guidance

In the case where no environmental disturbances act on the vessel (that is, $\theta_y = 0$ in (13)) the LOS guidance is formulated as follows:

$$\psi_d = \gamma_p - \beta_r + \arctan(-y_e/\Delta), \tag{14}$$

where $\Delta > 0$ is the user specified lookahead distance. The next section employs indirect adaptive control techniques and extends the conventional LOS algorithm in order to estimate the disturbance effects on the vehicle's motion and, finally, counteract them.

C. Indirect Adaptive Integral LOS Guidance

Due to the presence of unknown external forces, which can affect performance significantly, the LOS is augmented and takes the following form:

$$\psi_d = \gamma_p - \beta_r + \arctan\left(-\frac{1}{\Delta}(y_e + \alpha_y)\right) \tag{15}$$

where α_y is a control input, which can be designed to obtain integral action. Combining (13) and (15) gives:

$$\dot{y}_e = -\frac{U_r(y_e + \alpha_y)}{\sqrt{\Delta^2 + (y_e + \alpha_y)^2}} + \theta_y. \tag{16}$$

Since θ_y is unknown, we propose the following adaptive observer:

$$\dot{\hat{y}}_e = -\frac{U_r(\hat{y}_e + \alpha_y)}{\sqrt{\Delta^2 + (y_e + \alpha_y)^2}} + \hat{\theta}_y + k_1(y_e - \hat{y}_e), \tag{17}$$

$$\dot{\hat{\theta}}_y = \text{Proj}(\hat{\theta}_y, -k_2\tilde{y}_e). \tag{18}$$

Assume that the ocean current is constant and there exists a known constant $M_\theta > 0$ such that $|\theta| \le M_\theta < U_r$. Hence, the parameter adaptation law [1] denotes a parameter projection (Krstic et al. [23], App. E), which ensures that $|\hat{\theta}|$ remains smaller than some design constant $M_{\hat{\theta}} > M_\theta$.

Theorem 1 (Adaptive disturbance observer for θ_y): Assume that ψ_d is computed using (15), α_y is bounded and that $U > U_c$. Then the adaptive observer (17)–(18) with $0 < \Delta_{\min} \le \Delta \le \Delta_{\max}$, and adaptation gains $k_1 > 0$ and $k_2 > 0$ renders the equilibrium point $(\tilde{y}_e, \tilde{\theta}_y) = (y_e - \hat{y}_e, \theta_y - \hat{\theta}_y) = (0,0)$ globally $\kappa-$exponentially stable.

Proof: The observer error dynamics for the system (17)–(18) is a cascaded system:

$$\dot{\tilde{y}}_e = -\frac{U_r}{\sqrt{\Delta^2 + (y_e + \alpha_y)^2}}\tilde{y}_e + \tilde{\theta}_y - k_1\tilde{y}_e \tag{19}$$

$$\dot{\tilde{\theta}}_y = -\text{Proj}(\hat{\theta}_y, -k_2\tilde{y}_e) \tag{20}$$

In order to proof stability of (19)–(20) let $V_3 = (1/2)\tilde{y}_e^2 + 1/(2k_2)\tilde{\theta}_y^2$ be a Lyapunov function candidate. Consequently,

$$\begin{aligned}\dot{V}_3 &= -\frac{U_r\tilde{y}_e^2}{\sqrt{\Delta^2 + (y_e + \alpha_y)^2}} - k_1\tilde{y}_e^2 + \tilde{\theta}\left(\tilde{y}_e + \frac{1}{k_2}\dot{\tilde{\theta}}_y\right) \\ &= -\left(\frac{U_r}{\sqrt{\Delta^2 + (y_e + \alpha_y)^2}} + k_1\right)\tilde{y}_e^2 \le 0. \end{aligned} \tag{21}$$

Since $U_r \ge 0$ the signals $\tilde{\theta}_y$ and $\tilde{y}_e$ are bounded. For the case without projection, that is $\dot{\hat{\theta}} = -k_2\tilde{y}_e$, the equilibrium point $(\tilde{y}_e, \tilde{\theta}_y) = (0,0)$ of (19)–(20) is UGAS/ULES (global $\kappa-$exponential stable). This is seen by writing the error dynamics (19)–(20) in the following form:

$$\dot{x}_1 = f(x_1, t) + g(\mathbf{x}, t)x_2, \tag{22}$$

$$\dot{x}_2 = -k_2 g(\mathbf{x}, t)x_1, \tag{23}$$

where $x_1 = \tilde{y}_e$, $x_2 = \tilde{\theta}_y$, $\mathbf{x} = [x_1, x_2]^\top$ and

$$f(x_1, t) = -\left(\frac{U_r}{\sqrt{\Delta^2 + (y_e + \alpha_y)^2}} + k_1\right)x_1, \tag{24}$$

$$g(\mathbf{x}, t) = 1. \tag{25}$$

Since $g^2(\mathbf{x}, t) = 1 > 0$, the *persistency of excitation* condition is satisfied and consequently all conditions of Theorem 1 in Fossen et al. [24] (alternatively Panteley et al. [25]) are satisfied. Then we have proven that the system (19)–(20) is globally $\kappa-$exponentially stable. Note that if the projection algorithm is used, global convergence of y_e to zero can be proven. ■

