Chapter 7 – Autopilot Models for Course and Heading Control

7.1 Autopilot Models for Course Control
7.2 Autopilot Models for Heading Control

Automatic pilot, “also called autopilot, or autohelmsman, device for controlling an aircraft or other vehicle without constant human intervention.”  Ref. Encyclopedia Britannica

The first aircraft autopilot was developed by Sperry Corporation in 1912. It permitted the aircraft to fly straight and level on a compass course without a pilot's attention, greatly reducing the pilot's workload.

Lawrence Sperry (the son of famous inventor Elmer Sperry) demonstrated it in 1914 at an aviation safety contest held in Paris. Sperry demonstrated the credibility of the invention by flying the aircraft with his hands away from the controls. He was killed in 1923 when his aircraft crashed in the English channel.

In the early 1920s, the Standard Oil tanker J.A. Moffet became the first ship to use an autopilot.  Ref. Wikipedia
Chapter Goals

• Be able to explain the differences of course and heading controlled marine craft. In what applications are they used.
• Understand what the **crab angle** is for a marine craft:
  \[ \chi = \psi + \beta_c \]
• Understand why heading control is used instead of course control during stationkeeping.
• Be able to compute the **COG using two waypoints**
• Know what kind of sensors that give you a direct measurement of **SOG** and **COG** under water and on the surface.
• Understand the well-celebrated **Nomoto models** for heading and course control, and it is extension to nonlinear theory (maneuvering characteristics)

\[
\frac{r}{\delta}(s) = \frac{K(T_3 s + 1)}{(T_1 s + 1)(T_2 s + 1)} \\
\frac{r}{\delta}(s) = \frac{K}{T s + 1} \\
\chi(s) = \frac{K}{s(T s + 1)} \tau_6(s) + \beta_c(s)
\]

• Be able to explain what the **pivot point** is.
Chapter 7 – Autopilot Models for Course and Heading Control

It is important to stress the concepts for course and heading control since there are many conceptual misunderstandings regarding the course and heading of a marine craft.

• The course angle $\chi$ of a marine craft is the cardinal direction in which the craft is moving.
• The heading angle $\psi$, is the direction the craft’s bow ($x_b$ axis) is pointed.

How does this relate to bearing?

• Absolute bearing is the direction of the craft determined by a navigation system, usually a magnetic compass or a gyrocompass. Hence, a heading autopilot use “bearing” as a measurement to control its heading angle.
• Relative bearing refers to the angle between the craft’s forward direction and the location of another object.

The difference between the course and heading angles is the crab angle (see Chapter 2).

\[ \chi = \psi + \beta_c \]

\[ \beta_c = \sin^{-1}(v/U) \]
7.1 Autopilot Models for Course Control

Surface craft are usually equipped with a global navigation satellite system (GNSS) receiver, which measures

- COG course over ground, $\chi$
- SOG speed over ground, $U$

Underwater vehicles, however, use hydroacoustic reference systems to determine their position, velocity and course.

COG can also be computed from the track (slope of a straight line)

$$\chi[k] = \text{atan2} (y^n[k] - y^n[k - 1], x^n[k] - x^n[k - 1])$$

![Diagram of course angle autopilot and path-following control systems.](image)

**Figure 7.1**: Course angle autopilot and path-following control systems.

(\frac{\chi}{\chi_d} \approx 1)
7.1 Autopilot Models for Course Control

State-space model and measurement equations

\[
\dot{\eta} = R(\psi)\nu \\
\dot{\nu} = \begin{bmatrix}
r v_c \\
-r u_c \\
0
\end{bmatrix} + M^{-1} \left( \tau + \tau_{\text{wind}} + \tau_{\text{wave}} - C(\nu_r)\nu_r - D\nu_r - D_n(\nu_r)\nu_r \right)
\]

**Measurements:**
- GNSS x position
- GNSS y position
- SOG (optionally)
- COG (optionally)

The control law must compensate the drift force, that is the ocean current velocities, by adding integral action.

\[
\nu_r = \nu - [u_c, v_c, 0]^T
\]

The drift force must also be estimated by the Kalman filter by augmenting the current velocities as unknown states to the state-space model.