The adaptive observer (17)–(18) can be used together with a control signal α_y for cancellation of the drift term θ_y in (16) asymptotically. Let the control objective be to drive $y_e \to 0$ when $\hat{\theta}_y \to \theta_y$. From (16) it is seen that perfect asymptotic tracking $y_e = \tilde{\psi} = 0$ and cancellation of θ_y are obtained for:

$$\alpha_y = \text{sat}\left(\Delta\frac{\hat{\theta}_y/(U_r + \sigma)}{\sqrt{1 - (\hat{\theta}_y/(U_r + \sigma))^2}}\right) \tag{26}$$

where the saturation function is defined as:

$$\text{sat}(x) := \begin{cases} x & \text{if } |x| \le x_{\max} \\ \text{sgn}(x)x_{\max} & \text{if else,} \end{cases} \tag{27}$$

with $\text{sgn}(x)$ denoting the *signum function*. Moreover, we define:

$$\sigma := \begin{cases} 0 & \text{if } U_r > 0 \\ \varepsilon & \text{if } U_r = 0, \end{cases} \tag{28}$$

with $\varepsilon > 0$. The saturating element is used to avoid large values of α, which will give numerical instability, and σ is used to avoid that (26) blows up for when $U_r = 0$.

Remark 1: When deriving (16) we actually assumed perfect heading tracking, that is, $\psi = \psi_d$. This assumption is part of the guidance system design process, that is, finding suitable reference trajectories for minimizing x_e, y_e, and this is why in the next section we will also assume $u = u_{r_d}$. In reality, the convergence time of the control system ($\psi \to \psi_d$, $u_r \to u_{r_d}$) should be taken into account. This can be achieved elegantly using cascaded systems theory. Due to space limitations this analysis is not included in this paper, however the steps are similar to the ones in [21].

[1] $\text{Proj}(\ \cdot, \cdot)$

IV. Along-Track Error Minimization

In this section we combine the guidance system developed in [21] for minimizing the along-track error and the adaptive LOS guidance from the previous section for minimizing the cross-track error. As a result, the guidance system is extended so as to minimize also the along-track error when unknown ocean currents are present. We consider again the along-track error equation:

$$x_e = (x - x_t)\cos(\gamma_p) + (y - y_t)\sin(\gamma_p), \qquad (29)$$

and, similarly to the absolute velocities case, differentiate x_e w.r.t. time:

$$\begin{aligned}\dot{x}_e =& u_r\cos(\psi_d)\cos(\gamma_p) - v_r\sin(\psi_d)\cos(\gamma_p) \\ &+ u_r\sin(\psi_d)\sin(\gamma_p) + v_r\cos(\psi_d)\sin(\gamma_p) \\ &+ V_x\cos(\gamma_p) + V_y\sin(\gamma_p) - U_t, \qquad (30)\\ =& u_r\cos(\psi_d - \gamma_p) + v_r\sin(\gamma_p - \psi_d) \\ &+ \underbrace{U_c\sin(\gamma_p + \beta_{cx})}_{\theta_x} - U_t, \qquad (31)\end{aligned}$$

where β_{cx} is a function of the current velocity components:

$$\beta_{cx} = \operatorname{atan2}(V_x, V_y). \qquad (32)$$

We rewrite (31) in a more convenient form:

$$\dot{x}_e = u_r\cos(\gamma_p - \psi) + v_r\sin(\gamma_p - \psi) + \theta_x - U_t. \qquad (33)$$

Combining (33) and (15) yields:

$$\begin{aligned}\dot{x}_e =& u_r\cos\left(-\beta_r + \arctan\left(-\frac{1}{\Delta}(y_e + \alpha_y)\right)\right) \\ &+ v_r\sin\left(\underbrace{-\beta_r + \arctan\left(-\frac{1}{\Delta}(y_e + \alpha_y)\right)}_{\zeta_t}\right) \\ &+ \theta_x - U_t, \qquad (34)\end{aligned}$$

where ζ_t is computed as follows:

$$\zeta_t = \arctan\left(\frac{v_r\Delta + u_r(y_e + \alpha_y)}{v_r(y_e + \alpha_y) - u_r\Delta}\right). \qquad (35)$$