Not defined for \( u = 0 \). This term can be modeled as a disturbance/bias in the Kalman filter.
7.1 Autopilot Models for Course Control

Decoupled course/yaw dynamics (mass—damper system)

\[ \dot{\chi} = r + \dot{\beta}_e \]

\[(I_z - N_r)\dot{r} - N_r r = \tau_6\]

\(\beta_c\) is treated as a disturbance,

The control input \(\tau_6\) is a yaw moment generated by propellers/rudders

Yaw angle transfer function (Nomoto model)

\[ \frac{r}{\tau_6}(s) = \frac{K}{Ts + 1} \]

\(K = 1/(-N_r)\)

\(T = (I_z - N_r)/(-N_r)\)

Course angle transfer function

\[ \chi(s) = \frac{K}{s(Ts + 1)}\tau_6(s) + \beta_c(s) \]

The control law can compensate the crab angle by direct measurements but this solution is not robust at small speeds. Integral action is still needed due to model uncertainty so the preferred method is to model the crab angle as an uncertainty as well.
7.2 Autopilot Models for Heading Control

Second-order Nomoto (1957) model

\[
M \ddot{\nu}_r + N \nu_r = b \delta
\]

\[
r = c^T \nu_r, \quad c^T = [0, 1]
\]

\[
M = \begin{bmatrix}
m - Y_v & mx_g - Y_r \\
xm_g - Y_r & I - N_r
\end{bmatrix}
\]

\[
N = \begin{bmatrix}
-Y_v & (m - X_{\delta})U - Y_r \\
(X_{\delta} - Y_v)U - N_v & (mx_g - Y_r)U - N_r
\end{bmatrix}
\]

\[
b = \begin{bmatrix}
-Y_{\delta} \\
-N_{\delta}
\end{bmatrix}
\]

\[
\frac{r}{\delta}(s) = \frac{K(T_3s + 1)}{(T_1s + 1)(T_2s + 1)}
\]

The normalized model (see Appendix D) can be used as basis for gain-scheduling control by choosing \( U \) as scheduling variable. Consequently, the PID controller gains will be functions of the model parameters and the direct measurement \( U \).

\[
\left(\frac{L}{U}\right)^2 T'_1 T'_2 \psi^{(3)} + \left(\frac{L}{U}\right) (T'_1 + T'_2) \ddot{\psi} + \dot{\psi} = K' T'_3 \delta + \left(\frac{U}{L}\right) K' \delta
\]
7.2 Autopilot Models for Heading Control

Second-order Nomoto (1957) model

\[
\frac{r}{\delta}(s) = \frac{K(T_3s + 1)}{(T_1s + 1)(T_2s + 1)}
\]

\[T := T_1 + T_2 - T_3\]

First-order Nomoto (1957) model

\[
\frac{r}{\delta}(s) = \frac{K}{Ts + 1}
\]

\[\frac{\psi}{\delta}(s) = \frac{K}{s(Ts + 1)}\]

The normalized model (see Appendix D) can be used as basis for gain-scheduling control by choosing \(U\) as scheduling variable.

\[
\left(\begin{array}{l}
\frac{L}{U}
\end{array}\right) T' \ddot{\psi} + \dot{\psi} = \left(\begin{array}{l}
\frac{U}{L}
\end{array}\right) K' \delta
\]
7.2 Autopilot Models for Heading Control

Matlab:
The Bode diagram is generated by using the MSS toolbox commands:

\[
T_1 = 118; T_2 = 7.8; T_3 = 18.5; K = 0.185;
\]
\[
nomoto(T1, T2, T3, K)
\]
\[
T1 = 124.1; T2 = 16.4; T3 = 46.0; K = -0.019;
\]
\[
nomoto(T1, T2, T3, K);
\]

% nomoto(T1,T2,T3,K) generates the Bode plots for
% K K (1+T3s)
% H1(s) = --------------- H2(s) = ----------------------
% s(t+Ts) s(t+Ts)(1+Ts)
\[
T = T1 + T2 - T3;
\]
\[
d1 = [T 1 0]; n1 = K;
\]
\[
d2 = [T1+T2 T1+T2 1 0]; n2 = K*[T3 1];
\]
\[
[mag1,phase1,w] = bode(n1,d1);
\]
\[
[mag2,phase2] = bode(n2,d2,w);
\]

% shift phase with 360 deg for course-unstable ship
if K < 0
\[
\text{phase1} = \text{phase1}-360;
\]
\[
\text{phase2} = \text{phase2}-360;
\]
end

subplot(211), semilogx(w,20*log10(mag1)), grid;
xlabel('Frequency [rad/s]'), title('Gain [dB]');
hold on, semilogx(w,20*log10(mag2), '--'), hold off;

subplot(212), semilogx(w,phase1), grid;
xlabel('Frequency [rad/s]'), title('Phase [deg]');
hold on, semilogx(w,phase2, '--'), hold off;

Table 7.1: Parameters for a cargo ship and a fully loaded oil tanker

<table>
<thead>
<tr>
<th></th>
<th>L (m)</th>
<th>u_0 (m/s)</th>
<th>V (dwt)</th>
<th>K (l/s)</th>
<th>T_1 (s)</th>
<th>T_2 (s)</th>
<th>T_3 (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cargo ship</td>
<td>161</td>
<td>7.7</td>
<td>16622</td>
<td>0.185</td>
<td>118.0</td>
<td>7.8</td>
<td>18.5</td>
</tr>
<tr>
<td>Oil tanker</td>
<td>350</td>
<td>8.1</td>
<td>389100</td>
<td>-0.019</td>
<td>-124.1</td>
<td>16.4</td>
<td>46.0</td>
</tr>
</tbody>
</table>
7.2 Autopilot Models for Heading Control

Mariner class cargo ship

Course stable ship

Course unstable ship

Oil tanker

Frequency [rad/s]
7.2 Autopilot Models for Heading Control

The linear Nomoto models can be extended to include nonlinear effects by adding static nonlinearities referred to as maneuvering characteristics.

**Nonlinear Extension of Nomoto's 1st-Order Model (Norrbin, 1963)**

\[
T\ddot{r} + H_N(r) = K\dot{\delta}
\]

\[
H_N(r) = n_3r^3 + n_2r^2 + n_1r + n_0
\]

where \(H_N(r)\) is a nonlinear function describing the maneuvering characteristics. For \(H_N(r) = r\), the linear model is obtained.

**Nonlinear Extension of Nomoto's 2nd-order Model (Bech & Wagner-Smith, 1969)**

\[
T_1T_2\ddot{r} + (T_1 + T_2)\dot{r} + KH_B(r) = K(\delta + T_3\dot{\delta})
\]

\[
H_B(r) = b_3r^3 + b_2r^2 + b_1r + b_0
\]

where \(H_B(r)\) can be found from Bech's reverse spiral maneuver. The linear equivalent is obtained for \(H_B(r) = r\)
7.2 Pivot Point

The pivot point is a useful tool in ship handling and the location of the pivot point in a maneuvering situations is of great importance for the ship handler.

**Definition (Pivot Point)** A ship's pivot point $x_{np}$ is a point on the centerline measured from the CG from the CG at which sway, and yaw completely cancel each other (Tzeng 1998)

\[ v_{np} = v_{ng} + x_pr = 0 \]

The pivot point for a turning ship can be computed by measuring the velocity $v_{ng}(t)$ in CG and the turning rate $r(t)$ such that

\[ x_p(t) = -\frac{v_{ng}(t)}{r(t)}, \quad r(t) \neq 0 \]

This expression is not defined for a zero-yaw rate corresponding to a straight-line motion. This means that the pivot point is located at infinity when moving on a straight line or in a pure sway motion.

It is well known to the pilots that the pivot point of a turning ship is located at about $1/5$–$1/4$ ship length aft of bow.
7.2 Pivot Point

The Nomoto transfer functions in sway and yaw gives the steady-state gain

\[
\frac{\nu}{r} = \frac{K_v(1 + T_v s)}{K(1 + T_3 s)}
\]

\[
\lim_{s \to 0} \frac{\nu}{r} = \frac{K_v}{K}
\]

\[
\frac{\nu}{\delta} (s) = \frac{K_v(1+T_v s)}{(1+T_1 s)(1+T_2 s)}
\]

\[
r \frac{\delta}{s} = \frac{K(1 + T_3 s)}{(1 + T_1 s)(1 + T_2 s)}
\]

Steady-state location of the pivot point

\[
x_{p,ss} = -\frac{K_v}{K}
\]

\[
x_{p,ss} = -\frac{N_r Y_\delta - (Y_r - m U) N_\delta}{Y_v N_\delta - N_v Y_\delta}
\]

Example: The Mariner Class vessel where the nondimensional linear maneuvering coefficients are given as:

\[
Y_v' = -1160 \cdot 10^{-5} \quad N_v' = -264 \cdot 10^{-5}
\]

\[
Y_r' = -499 \cdot 10^{-5} \quad N_r' = -166 \cdot 10^{-5}
\]

\[
Y_\delta' = 278 \cdot 10^{-5} \quad N_\delta' = -139 \cdot 10^{-5}
\]

Location of the pivot point: \( x_{p,ss} = 0.4923 L_{pp} \)
• Be able to explain the differences of course and heading controlled marine craft. In what applications are they used.
• Understand what the crab angle is for a marine craft:
\[ \chi = \psi + \beta_c \]
• Understand why heading control is used instead of course control during stationkeeping.
• Be able to compute the COG using two waypoints
• Know what kind of sensors that give you a direct measurement of SOG and COG under water and on the surface.
• Understand the well-celebrated Nomoto models for heading and course control, and it is extension to nonlinear theory (maneuvering characteristics)
\[
\frac{r}{s} \left( \frac{T_3 s + 1}{T_1 s + 1} \right) \\
\frac{r}{s} \left( \frac{T s + 1}{s(T s + 1)} \right) \\
\chi(s) = \frac{K}{s(T s + 1)} \tau_6(s) + \beta_c(s)
\]
• Be able to explain what the pivot point is.