Note that $\cos(\zeta_t) \neq 0$, the justification can be found in [21]. For the sake of notational brevity we write $\zeta_t := \arctan(\xi_t)$ and rewrite the along-track error propagation equation:

$$\dot{x}_e = \frac{u_r}{\sqrt{1+\xi_t^2}} + \frac{v_r\xi_t}{\sqrt{1+\xi_t^2}} - U_t + \theta_x. \qquad (36)$$

Therefore, a suitable relative velocity reference trajectory u_{r_d} must be computed in order to minimize x_e. Assuming $u_r = u_{r_d}$ (see Remark 1), we observe that choosing:

$$u_{r_d} = \sqrt{1+\xi_t^2}\left(-v_r\frac{\xi_t}{\sqrt{1+\xi_t^2}} + U_t + \alpha_x - k_x x_e\right), \qquad (37)$$

where $\alpha_x = -\hat{\theta}_x$, gives the expression:

$$\dot{x}_e = -k_x x_e + \alpha_x + \theta_x. \qquad (38)$$

Since θ_x is unknown, we proceed with proposing the following observer:

$$\dot{\hat{x}}_e = -k_x\hat{x}_e + \hat{\theta}_x + \alpha_x + k_3(x_e - \hat{x}_e), \qquad (39)$$

$$\dot{\hat{\theta}}_x = k_4(x_e - \hat{x}_e). \qquad (40)$$

Theorem 2 (Adaptive disturbance observer for θ_x): Assume that u_{r_d} is computed using (37) and α_x is bounded. Then the observer (39)–(40), with the gains k_3, k_4 and k_x selected so that the matrix

$$\mathbf{A} = \begin{bmatrix} -(k_x + k_3) & 1 \\ -k_4 & 0 \end{bmatrix} \qquad (41)$$

is Hurwitz, renders the equilibrium point $(\tilde{x}_e, \tilde{\theta}_x) = (x_e - \hat{x}_e, \theta_x - \hat{\theta}_x) = (0, 0)$ GES.

Proof: This follows from linear system theory since the error dynamics for (31), (39) and (40) is GES. ■

V. Estimation of the Ocean Current Parameters

Having estimated both θ_y and θ_x, it is now possible to estimate all the parameters of the current. Starting from:

$$\theta_y = U_c\sin(\gamma_p - \beta_c), \qquad (42)$$

$$\theta_x = U_c\cos(\gamma_p - \beta_c), \qquad (43)$$

first we compute:

$$\hat{U}_c = \sqrt{\hat{\theta}_y^2 + \hat{\theta}_x^2}. \qquad (44)$$

And then, for $U_c \neq 0$, we divide (42) by (43), which gives:

$$\hat{\beta}_c = \gamma_p + \operatorname{atan2}(\hat{\theta}_y, \hat{\theta}_x). \qquad (45)$$

VI. Simulations

The virtual vehicle is moving with speed $U_t = 5$ m/s along the line connecting the waypoints $(0, 0) - (60, 200)$, which gives the path-tangential angle $\gamma_p = 73.3$ deg. The ocean current velocity vector magnitude is $U_c = 1$ m/s and its orientation w.r.t. the inertial frame is $\beta_c = -40$ deg. The initial position and heading of the marine vessel is $(x, y, \psi) = (-20, 10, 0)$ and the lookahead distance for the LOS algorithm $\Delta = 50$ m. The gain values are $k_x = 0.5$, $k_1 = 10$, $k_2 = 0.8$, $k_3 = 10$, and $k_4 = 1$. We assume no prior estimation or measurement of the ocean current is available, therefore the observers' initial conditions are $(\hat{x}_e, \hat{\theta}_x, \hat{y}_e, \hat{\theta}_y) = (0, 0, 0, 0)$. Finally, the simulation duration is 250 seconds.

Fig. 2 shows that the observer is successful in estimating the current effect θ_x in the direction tangent to the path, which is constant in this case because the vehicle is tracking a straight line. As a result, the guidance system generates surge speed commands capable of compensating for the current effect and minimizing the along-track error x_e, which converges to zero. This can be verified by Fig. 4 where it can be seen that, in order to track the virtual vehicle moving with speed $U_t = 5$ m/s, the steady-state relative surge velocity required is $u_{r_d} = 5.47$ m/s.

The same is true for the adaptive nonlinear observer designed to estimate the ocean current effect θ_y in the

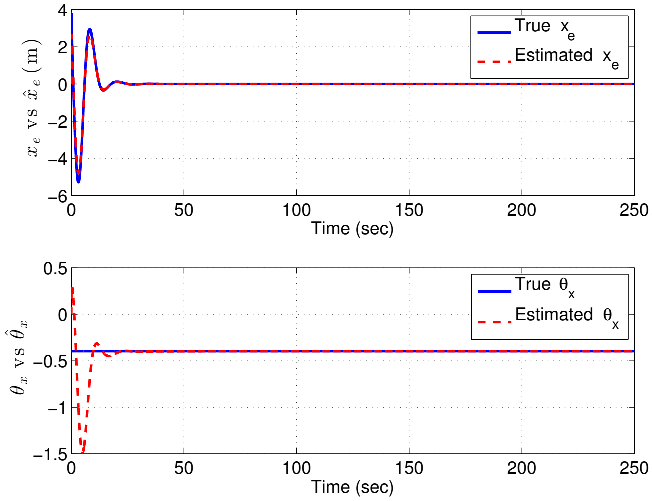

Fig. 2. The adaptive nonlinear observer estimates the ocean current effect tangent to the path, θ_x. The along-track error x_e converges to zero.

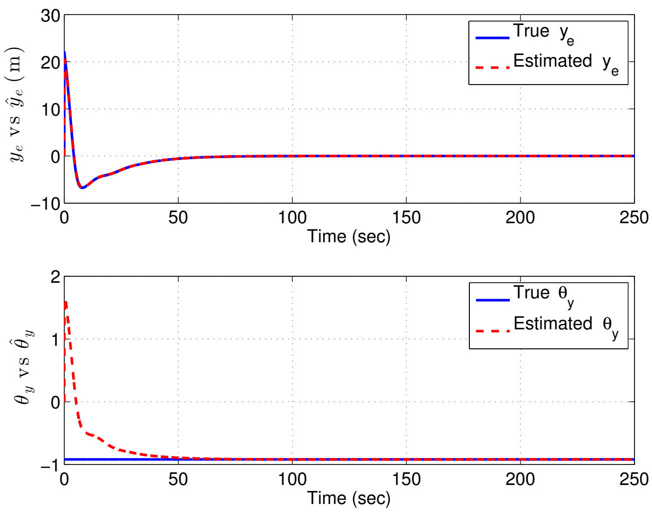

Fig. 3. The adaptive nonlinear observer estimates the ocean current effect normal to the path, θ_y. The cross-track error y_e converges to zero.

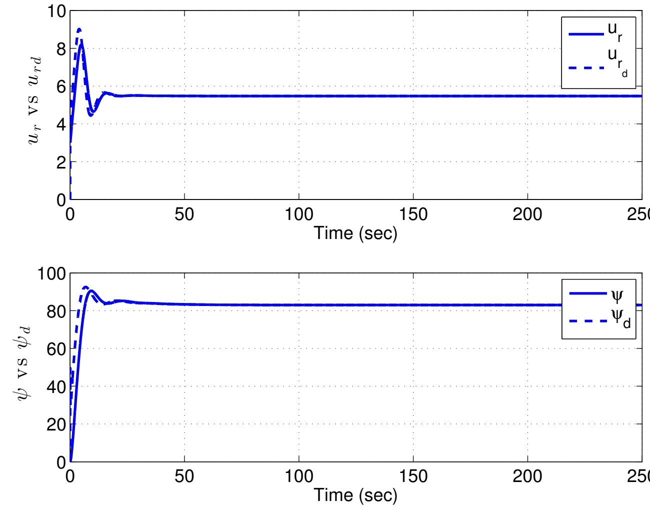

Fig. 4. Comparison between the desired commands, u_{r_d}, ψ_d and the true values u_r, ψ.

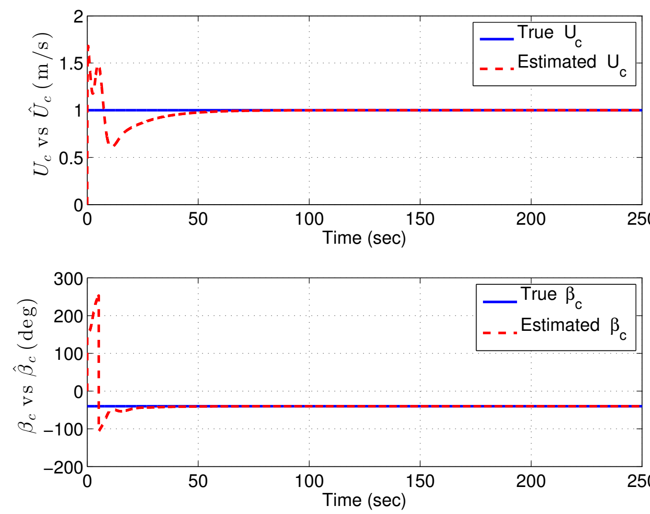

Fig. 5. The methodology presented results in the estimation of both the velocity and orientation of the ocean current w.r.t. the NED frame.

direction normal to the path, see Fig. 3. The augmented LOS guidance algorithm compensates for θ_y and minimizes the cross-track error, which also converges to zero. This is verified by observing Figs 4 and 6. If no disturbance was acting on the vehicle, the steady-state heading angle would have been $\psi_{ss} = 73.3$ deg. However, the vehicle now has to adjust its heading so as to eliminate the current effect θ_y and reaches $\psi_{ss} = 82.96$ deg (Fig. 4), hence resulting in a sideslip angle $\beta_{ss} = -9.66$ deg (Fig. 6).

As mentioned in previous sections, θ_x and θ_y are the tangent and normal, respectively, ocean current components w.r.t. the path-fixed frame. If the vehicle were to track a different straight line, the values for θ_x and θ_y would be different as well. Moreover, if the path was curved, θ_x and θ_y would be time-varying. It is therefore useful to estimate the ocean current parameters, U_c and β_c, in the NED frame using (45)–(44), the plot can be seen in Fig. 5. Fig. 6 shows plots of the relative sideslip angle and absolute sideslip angle. In the steady state, β is nonzero because the heading is not aligned with the course angle, whereas β_r is zero because the vehicle is following a straight line without turning. Finally, Fig. 7 plots the control inputs α_x, α_y and the ratio $\theta_n = \hat{\theta}_y / U_r$.

VII. Conclusions

A guidance system suitable for path-tracking of marine vessels exposed to unknown constant ocean currents was developed. Two adaptive nonlinear observers were designed in order to estimate the the current components normal and tangential to the desired straight path. These estimates are exploited by the guidance system, which incorporates them so as to generate reference trajectories for the speed and heading controllers such that the along-track and cross-track

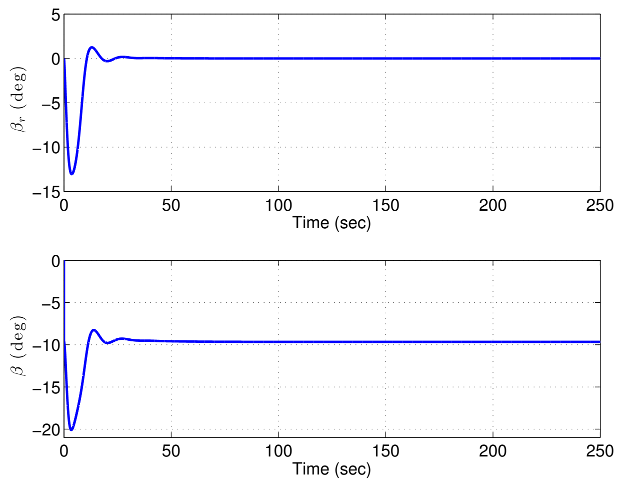

Fig. 6. Plots of the relative sideslip angle β_r and sideslip angle β.

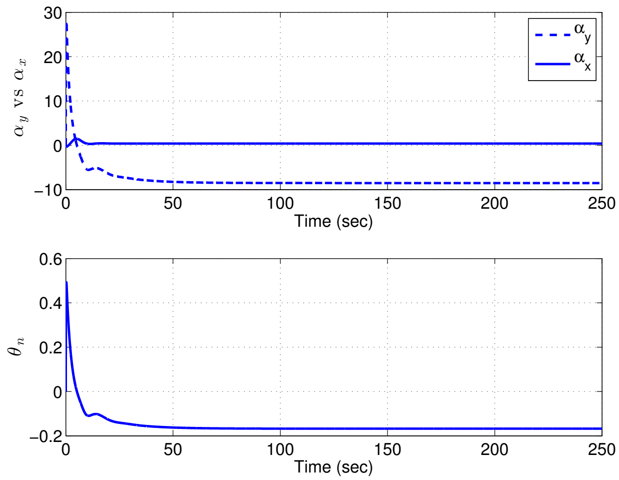

Fig. 7. Plots of θ_n and the control inputs α_x, α_y.

errors (and, therefore, the position error) converge to zero. The approach allows to compute the ocean current parameters in the inertial frame without requiring any absolute velocities measurements.

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